

portion of the grade would be dependent on actual participation in class, coming to class, and completing an assigned IQI, (Interesting Point, Question to Ponder, or Important Issue). Basically, each student would be assigned to do an IQI for one chapter out of the textbook, as a way to monitor reading and begin discussions.

I was one of the three students assigned an IQI for chapter one. This assignment seemed easy enough. I read through the chapter, making notes as usual. After I finished, I looked back at what I had written down, and tried to think of what Interesting Point was made by the author, what Question I had about the content of the chapter, and what Important Issue the author brings up. Many ideas came to mind. I chose one of each, and marked them so I would be ready to bring them up in class.

I returned to class ready to share my IQI, but not expecting much discussion or class input. Another student shared hers first. Dr. Mohr spoke very little, mostly to facilitate the discussion. The students contributed to the first student's IQI, and added their own perceptions, understandings, and questions. As the discussion started to die down, I shared my IQI with the class. The same thing happened. The third student shared her IQI. She took the IQI assignment in a unique and interesting direction. She had, completely on her own initiative, created a questionnaire to go along with her IQI. Dr. Mohr happily

handed the "stage" over to her, and allowed her, and the class, to learn more about the text through the worksheet/activity task she had prepared. Finally, Dr. Mohr concluded the discussion by pointing out additional items and answering any remaining questions.

We have now done this process multiple times. Each student brings his/her own knowledge and unique personality to the class through his/her own IQI, as well as responding to those of others. As a result, I think more students are reading the textbook, and thinking about the content. I have been in many classes where students, including myself, did not read the textbook and did fine in the class. In a college classroom, especially with many students, only a select few will contribute to any given discussion. In my opinion, I learn less in these classes, and ultimately, gain little from them. IQI's have been a way to allow varied comments generated by every student at least once.

As a student, I would like to see the IQI method used more often, especially in classes where the textbook is a major source of information. It is nice to hear opinions of all the students rather than just the professor and a few students. The IQI's have made me think about things differently than when I just read the text. I would like to take this opportunity to encourage other professors to implement IQI's and modify it for their own courses.

## Teaching Mathematics from a Chemist's Viewpoint\*

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### Introduction

To better appreciate the approaches to teaching math discussed here, it's important to know that I am not a mathematician by profession. I have no special training or academic degrees in math, but my formal education (physical chemistry, Ph.D.) does require a math background. I have been teaching chemistry for about twenty-five years. Three years ago, I was asked to teach Calculus III. Because of my background as a scientist, i.e., as a practitioner of math rather than as a mathematician, my approaches are somewhat different than those of most math instructors.

### Main Goals of the Educational Process

Waterman (1980) and DeLorenzo (1981) state that the main goals of the educational process are to teach students to: (1) communicate clearly, (2) study regularly, (3) master basic math skills, and (4) think logically. The acquisition of these four fundamental skills is more important than the subject matter. In fact, one might profess that the subject matter of many courses serves primarily as a vehicle for the mastering of the four fundamental skills.

For many generations, we as educators have been

trying to insure our species' survival by requiring each new generation to hone various fundamental skills in as many different areas as possible. We firmly believe that this approach will equip future generations with the ability to solve unforeseen problems. We understand that although the majority of specific information will be forgotten, become obsolete, or simply never be used, the basic skills developed in the process will be retained. (Employers are also looking for students who have minds with these capabilities.) In this article, I will show how I attempt to develop in my students communication skills, regular study habits, a basic math proficiency, and a talent for logical thinking.

### Communication

Cipra (1988) reports that David Smith of Duke University lowers one-fourth of his calculus students' grades by one letter because of their writing. Smith tells his students that if they don't write well as professionals, they will lose more than just letter grades. Stacy (1978) revealed that the official view of the American Chemical Society is that the ability to communicate is as important as the knowledge of chem-

istry. Accordingly, the American Chemical Society recommends that instructors make writing skills (sentence structure, spelling, punctuation, and style) a significant part of examinations. Paradis (1983) reinforces the need to develop writing skills with an MIT study showing that about 30% of all industrial professional activities involve writing. If students can't explain material in complete grammatically correct sentences, their mastery of the material is incomplete. Thurman (1988) states, "The fact is, of course, that the abilities to read carefully, to think analytically, to abstract generalizations from a mass of particulars and to communicate with both economy and precision have always been marketable skills" (p. 7A).

I try to develop communication skills in part by incorporating essay questions on all one-hour examinations and on about one-third of the daily quizzes. The essay questions are worth 35% of the one-hour tests. My essay questions are usually one of two types: (1) Write a paragraph explaining how to solve a problem, and (2) Write a paragraph explaining how to derive a formula. An example of the first type of essay question is, "Given the parametric equations  $x(t)$ ,  $y(t)$ , and  $z(t)$  describing the Cartesian coordinates of a particle, write a paragraph explaining how the curvature of the particle's path can be derived." An example of the second type of essay question is, "Given the coordinates of the point P on line L and the equation of a vector parallel to line L, write a paragraph explaining how the equation of line L can be determined."

Student communication skills can be further sharpened when students recognize the importance of and master the use of significant digits and units. The number of significant digits in an answer communicates information concerning the number of significant digits in the original data. Multiplying 4 cm by 4 cm doesn't yield 16 cm<sup>2</sup> because such an answer incorrectly communicates that the original measurements were made to two significant digits (i.e., 4.0 cm). (The correct answer would be  $2 \times 10^1$  cm<sup>2</sup> which, in lay language, is verbally expressed as, "about 20 cm<sup>2</sup>". We round to one significant digit.) The units associated with numbers communicate additional meaning. Not only are answers such as 16 cm<sup>2</sup> incomplete and less clear without the unit cm<sup>2</sup>, all numbers written during calculations are similarly incomplete. In my chemistry classes, I go as far as requiring that even the numbers used in calculations include nouns and verbs, e.g.,

(10.0 grams salt form)(27.0 grams HCl react/9.00 grams salt form) = 30.0 grams HCl react as opposed to the more typical calculation (10 g)(27 g/9.0 g) = 30 g, which doesn't communicate as much information as the former and which also contains the incorrect number of significant digits.

The use of units can also provide additional insight for students. For example, consider what happens if we take the cross product of two vectors whose lengths are measured in centimeters. The  $i$ ,  $j$ , and  $k$  components of these two vectors would also have to have the units of centimeters. This leads to an interesting result: the cross product of two

vectors, whose components are measured in cm, produces a third vector orthogonal to the first two but with the strange units of cm<sup>2</sup> and with a magnitude of length also measured in cm<sup>2</sup>. By asking my students how this can be, I am hoping to both stimulate their thinking and to lead them to a better understanding of the material being discussed. Since the majority of my students will be engineers, i.e., practitioners of mathematics, it is essential for them to be aware of these concepts.

### Regular Study

I try to develop regular study habits in part by giving daily quizzes which count 30% of the final grades. When I began teaching, I knew of other teachers who gave daily quizzes, but I resisted the idea as "high schoolish" for many years. It was sobering for me to later learn that some students were bright enough to confine their studying to cram sessions before exams and still do well. I now tell my students that I wouldn't want a medical doctor who learned all of his knowledge of medicine in a one-evening cram session. Knowledge is best learned and retained by being absorbed in small chunks over an extended period of time.

Daily quizzes are less troublesome for instructors than you might think. Daily quizzes can be easy to grade (students must circle their answers, and the given grade can be 1 or 0). Daily quizzes can be easy to record (collect quizzes alphabetically, and record correct quiz grades with slashes / / / / in the grade book). Daily quizzes can be easy to administer (use transparencies to project quizzes onto a screen with students providing their own paper). To emphasize to my students my strong belief in the importance of daily study, I tell them that I will give them the benefit of the doubt for final grades that are on the borderline between two letter grades if the quiz grades are good. No such benefit of the doubt is given to students who consistently do poorly on daily quizzes.

### Basic Math Skills

I try to develop basic math skills in part by developing both estimation skills and scientific calculator and computer proficiency. Examples of estimation skills that should be mastered include performing calculations such as  $398 \text{ ft} \times 2076 \text{ ft} = 8 \times 10^5 \text{ ft}^2$  (by converting to exponential notation with one significant digit), and determining that  $\sin(1) = 0.8$  (by sketching).

Because so many students take to using calculators like fish take to water, it is all the more important that these students know how to estimate answers if for no other reason than to efficiently check their work by arriving at answers through two different routes. Much verification in science is based upon solutions arrived at through mutually independent approaches. For example, the exact amount of water is determined by weighing the sample (e.g., 5.00 grams) and measuring its volume (e.g., 5.00 mL). Likewise, the date of the universe is based on abundances of isotopes, the evolution of the oldest galactic stars, and the Hubble relation between the velocities of galaxies and their distances from us. In like spirit, students need to be able to

obtain answers by both estimation and by electronic computational devices to help ensure correctness and a feeling of confidence.

There has been much debate as to allowing students the use of calculators in calculus courses. Cipra (1988) reports that some math educators believe that the more computer power students have, the less students know what they're doing. Others believe that we are doing a disservice to students by not allowing them to make use of the current computational devices which would allow them more time to concentrate on the subject material.

I believe that there is a middle ground. My school, Middle Georgia College, is somewhat unusual among two-year colleges: our Math Department has ordered enough HP 28S calculators so that there is at least one available for each pair of students in Calculus III. I teach my Calculus III students how to use the Hewlett Packard HP 28S calculator to perform course-related calculations, but I do so primarily for their future reference. I disallow the use of electronic computational devices on exams. This approach is not inconsistent with my views that students need to be able to use computational devices and to be able to estimate to check their calculator answers. Let me explain. Once students observe the ease, power, and speed of computational devices, they naturally drift toward their use. On the other hand, estimation skills require time and effort, and mastering them is met with resistance. When students are told that they can use a calculator but they must also check their answers with mental estimation, the vast majority simply use their calculators and fudge estimated answers to resemble their calculator answers. By forcing students to rely solely on estimation skills during examinations, these students slowly, painfully, and begrudgingly develop these more important skills. Once students have mastered estimation skills, they begin to call upon these skills with increasing frequency.

Timnick (1982) discusses national studies which show that even the brightest students have blind faith even in machines deliberately designed to give incorrect answers. I (DeLorenzo, 1987) had a similar experience when I once gave a fast-food cashier a \$10 bill for an order that came to \$2 plus sales tax. I knew (estimated) that my change would be around \$8 and was surprised to receive \$1.55 in change. When I mentioned this discrepancy to the cashier, the cashier calmly said, "I'm sorry, sir, but the machine says that your change is \$1.55."

### Logical Thinking

I try to develop communication skill in part by implementing some of the aforementioned basic educational goals related to communication. For example, when students express themselves with grammatically correct paragraphs, they undergo thought processes quite different from those that they experience when they travel through more conventional mathematical channels of thought.

Additional student thinking can be stimulated by asking students simple questions such as the following:

A pre-solution question: When students are presented with problems, a simple question that can be asked is, "Does the problem make sense?" For example, given vector  $A = 2i+3j+4k$ , vector  $B = 3i+5j+2k$ , and vector  $C = i+7j+6k$ , is it possible to determine  $A \times (B \cdot C)$ ? Such an operation cannot be done because vectors cannot be crossed with scalars. Likewise, a  $3 \times 4$  matrix cannot be multiplied by a  $5 \times 3$  matrix. Occasionally, on quizzes and on examinations, I ask questions that are impossible to answer.

A second pre-solution question: Students can also be taught logical thinking in terms of developing successful strategies for taking examinations. For example, if students are asked to find the determinant of a  $6 \times 6$  matrix, students should realize that solving such problems takes an excessive amount of examination time and makes it difficult to solve the remaining questions in the fifty-minute examination. Perhaps there is a shortcut? Does the matrix contain two identical rows or two identical columns? Was the value of the transpose of this matrix given in an earlier problem?

A post-solution question: Additional student thinking can be stimulated by asking students simple questions such as, "What does this answer mean?" For example, after finding the magnitude of  $dT/ds = aw^2/(a^2w^2+b^2)$  for a vector valued function  $F(t)$ , most students are overwhelmed by the apparently meaningless mix of letters in the solution and don't think what the solution conveys, i.e., that the curvature is independent of time and is in fact a constant. After drawing this out of them, I follow with the question, "What kind of curve has a constant curvature that is independent of time?"

A second post-solution question: Another simple question that can be asked after solving problems is, "Does this answer make sense?" For example, when finding the unit normal vector  $N$  from a positive vector  $R(t)$ , several steps are involved. These steps include finding the velocity vector, the magnitude of the velocity vector, the unit tangent vector  $T$ , the first derivative of the unit tangent vector with respect to time, the vector  $dT/ds$ , and curvature. Finally,  $N$  is calculated by dividing  $dT/ds$  by the curvature. Does the calculated  $N$  make sense? A correct  $N$  must be a unit vector (check its magnitude). Also, a correct  $N$  should be orthogonal to the previously calculated  $T$  (compare the slopes of these two vectors to see if they are negative reciprocals of one another).

### Summary

In summary, as math teachers, we must realize that the rigors of our discipline are but one facet of our students' overall education. There are practical sides to math that need to be taught, e.g., the recognition that measurements contain a finite number of significant digits. Also, there are general and universal concerns that need to be addressed, e.g., the development of regular study habits and the honing of communication and thinking skills.

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