1-2011

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Recommended Citation

Rosengrant D. 2011. Impulse-momentum diagrams. Phys Teach 49(1):36-9.

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Citation: The Physics Teacher **49**, 36 (2011); doi: 10.1119/1.3527754
View online: http://dx.doi.org/10.1119/1.3527754
View Table of Contents: http://scitation.aip.org/content/aapt/journal/tpt/49/1?ver=pdfcov
Published by the American Association of Physics Teachers

Impulse-Momentum Diagrams

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Multiple representations are a valuable tool to help students learn and understand physics concepts.[1] Furthermore, representations help students learn how to think and act like real scientists.[2] These representations include: pictures, free-body diagrams,[3] energy bar charts,[4] electrical circuits, and, more recently, computer simulations and animations.[5] However, instructors have limited choices when they want to help their students understand impulse and momentum. One of the only available options is the impulse-momentum bar chart.[6] The bar charts can effectively show the magnitude of the momentum as well as help students understand conservation of momentum, but they do not easily show the actual direction. This paper highlights a new representation instructors can use to help their students with momentum and impulse—the impulse-momentum diagram (IMD).

What are they?

Momentum is the product of an object's mass and its velocity, written mathematically as shown in Eq. (1):

$$\mathbf{p} = m\mathbf{v}. \tag{1}$$

Momentum is the product of a scalar quantity (mass) and a vector quantity (velocity). Thus, a representation for momentum must take both of these quantities into account. Impulse-momentum diagrams do this by combining motion diagrams with basic geometry.

Physicists use motion diagrams, similar to the one in Fig. 1, to describe an object's velocity. The motion diagram contains key pieces of information. The length of the arrow refers to the magnitude of the object's velocity and the direction of the arrow refers to the direction of the object. The motion diagram in Fig. 1 shows that this object is initially moving to the right. Since the lengths of the arrows are decreasing, the object is slowing down to a stop. Next, the spacing of the dots shows the relative location of an object at equal points in time. Finally, the direction of the object's acceleration is to the left.

The key pieces of information needed from motion diagrams to construct an IMD are the direction and magnitude of the object's velocity. To make this a representation for an object's momentum, we need to multiply the object's velocity by a scalar, in this case, its mass. Thus, instead of using a straight line as in the motion diagram, we give the line a thickness. The thickness corresponds to the mass as shown in Fig. 2.

The diagram in Fig. 2 contains some very important pieces of information. An object of 10 kg is moving to the right at a speed of 3 m/s. The magnitude of the velocity (3 m/s) is shown by the length of the arrow. The direction of the velocity is shown by the direction that the arrow points. The height or the thickness of the vector would represent the mass (10 kg). The area of the square is the magnitude of the momentum. We can use the same strategy for impulse.

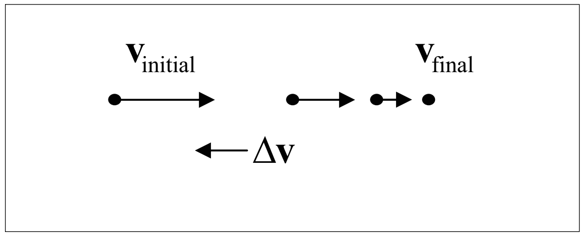

Fig. 1. Example of a motion diagram.

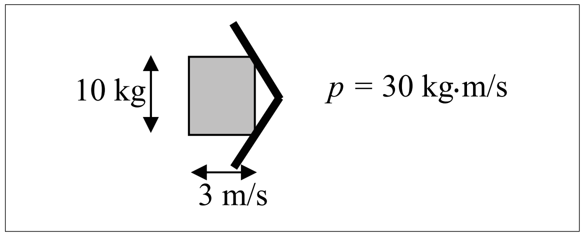

Fig. 2. Example of a momentum diagram.

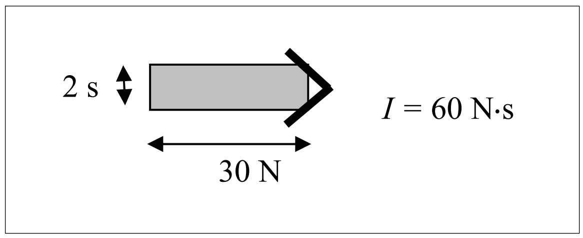

Fig. 3. Example of an impulse diagram.

Impulse is defined as the change in momentum, thus another vector quantity. Impulse can also be found by multiplying the net force exerted on an object (a vector quantity) by the time of the interaction (a scalar quantity). We repeat the process from before and show that the direction of the force is shown by the direction of the arrow and the magnitude of the force is represented by the length of the arrow. The time interval is shown by the thickness of the arrow. The diagram in Fig. 3 shows a 30-N force being exerted on an object for 2 s. The area of the square is the magnitude of the impulse.

The purpose of the representations is to help students develop a fundamental qualitative understanding of momentum and impulse. Students are usually novice problem solvers and, unlike experts, see a verbal description of a scenario as abstract. This means that they cannot form a mental picture of the situation. They tend to think of the problem as just a list of variables that they identify and others they need to find.[7] To them, the answer is typically a number devoid of any meaning. Thus, the answer is also abstract. These representations act like free-body diagrams in that they serve as the link between an abstract verbal statement and an abstract mathematical answer by helping students visualize the situation. Thus,

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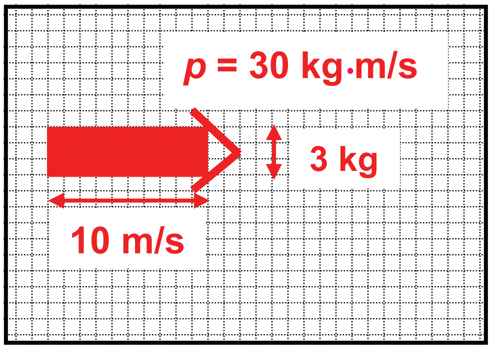

Fig. 4. Momentum diagram on graph paper.

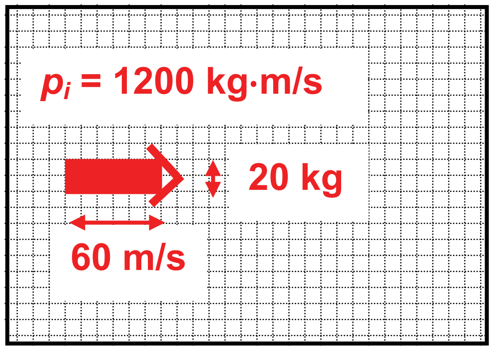

Fig. 5. Example problem 1; initial situation.

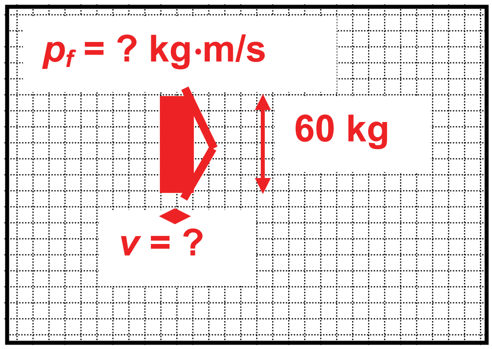

Fig. 6. Example problem 1; final situation.

IMDs also serve to help develop a quantitative understanding of momentum and impulse.

When you introduce these representations for the first time, students should construct them on graph paper. As shown in Fig. 4, the side of each block is scaled depending on the problem. In this case, the one block on the horizontal axis is 1 m/s and one block on the vertical is 1 kg. Ultimately, you need to show a relationship between the area of the rectangular portion of the diagram and the magnitude of the momentum or impulse of the object. The area of the diagram gives the students the ability to quickly gauge the relative magnitude of momentum of the object.

Students in introductory courses have found these diagrams very helpful when they begin to investigate one-dimensional momentum situations. They found them useful because it helped them visualize momentum without having to use numbers.[8] The students were able to see how the mass and velocity affected the momentum of the object while at the same time the arrows depicted the direction. The students showed no difficulty associating the magnitude of the momentum with the area of the diagram. The biggest detractor the students had with the diagrams was that they had never seen them before. Thus, if students have more practice with and exposure to these diagrams, then they will be more helpful. We present four example problems with a description of how a student could use an IMD to help solve the problem in the next section.

Example problems

Problem 1

A block of ice of 20 kg is sliding on a sheet of ice, collides with and sticks to a stationary 40-kg chunk of ice. If the ice was initially moving at 60 m/s, what is the final speed of the two chunks of ice stuck together?

The first step in drawing a diagram is choosing a suitable scale. For this problem we chose 10 kg per square on the vertical axis and 10 m/s per square on the horizontal axis (Fig. 5). Thus, each block has an area of 100 kg·m/s. The second step is to draw a diagram for each chunk of ice at the beginning of the problem. Since the second (40 kg) chunk is not moving initially, we will not construct a diagram for it. We see that the area (momentum) of the system initially is 12 blocks or 1200 kg·m/s (20 kg * 60 m/s = 1200 kg·m/s).

Next, we focus on the final situation (Fig. 6). We increase the length of the arrow by four blocks to represent the additional mass. Since momentum is conserved, we need to have the 12 blocks (or an area of 1200 kg·m/s) in our final IMD. The six blocks on the vertical axis (the chunks of ice stuck together) multiplied by 2 will give the area of 12 blocks. This 2 represents a speed of 20 m/s.

Another popular type of momentum problem involves a collision of two vehicles from opposite directions. Problem 2 is an example of this situation.

Problem 2

A 1200-kg Chrysler Sebring is traveling west at 20 m/s and collides head on with an 800-kg Prius moving east (out of a parking lot) at 5 m/s. The two collide and stick together. What is the velocity of the mangled cars?

Initial inspection of Fig. 7 gives some information about the scenario. First, students can visually see that the magnitude of the momentum of the Sebring is much larger than that of the Prius. From this, the final velocity of the two will be to the west. To determine the magnitude of the velocity, you must calculate the area of the graph. Initially, the Sebring has 24 blocks to the west and the Prius has four blocks to the east. Thus, the net momentum is 20 blocks to the west, as shown

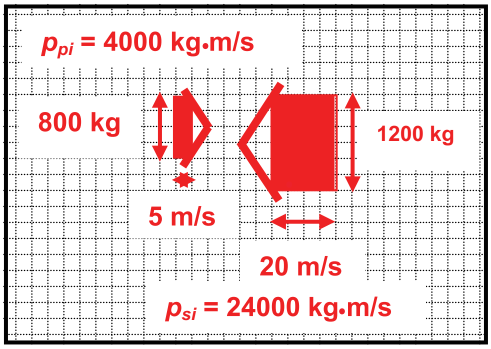

Fig. 7. Example problem 2; initial situation.

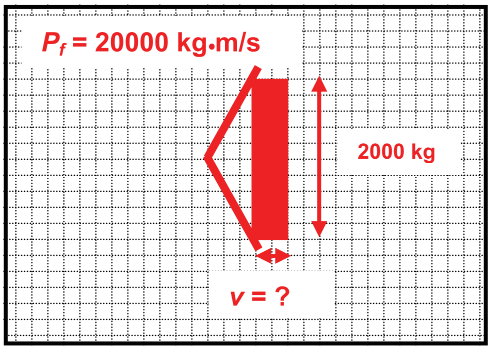

Fig. 8. Example problem 2; final situation.

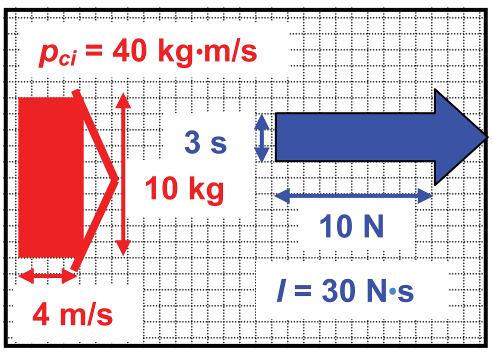

Fig. 9. Example problem 3; initial situation.

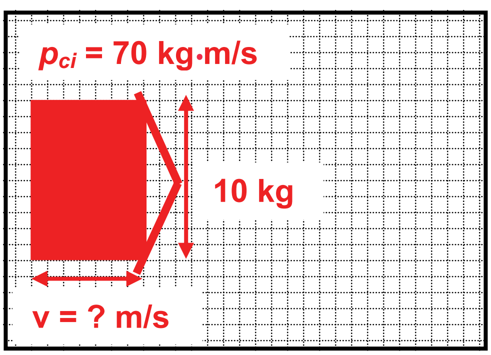

Fig. 10. Example problem 3; final situation.

in Fig. 8. The diagram must have a vertical height of 10 blocks (2000 kg), which means that the horizontal length must be two blocks. This corresponds to a velocity of 10 m/s westward.

Instructors can also use these diagrams when combining impulse and momentum. However, the horizontal scale must have the same magnitude (i.e., 1 block equals 1 m/s and 1 N or 1 block equals 15 m/s and 15 N) as well as the vertical scale (i.e., 1 block equals 3 kg and 3 s).

Problem 3

A 10-kg model rocket car is moving at 4.0 m/s after its first rocket has fired. The second rocket fires late. The rocket exerts 10 N of thrust on the car over a time of 3 s. What is the final speed of the car?

The blue arrow in Fig. 9 represents the rocket's impulse and is in the same direction as the rocket's momentum. Since the mass of the rocket does not change, the height of the arrow will not change. Thus, we can only change the length of the arrow as seen in Fig. 10 to seven blocks, which corresponds to 7 m/s.

Problem 4

Two balls are released from rest at the top of a ramp and hit a plank. Ball A hits the plank and drops straight down. The plank remains standing. Ball B hits the plank and bounces straight off of the plank. The plank gets knocked over. Ball A has a slightly larger mass than B, yet it was not able to knock the plank over. Why?[9]

There are many conceptual problems IMDs can be used with to help student understanding. In the above problem, the reason the plank is knocked over is a combination of Newton's third law with impulse. Students may want to believe that since ball A has a larger mass, it will exert a larger force on the plank. However this is not the case. Figure 11 shows that the change in momentum for ball B (though a smaller mass) is much larger. If we assume that the times the balls are in contact with the plank are roughly equal, then the force the plank needs to exert on ball B is much greater. Thus, by New-

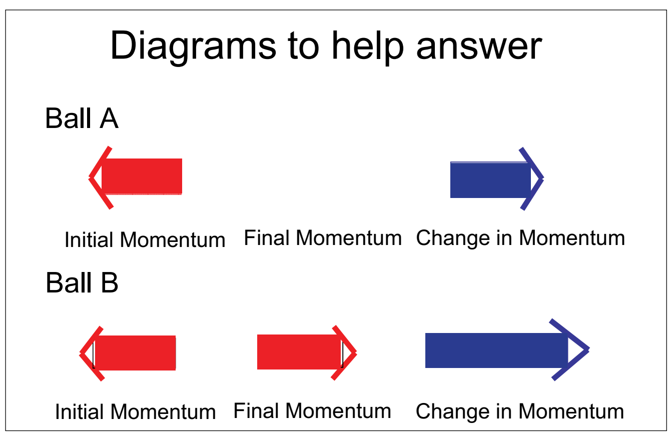

Fig. 11. Solution to example problem 4.

ton's third law the force that ball B exerts on the plank is much greater than the force that ball A exerts on the plank.

Conclusion

There are many different ways in which you can utilize these diagrams in the classroom. The main purpose for these diagrams is to help students qualitatively understand momentum and impulse. Furthermore, they can act as a link between abstract verbal descriptions of a scenario and the abstract mathematical answer to the problem, thus acting in a quantitative fashion much the same way that free-body diagrams serve as a link to help students construct Newton's second law in component form.[6]

In addition to highlighting the positive aspects of this representation, it is important to highlight the limitations. The largest limitation of these diagrams is that they cannot easily be applied to two-dimensional cases. They work well for one-dimensional motion with simple addition and subtraction of the area of the arrow; however, a two-dimensional case would involve much more complicated geometry. Thus, IMDs should be used in the beginning to help lay the foundation for the students to understand momentum and impulse. Furthermore, some students may have difficulties in determining appropriate scales when constructing the diagrams.

Acknowledgments

Special thanks go out to Taha Mzoughi from Kennesaw State University and Eugenia Etkina, Alan Van Heuvelen, and the entire Rutgers Physics Education Research group for their helpful feedback on this paper and for their continued support over the years.

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9. Problem is adapted from the following website: Rutgers, The State University of New Jersey, Physics Teaching Technology Resource, Video Website, http://paer.rutgers.edu/pt3/experiment.php?topicid=4&exptid=140.

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