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Second-order Approximate Corrections for QCD Processes

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I present generalized formulas for approximate corrections to QCD hard-scattering cross sections through second order in the perturbative expansion. The approximate results are based on recent two-loop calculations for soft and collinear emission near threshold and are illustrated by several applications to strong-interaction processes in hadron colliders.

1. Introduction

Higher-order perturbative QCD corrections are necessary to produce reliable estimates of theoretical cross sections and reduce theoretical uncertainties for hard-scattering processes. Collinear and soft-gluon corrections are an important subset of the QCD corrections and they can be significant for many processes, particularly near partonic threshold. The soft and collinear corrections can be derived from factorization theorems and resummation formalisms and they require loop calculations in the eikonal approximation. In this proceedings I discuss resummation, recent calculations of soft anomalous dimensions through two loops for a number of processes, and I present master formulas for NNLO expansions of the resummed cross section. Collinear and soft-gluon corrections have been calculated for many processes and I present some explicit applications to top quark and electroweak boson production.

In Section 2 the resummation formalism is described and expressions for the resummed cross section are presented, including two-loop expressions for some universal functions in the resummation. In Section 3 we use the expansion of the resummed cross section to derive master formulas for the approximate NLO and NNLO corrections. In section 4 we describe in general the calculation of soft anomalous dimensions, and in particular the massive cusp anomalous dimension. In Section 5 we present results for the soft anomalous dimension matrices for top-antitop production, in Section 6 for t-channel single top quark production, in Section 7 for s-channel single top quark production, in Section 8 for the associated production of a top quark with a W boson (or a charged Higgs), and in Section 9 for electroweak boson (W, Z, γ) production at large transverse momentum.

2. Higher-order collinear and soft corrections

Soft-gluon corrections arise from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft gluons, i.e. low-energy gluons. These soft corrections take the form $[(\ln^k(s_4/M^2))/s_4]_+$ where $k \le 2n-1$ for the α_s^n corrections, M is a hard scale, and s_4 is the kinematical distance from partonic threshold. The leading logarithms are of double collinear and soft emission origin. There are also purely collinear terms of the form $(1/M^2)\ln^k(s_4/M^2)$.

These corrections can be resummed to all orders in perturbation theory. An essential ingredient in these calculations are the soft anomalous dimensions. At NLL accuracy the resummation requires one-loop calculations for the soft anomalous dimensions in the eikonal approximation. There are several recent results at NNLL with two-loop calculations completed.

Approximate NNLO cross sections are derived from the expansion of the resummed cross section. These are useful because soft-gluon corrections are dominant near threshold and thus the NNLO soft-gluon corrections are expected to approximate well the complete NNLO corrections for many processes of interest. Also, typically corrections beyond NNLO are small.

Resummation follows from factorization properties of the cross section, performed in moment space [1]: $\sigma = (\prod \psi)\; H_{IL}\, S_{LI}\; (\prod J)$ where ψ are distributions for the incoming partons that absorb universal collinear singularities, H is the hard-scattering matrix (IL are color indices), S is a soft-gluon matrix describing non-collinear soft-gluon emission, and J are jet functions for the final state. We use renormalization group evolution (RGE) to evolve the soft-gluon function

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g_s)\frac{\partial}{\partial g_s}\right) S_{LI} = -(\Gamma^\dagger_S)_{LB}S_{BI} - S_{LA}(\Gamma_S)_{AI}$$

where $\beta(g_s)$ is the QCD beta function, with $g_s^2 = 4\pi\alpha_s$, and Γ_S is the soft anomalous dimension [1], a matrix in color space and a function of kinematical invariants s, t, u.

The resummed cross section follows from RGE and can be written as

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N_i)\right] \exp\left[\sum_j E'_j(N')\right] \exp\left[\sum_i 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\, \gamma_{i/i}\left(\tilde{N}_i, \alpha_s(\mu)\right)\right]$$
$$\times \, \mathrm{tr}\left\{H\left(\alpha_s(\sqrt{s})\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu}\, \Gamma_S^\dagger\left(\alpha_s(\mu)\right)\right] S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}'}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu}\, \Gamma_S\left(\alpha_s(\mu)\right)\right]\right\} \quad (1)$$

with N the Melin moment variable.

The collinear and soft radiation from incoming partons is resummed by the first exponential in Eq. (1) with

$$E_i(N_i) = \int_0^1 dz \frac{z^{N_i-1}-1}{1-z}\left\{\int_1^{(1-z)^2} \frac{d\lambda}{\lambda} A_i\left(\alpha_s(\lambda s)\right) + D_i\left[\alpha_s((1-z)^2 s)\right]\right\}. \quad (2)$$

Purely collinear terms can be derived by replacing $(z^{N-1}-1)/(1-z)$ in the above equation by $-z^{N-1}$. In Eq. (2), A_i has the perturbative expansion $A_i = (\alpha_s/\pi)A_i^{(1)} + (\alpha_s/\pi)^2 A_i^{(2)} + \cdots$, where $A_i^{(1)} = C_i$ [2] with $C_i = C_F = (N_c^2-1)/(2N_c)$ for a quark or antiquark and $C_i = C_A = N_c$ for a gluon, with $N_c = 3$ the number of colors, while $A_i^{(2)} = C_i K/2$ [3] with $K = C_A\,(67/18 - \pi^2/6) - 5n_f/9$, where n_f is the number of quark flavors. Also $D_i = (\alpha_s/\pi)D_i^{(1)} + (\alpha_s/\pi)^2 D_i^{(2)} + \cdots$, with $D_i^{(1)} = 0$ in Feynman gauge ($D_i^{(1)} = -C_i$ in axial gauge). In Feynman gauge the two-loop result is (c.f. [4])

$$D_i^{(2)} = C_i C_A\left(-\frac{101}{54} + \frac{11}{6}\zeta_2 + \frac{7}{4}\zeta_3\right) + C_i n_f\left(\frac{7}{27} - \frac{\zeta_2}{3}\right). \quad (3)$$

Collinear and soft radiation from outgoing massless quarks and gluons is resummed by the second exponential in Eq. (1) with

$$E'_j(N') = \int_0^1 dz \frac{z^{N'-1}-1}{1-z}\left\{\int_{(1-z)^2}^{1-z} \frac{d\lambda}{\lambda} A_j\left(\alpha_s(\lambda s)\right) + B_j\left[\alpha_s((1-z)s)\right] + D_j\left[\alpha_s((1-z)^2 s)\right]\right\}. \quad (4)$$

Here $B_j = (\alpha_s/\pi)B_j^{(1)} + (\alpha_s/\pi)^2 B_j^{(2)} + \cdots$ with $B_q^{(1)} = -3C_F/4$ and $B_g^{(1)} = -\beta_0/4$ [2, 3], where β_0 is the lowest-order β-function, $\beta_0 = (11C_A - 2n_f)/3$. Also

$$B_q^{(2)} = C_F^2\left(-\frac{3}{32} + \frac{3}{4}\zeta_2 - \frac{3}{2}\zeta_3\right) + C_F C_A\left(-\frac{1539}{864} - \frac{11}{12}\zeta_2 + \frac{3}{4}\zeta_3\right) + n_f C_F\left(\frac{135}{432} + \frac{\zeta_2}{6}\right) \quad (5)$$

$$B_g^{(2)} = C_A^2\left(-\frac{1025}{432} - \frac{3}{4}\zeta_3\right) + \frac{79}{108} C_A n_f + C_F \frac{n_f}{8} - \frac{5}{108} n_f^2. \quad (6)$$

The factorization scale, μ_F, dependence is controlled by the moment-space anomalous dimension $\gamma_{i/i} = -A_i \ln \tilde{N}_i + \gamma_i$ with γ_i the parton anomalous dimensions [5, 6]. We have

$$\gamma_i = (\alpha_s/\pi)\gamma_i^{(1)} + (\alpha_s/\pi)^2 \gamma_i^{(2)} + \cdots \quad (7)$$

with $\gamma_q^{(1)} = 3C_F/4$, $\gamma_g^{(1)} = \beta_0/4$,

$$\gamma_q^{(2)} = C_F^2\left(\frac{3}{32} - \frac{3}{4}\zeta_2 + \frac{3}{2}\zeta_3\right) + C_F C_A\left(\frac{17}{96} + \frac{11}{12}\zeta_2 - \frac{3}{4}\zeta_3\right) + n_f C_F\left(-\frac{1}{48} - \frac{\zeta_2}{6}\right), \quad (8)$$

and

$$\gamma_g^{(2)} = C_A^2\left(\frac{2}{3} + \frac{3}{4}\zeta_3\right) - n_f\left(\frac{C_F}{8} + \frac{C_A}{6}\right). \quad (9)$$

The noncollinear soft gluon emission is controlled by the soft anomalous dimension Γ_S. We determine Γ_S from the coefficients of ultraviolet poles in dimensionally regularized eikonal diagrams [1, 7]. The hard H and soft S matrices as well as Γ_S are process-dependent and thus have to be calculated separately for each hard-scattering process.

3. NNLO approximate cross sections

The resummed cross section in Eq. (1) can be expanded to any finite order. Here we present master fromulas for the NLO and NNLO expansions.

We define $\mathcal{D}_k(s_4) \equiv \left[(\ln^k(s_4/M^2))/s_4\right]_+$. Then the NLO approximate corrections are

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu_R)}{\pi} \{c_3\, \mathcal{D}_1(s_4) + c_2\, \mathcal{D}_0(s_4) + c_1\, \delta(s_4)\} + \frac{\alpha_s^{d_{\alpha_s}+1}(\mu_R^2)}{\pi} [A^c\, \mathcal{D}_0(s_4) + T_1^c\, \delta(s_4)] \tag{10}$$

where σ^B is the Born term, μ_R is the renormalization scale, and d_{α_s} denotes the power of α_s for the leading-order cross section. Also $c_3 = \sum_i 2\, A_i^{(1)} - \sum_j A_j^{(1)}$ and $c_2 = c_2^\mu + T_2$, with $c_2^\mu = -\sum_i A_i^{(1)} \ln\left(\mu_F^2/M^2\right)$ and

$$T_2 = \sum_i \left[-2\, A_i^{(1)}\, \ln\left(\frac{-t_i}{M^2}\right) + D_i^{(1)} - A_i^{(1)}\, \ln\left(\frac{M^2}{s}\right)\right] + \sum_j \left[B_j^{(1)} + D_j^{(1)} - A_j^{(1)}\, \ln\left(\frac{M^2}{s}\right)\right]. \tag{11}$$

Also

$$A^c = \mathrm{tr}\left(H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma_S^{(1)}\right). \tag{12}$$

We split the c_1 coefficient as $c_1 = c_1^\mu + T_1$, with

$$c_1^\mu = \sum_i \left[A_i^{(1)}\, \ln\left(\frac{-t_i}{M^2}\right) - \gamma_i^{(1)}\right] \ln\left(\frac{\mu_F^2}{M^2}\right) + d_{\alpha_s} \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{M^2}\right) \tag{13}$$

the terms involving the factorization scale μ_F and renormalization scale μ_R dependence. On the other hand T_1 as well as T_1^c are not derivable from resummation but can be extracted from complete NLO calculations.

The NNLO approximate corrections are

$$\begin{aligned}
\hat{\sigma}^{(2)} \;=\; & \sigma^B \frac{\alpha_s^2(\mu_R)}{\pi^2} \left\{ \frac{1}{2} c_3^2\, \mathcal{D}_3(s_4) + \left[\frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + \sum_j \frac{\beta_0}{8} A_j^{(1)}\right] \mathcal{D}_2(s_4) \right. \\
& + \left[c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln\left(\frac{\mu_R^2}{M^2}\right) + \sum_i 2A_i^{(2)} - \sum_j A_j^{(2)} + \sum_j \frac{\beta_0}{4} B_j^{(1)}\right] \mathcal{D}_1(s_4) \\
& + \left[c_2 c_1 - \zeta_2 c_3 c_2 + \zeta_3 c_3^2 + \frac{\beta_0}{4} c_2 \ln\left(\frac{\mu_R^2}{s}\right) - \sum_i \frac{\beta_0}{2} A_i^{(1)} \ln^2\left(\frac{-t_i}{M^2}\right)\right. \\
& + \sum_i [\left(-2A_i^{(2)} + \frac{\beta_0}{2} D_i^{(1)}\right) \ln\left(\frac{-t_i}{M^2}\right) + D_i^{(2)} + \frac{\beta_0}{8} A_i^{(1)} \ln^2\left(\frac{\mu_F^2}{s}\right) - A_i^{(2)} \ln\left(\frac{\mu_F^2}{s}\right)] \\
& \left.\left. + \sum_j [B_j^{(2)} + D_j^{(2)} - \left(A_j^{(2)} + \frac{\beta_0}{4}(B_j^{(1)} + 2D_j^{(1)})\right) \ln\left(\frac{M^2}{s}\right) + \frac{3\beta_0}{8} A_j^{(1)} \ln^2\left(\frac{M^2}{s}\right)]\right] \mathcal{D}_0(s_4) \right\} \\
& + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R)}{\pi^2} \left\{ \frac{3}{2} c_3 A^c\, \mathcal{D}_2(s_4) + \left[\left(2c_2 - \frac{\beta_0}{2}\right) A^c + c_3 T_1^c + F^c\right] \mathcal{D}_1(s_4) \right. \\
& \left. + \left[\left(c_1 - \zeta_2 c_3 + \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{s}\right)\right) A^c + c_2 T_1^c + F^c \ln\left(\frac{M^2}{s}\right) + G^c\right] \mathcal{D}_0(s_4) \right\}
\end{aligned} \tag{14}$$

where

$$F^c = \mathrm{tr}\left[H^{(0)} \left(\Gamma_S^{(1)\,\dagger}\right)^2 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma_S^{(1)}\right)^2 + 2H^{(0)} \Gamma_S^{(1)\,\dagger} S^{(0)} \Gamma_S^{(1)}\right] \tag{15}$$

and

$$\begin{aligned}
G^c \;=\; & \mathrm{tr}\left[H^{(1)}\Gamma_S^{(1)\,\dagger} S^{(0)} + H^{(1)} S^{(0)} \Gamma_S^{(1)} + H^{(0)} \Gamma_S^{(1)\,\dagger} S^{(1)} + H^{(0)} S^{(1)} \Gamma_S^{(1)}\right. \\
& \left. + H^{(0)} \Gamma_S^{(2)\,\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma_S^{(2)}\right].
\end{aligned} \tag{16}$$

In Eq. (14), c_3, c_2, c_1, etc are defined from the NLO expansion. The two-loop universal quantities $A^{(2)}$, $B^{(2)}$, $D^{(2)}$ are known and were presented in the previous section. The functions H, S, Γ_S are process dependent and have been calculated for many processes.

4. Soft anomalous dimensions

The two-loop process-dependent soft anomalous dimensions, $\Gamma_S^{(2)}$, have been recently calculated for several processes. The calculations use the eikonal approximation where the rules for soft gluon emission simplify. For the emission of a gluon with momentum k off a quark with momentum p the Feynman rules simplify as follows in the limit $k \to 0$:

$$\bar{u}(p)\,(-ig_s T_F^c)\,\gamma^\mu \frac{i(\not{p}+\not{k}+m)}{(p+k)^2-m^2+i\epsilon} \to \bar{u}(p)\,g_s T_F^c\,\gamma^\mu \frac{\not{p}+m}{2p\cdot k+i\epsilon} = \bar{u}(p)\,g_s T_F^c\,\frac{v^\mu}{v\cdot k+i\epsilon}$$

with v a dimensionless vector proportional to the momentum p, and T_F^c the generators of $SU(3)$ in the fundamental representation.

We perform the calculations of eikonal diagrams in momentum space and Feynman gauge. Complete two-loop results for the soft (cusp) anomalous dimension for $e^+e^- \to t\bar{t}$ were presented in [7].

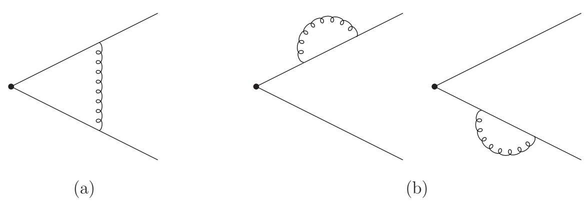

Figure 1: One-loop cusp diagrams.

In Fig. 1 we show the one-loop diagrams for the massive cusp anomalous dimension, with the lines representing a top and antitop pair in the process $e^+e^- \to t\bar{t}$. From the UV poles of these diagrams we find the one-loop result

$$\Gamma_S^{(1)} = C_F \left[-\frac{(1+\beta^2)}{2\beta} \ln\left(\frac{1-\beta}{1+\beta}\right) - 1 \right] \tag{17}$$

where $\beta = \sqrt{1-4m^2/s}$.

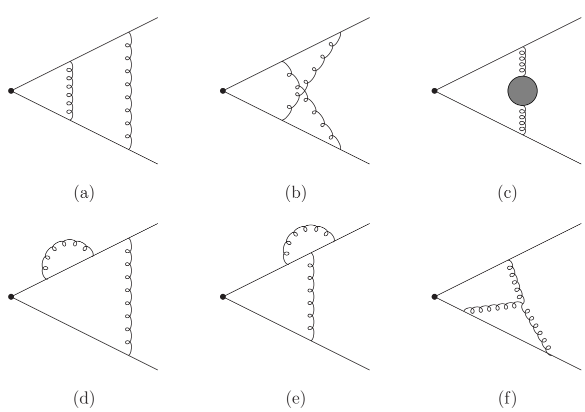

Figure 2: Two-loop cusp diagrams.

In Fig. 2 we show two-loop cusp diagrams (there are additional ones involving top-quark self energies). From the UV poles of these diagrams we find the two-loop result [7]

$$\Gamma_S^{(2)} = \frac{K}{2}\,\Gamma_S^{(1)} + C_F C_A \left\{ \frac{1}{2} + \frac{\zeta_2}{2} + \frac{1}{2}\ln^2\left(\frac{1-\beta}{1+\beta}\right) \right.$$
$$- \frac{(1+\beta^2)^2}{8\beta^2}\left[\zeta_3 + \zeta_2 \ln\left(\frac{1-\beta}{1+\beta}\right) + \frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right) + \ln\left(\frac{1-\beta}{1+\beta}\right)\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right) - \mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]$$
$$- \frac{(1+\beta^2)}{4\beta}\left[\zeta_2 - \zeta_2\ln\left(\frac{1-\beta}{1+\beta}\right) + \ln^2\left(\frac{1-\beta}{1+\beta}\right) - \frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right) + 2\ln\left(\frac{1-\beta}{1+\beta}\right)\ln\left(\frac{(1+\beta)^2}{4\beta}\right)\right.$$
$$\left.\left. - \mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]\right\}\,. \tag{18}$$

The results in Eqs. (17) and (18) can also be expressed in terms of the cusp angle. Such expressions have been presented in [7] and are more analytically explicit than earlier results in [8].

More recently results have appeared for $t\bar{t}$ hadroproduction, t-channel single top production, s-channel single top production, $bg \to tW^-$ and $bg \to tH^-$, and direct photon, W, and Z production at large p_T. The color structure gets more complicated with more than two colored partons in the process. The cusp anomalous dimension is an essential component of other calculations.

5. Top-antitop production in hadron colliders

The soft anomalous dimension matrix for $q\bar{q} \to t\bar{t}$ is a 2×2 matrix:

$$\Gamma_{S\,q\bar{q}} = \left[\begin{array}{cc} \Gamma_{q\bar{q}\,11} & \Gamma_{q\bar{q}\,12} \\ \Gamma_{q\bar{q}\,21} & \Gamma_{q\bar{q}\,22} \end{array}\right]\,.$$

Results have been presented in [9] in a singlet-octet color exchange basis through two loops. At one loop

$$\Gamma_{q\bar{q}\,11}^{(1)} = -C_F\,[L_\beta + 1] \qquad \Gamma_{q\bar{q}\,21}^{(1)} = 2\ln\left(\frac{u_1}{t_1}\right) \qquad \Gamma_{q\bar{q}\,12}^{(1)} = \frac{C_F}{C_A}\ln\left(\frac{u_1}{t_1}\right)$$
$$\Gamma_{q\bar{q}\,22}^{(1)} = C_F\left[4\ln\left(\frac{u_1}{t_1}\right) - L_\beta - 1\right] + \frac{C_A}{2}\left[-3\ln\left(\frac{u_1}{t_1}\right) + \ln\left(\frac{t_1 u_1}{s m^2}\right) + L_\beta\right]$$

where $L_\beta = \frac{1+\beta^2}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right)$ with $\beta = \sqrt{1-4m^2/s}$.

We write the two-loop cusp anomalous dimension, Eq. (18), from the previous section as $\Gamma_S^{(2)} = \frac{K}{2}\,\Gamma_S^{(1)} + C_F C_A M_\beta$ where M_β denotes the terms in curly brackets in Eq. (18). We use M_β below in the two-loop matrix for $q\bar{q} \to t\bar{t}$. Then at two loops for $q\bar{q} \to t\bar{t}$ we find [9]

$$\Gamma_{q\bar{q}\,11}^{(2)} = \frac{K}{2}\Gamma_{q\bar{q}\,11}^{(1)} + C_F C_A\,M_\beta \qquad \Gamma_{q\bar{q}\,22}^{(2)} = \frac{K}{2}\Gamma_{q\bar{q}\,22}^{(1)} + C_A\left(C_F - \frac{C_A}{2}\right)M_\beta$$
$$\Gamma_{q\bar{q}\,21}^{(2)} = \frac{K}{2}\Gamma_{q\bar{q}\,21}^{(1)} + C_A N_\beta \ln\left(\frac{u_1}{t_1}\right) \qquad \Gamma_{q\bar{q}\,12}^{(2)} = \frac{K}{2}\Gamma_{q\bar{q}\,12}^{(1)} - \frac{C_F}{2}N_\beta\ln\left(\frac{u_1}{t_1}\right)$$

with N_β a subset of terms of M_β,

$$N_\beta = \frac{1}{2}\ln^2\left(\frac{1-\beta}{1+\beta}\right) - \frac{(1+\beta^2)}{4\beta}\left[\ln^2\left(\frac{1-\beta}{1+\beta}\right) + 2\ln\left(\frac{1-\beta}{1+\beta}\right)\ln\left(\frac{(1+\beta)^2}{4\beta}\right) - \mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]. \tag{19}$$

Corresponding results for the $gg \to t\bar{t}$ channel are given in [9].

6. Single top quark production - t channel

The dominant single top production channel at both Tevatron and LHC energies is the t channel, which at lowest order involves the partonic processes $qb \to q't$ and $\bar{q}b \to \bar{q}'t$. Here we show some of the elements of the one-loop and two-loop [10] soft anomalous dimension matrices that are used in the NNLO expansions. At one loop

$$\Gamma_{S\,11}^{(1)} = C_F \left[\ln\left(\frac{-t}{s}\right) + \ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2} \right]$$

$$\Gamma_{S\,21}^{(1)} = \ln\left(\frac{u(u - m_t^2)}{s(s - m_t^2)}\right) \qquad \Gamma_{S\,12}^{(1)} = \frac{C_F}{2N_c} \Gamma_{S\,21}^{(1)} .$$

At two loops we find

$$\Gamma_{S\,11}^{(2)} = \frac{K}{2} \Gamma_{S\,11}^{(1)} + C_F C_A \frac{(1 - \zeta_3)}{4} .$$

7. Single top quark production - s channel

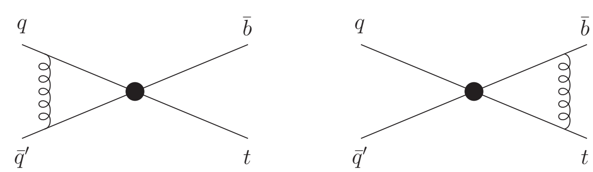

Figure 3: One-loop s-channel diagrams.

The s-channel processes are of the form $q\bar{q}' \to \bar{b}t$. One-loop eikonal diagrams for s-channel production are shown in Fig. 3.

Two-loop eikonal diagrams are shown in Figs. 4 and 5.

The 11 elements of the soft anomalous dimension for s-channel single top production at one and two loops are [11]

$$\Gamma_{S\,11}^{(1)} = C_F \left[\ln\left(\frac{s - m_t^2}{m_t\sqrt{s}}\right) - \frac{1}{2} \right] , \qquad \Gamma_{S\,11}^{(2)} = \frac{K}{2} \Gamma_{S\,11}^{(1)} + C_F C_A \frac{(1 - \zeta_3)}{4} .$$

8. Associated production of a top quark with a W^- or H^-

The associated production of a top quark with a W boson has the second largest cross section among single-top quark processes at the LHC. The NNLL resummation for this process was derived in [12]. The two-loop eikonal diagrams that are evaluated in the calculation of the two-loop soft anomalous dimension are shown in Ref. [12].

The soft anomalous dimension for $bg \to tW^-$ (or $bg \to tH^-$) is at one loop

$$\Gamma_{S,\,tW^-}^{(1)} = C_F \left[\ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2} \right] + \frac{C_A}{2} \ln\left(\frac{m_t^2 - u}{m_t^2 - t}\right)$$

and at two loops

$$\Gamma_{S,\,tW^-}^{(2)} = \frac{K}{2} \Gamma_{S,\,tW^-}^{(1)} + C_F C_A \frac{(1 - \zeta_3)}{4} .$$

9. W, Z, and direct photon production at large p_T

Threshold corrections for electroweak boson production dominate the p_T distribution at large transverse momentum. NLL resummation for W production was studied in [13, 14].

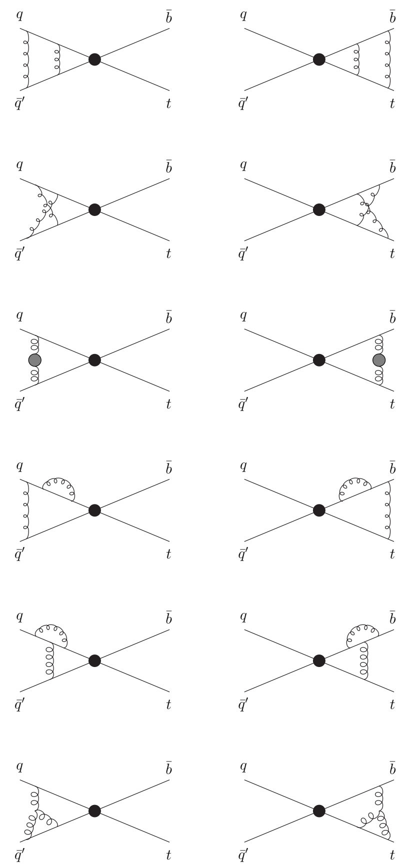

Figure 4: Two-loop s-channel diagrams.

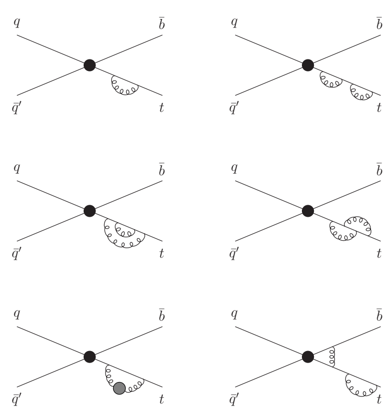

Figure 5: One- and two-loop top-quark self-energy graphs in the s channel.

The two loop soft anomalous dimensions for NNLL resummation can be derived from two-loop diagrams in the eikonal approximation [15]. For $qg \rightarrow Wq$ (or $qg \rightarrow Zq$ or $qg \rightarrow \gamma q$) the diagrams are similar to those for tW production. The results for the soft anomalous dimension at one and two loops [15] are

$$\Gamma^{(1)}_{S,\,qg\to Wq} = C_F \ln\left(\frac{-u}{s}\right) + \frac{C_A}{2}\ln\left(\frac{t}{u}\right)$$

and

$$\Gamma^{(2)}_{S,\,qg\to Wq} = \frac{K}{2}\Gamma^{(1)}_{S,\,qg\to Wq}\,.$$

For $q\bar{q} \to Wg$ (or $q\bar{q} \to Zg$ or $q\bar{q} \to \gamma g$) the corresponding one-loop and two-loop expressions are [15]

$$\Gamma^{(1)}_{S,\, q\bar{q}\to Wg} = \frac{C_A}{2} \ln\left(\frac{tu}{s^2}\right)$$

and

$$\Gamma^{(2)}_{S,\, q\bar{q}\to Wg} = \frac{K}{2} \Gamma^{(1)}_{S,\, q\bar{q}\to Wg} \, .$$

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