Fall 2008

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Recommended Citation
Mazzotta, Stefano. "Performance, Bias, and Efficiency of Foreign Exchange Correlation Forecasts." Review of Futures Markets 17.2 (2008): 129-166.

Performance, Bias, and Efficiency of Foreign Exchange Correlation Forecasts[*]

This version January 2008

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Abstract: This paper evaluates the performance, bias, and the efficiency of option-implied, and return-based correlation measures using twelve years of daily data on foreign exchange and over-the-counter (OTC) currency option. The sample includes five years of rates for the Polish zloty and the Czech koruna with respect to the euro and the U.S. dollar. The results show that implied correlation is a good predictor of realized correlation and is, generally, unbiased and efficient.

JEL classification: F31, F37, G15, E58.
Key words: Foreign exchange implied correlation, forecasting bias, correlation risk premium

[*] Acknowledgements: The OTC options data were provided by Citibank N.A. I am grateful to Olli Castren , Peter Christoffersen and Lorenzo Cappiello who provided many ideas and suggestions to this work, and Stelios Makrydakis, Simone Manganelli, Filippo Di Mauro, Sam Nasypbek, the participants in an internal ECB seminar and the participant in the "Hedging and Market Efficiency in Foreign Exchange" session at the 2005 Eastern Finance Association conference for useful comments. The usual disclaimer applies. The financial support of Kennesaw State University is gratefully acknowledged.
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Introduction

This paper investigates the extent to which it is possible to use foreign exchange options implied measures and returns based measures to predict the correlation between bilateral exchange rates. In particular, we study the *bias* and *efficiency* properties of returns-based, and options based measures of correlation. We use a large data set that includes unique time series of currency options data covering not only the major currencies, but also the Polish zloty (PLZ) and Czech koruna (CKZ) with respect to the euro and the U.S. dollar.[1]

We study in particular whether the forward-looking information contained in the Over-The-Counter (OTC) currency options data can provide good forecasts of the future realized correlation between exchange rates by themselves or in addition to various correlation forecasts derived from returns based measures. We extend upon the results in the literature by looking at not only the informational content of options for the purpose of correlation forecast, but also at the *bias* and *efficiency* properties of such forecast for eight currency pairs. We argue that when evaluating implied correlation forecasts, bias and efficiency are two dimensions that should be taken into account. Implied correlations are defined under the risk neutral probability measure, whereas return based correlations are defined under the objective probability measure. If volatility risk is priced in the currency options markets, then risk neutral volatility and objective volatility diverge and, in principle, so do correlations. It follows that, although in theory the two types of correlation measures are very different, whether they diverge in practice is an empirical question. This question is of interest for academics and practitioners alike as the presence of bias in implied correlation forecasts may indicate the existence of a correlation risk premium.[2]

In addition, sometimes the forecaster may be willing to accept a little bias in exchange for higher efficiency. At the time of writing this paper, we are not aware of any study looking at bias and efficiency of correlation forecasts of foreign exchange rates. We aim at filling this gap in the literature by presenting results on the bias and inefficiency properties of correlation forecasts in a Mincer and Zarnowitz (1969) framework.

There is a substantial literature investigating the informational content of options in relation to asset price returns.[3] However, most of the studies focus on the informational content with regard to *volatility* forecasts. Studies investigating exchange rate *correlations* implied by market data are, on the contrary, rather sparse.[4] The contributions perhaps closest related to our work are Siegel (1997), Bodurtha and Shen (1999), Campa and Chang (1998), and Lopez and Walter (2000), who specifically focus on exchange rate correlations. For correlations between DEM/USD-JPY/USD, USD/DEM-JPY/DEM, and USD/JPY- DEM/JPY, Lopez and Walter (2000) find that implied correlations from October 1990 to April 1997 are useful in forecasting observed correlations, but they do not fully incorporate all the information in the historical data. Campa and Chang (1998) find that implied correlation among the DEM/USD, USD/JPY and DEM/JPY currency pairs from January 1989 to May 1995 outperform alternative forecasts at one- and three-month horizons. In addition, they find that when included in joint forecast regressions, implied correlation always incrementally improves the performance of other forecasts, but not the converse. None of these papers investigates forecasting bias or efficiency, however.

A brief summary of our results – that derive from time series from April 1992 to November 2005 – is as follows. We find that the implied correlation calculated from currency options prices shows substantial predictive power, both in terms of adjusted R^2 and forecasting error, for the future realized correlation, especially in the most recent years. The predictive power, as measured by the predictive regressions adjusted R^2, varies from a minimum of 5 per cent for the GBP/EUR-JPY/EUR to a maximum of 51 per cent for USD/EUR-PLZ/EUR, over the same sample period. Moreover, for the exchange rate pairs that show correlation predictability, implied correlation is not the only predictor that produces good forecasts. Both GARCH and RiskMetrics correlation forecasts show substantial predictive power in some cases. In substance, option-based and returns-based correlations forecasts appear to complement each other. The best forecasts often obtain when implied and return-based correlations are used jointly. The highest adjusted R^2 is invariably obtained from the encompassing (multivariate) regressions. The total predictability obtained using a combination of forecasts ranges from 21 to 36 per cent for the entire sample, (April 1992 to November, 2005), and from 14 to 52 per cent for the post-1999 sub-sample.

Importantly, when we examine forecasting error, we document that when the mean squared forecasting error (MSE) is lowest for implied correlation, implied correlation is also a substantially unbiased forecast. This is so especially after the introduction of the single currency. This suggests that there is no correlation risk premium in the foreign exchange market. Implied correlations also score favorably in term of efficiency when compared to return based correlations.

The rest of this study is organized as follows. Section 2 introduces the framework in which the various correlation measures are analyzed. Section 3 specifies the estimated equations and reports the results, and Section 4 concludes.

2 CORRELATION FORECAST EVALUATION

2.1 Data Issues

The convention of Over-the-Counter currency options market is that quotes are not made on prices, but on the Garman and Kohlhagen (1983) implied volatilities at a fixed delta. The dealers then calculate the price using agreed-upon underlying price and interest rates.[5] Our data set includes at-the-money (ATM) implied volatilities for the currency exchange options on USD/EUR, GBP/EUR, JPY/EUR, JPY/USD, USD/GBP, and JPY/GBP. In addition, our sample includes implied volatilities of the following new EU Member States' currencies exchange options: CZK/EUR, CZK/USD, PLZ/EUR, and PLZ/USD. To our knowledge, ours is the fist study that incorporates such a large set of European currencies for a sample period after the launch of the euro. These quotes are expressed in terms of Garman and Kohlhagen (1983) 1-month at-the-money (ATM) implied volatilities. The options are European style. We emphasize that the use of the Garman and Kohlhagen pricing formula is a convenient market convention that does not imply subscribing to its accuracy or underlying economic assumptions. In particular, the facts that volatility is not constant, and that currency returns are fat tailed and skewed are well established in the literature and known to market participants.[6] Nonetheless, since implied volatility quotes are readily available, it is interesting to use them to study their properties in terms of correlation forecasting power, bias and efficiency in an empirical fashion.

Traditionally, the bulk of trading in options is on OTC basis and not at centralized futures/options exchanges. Christensen, Hansen and Prabhala (2001) argue that in terms of forecasting properties, OTC options data could be of superior quality relative to exchange-traded options. This is because OTC prices are quoted daily with fixed “moneyness“ (the distance between the forward rate and the option’s strike price) in contrast with market-traded options, which have fixed strike prices and thus time-varying moneyness as the forward exchange rate changes. Moreover, the trading volume in OTC options is often much larger than in the corresponding market traded contracts.[7] The underlying liquidity on OTC quotes is therefore deeper, which makes the OTC quotes a more reliable source for information extraction. The fact that the currency options market is heavily concentrated on a few global players does that the liquidity problems can be reduced further if data from these institutions is available. Citigroup, the provider of our implied volatility data, has a significant market share both in options on major exchange rates as well as on the emerging currencies.

The spot exchange rates are from the Bank of International Settlements (BIS). Our sample starts in March 1992 and ends in November 2005, except for the Polish zloty and the Czech koruna currency pairs for which the sample period commences at January 2001. In light of the enlargements of the euro area, these latter currencies have become object of increased attention in the last few years. Nonetheless, they are still very little studied. During the period of their gradual return to convertibility throughout the 1990s, the Polish zloty and the Czech koruna have gone through several regime shifts. In Poland, the authorities devised a crawling peg regime for the zloty in October 1991 with initially a narrow fluctuation band. By the time of the launch of the euro in January 1999, the fluctuation band had been gradually widened, and in April 2000 Poland switched to a floating exchange rate regime with no formal restrictions for currency movements. In the Czech Republic, the initial exchange rate peg was abandoned in favor of an inflation-targeting regime with a floating exchange rate in January 1998 after an exchange rate crisis in May 1997. Over the years, the Czech authorities have occasionally intervened in the market to smooth out excessive fluctuations. As illustrated in figure 1 in appendix, after the launch of the euro in January 1999, the Polish zloty and the Czech koruna both initially appreciated against the euro. Between mid-2001 and early 2004 the zloty and the koruna depreciated before starting a new appreciating trend that lasted until late 2005.

[Place Table 1and 2 about here]

To compute the foreign exchange returns prior to the launch of the euro in January 1999 we use a synthetic euro computed from the DEM and the official conversion rate. We then splice the log return series. Descriptive statistics of the foreign exchange returns before and after the introduction of the euro are separately shown in table 1 and 2. The tables show that, regardless of the sample period, all exchange rates returns are skewed, leptokurtic and highly non-normal.

2.2 The Forecasting Object of Interest

The particular object of interest of our study is forecasting the realized future sample correlation of an exchange rate pair over the horizon of the following 21 trading days.

There exists substantial literature regarding the use of realized volatility as a measure of equity and foreign exchange variability (see e.g. Andersen and Bollerslev (1998) and Andersen et al. (2001a, 2001b, 2003)). The common thread of this literature is the idea that one can sum squared log returns at a frequency higher than that of interest to obtain a measure of the realized quadratic variation over the frequency of interest. For instance, one can compute the monthly variance as the sum of squared daily log returns, or the daily variance as the sum of intraday squared log returns. In this theoretical framework, by increasing the sampling frequency it is possible to construct ex post realized volatility measures for the integrated latent volatilities that are asymptotically free of measurement error. In practice, the benefit of increasing the frequency is offset by the microstructure noise that is invariably included in the observed market quotes.

One approach commonly taken is to strike a balance between the horizon of interest and the number of sub-periods in which such horizon is divided for the purpose of computing the squared returns. In the case of *daily* variance estimates, whereas early works use 5-minute returns, recent contributions indicate that 30-minute returns (i.e. about 16-18 data points per trading day) provide a measure of daily volatility relatively robust to microstructure noise. In our case, since we want a measure of *monthly* correlation, the sum of own and cross products of demeaned daily log return over the 21 trading days can be considered a measure of monthly realized co-variation that is robust to microstructure noise and sufficiently precise. We choose this measure of realized correlation also to preserve the comparability of our results with most of earlier studies. The measure of correlation we obtain is the ex-post sample correlation over the next 21 trading days. Following the conventions established in the above-mentioned literature, we call this measure "realized correlation", henceforth RC. Let A, B, and C be three currencies, and $S_t^{A/B}$ the price of currency B in terms of currency A at time t. The continuously compounded return on the rate $S_t^{A/B}$ is defined as

$$R_t^{A/B} = \ln\left(S_t^{A/B} \;/ S_{t-1}^{A/B}\right). \tag{1}$$

We define RC for the next *h* days as follows

$$\rho\left(R^{A/C}, R^{B/C}\right)_{t,h}^{RC} = \frac{\frac{1}{h}\sum_{i=1}^{h}\left(R_{t+i}^{A/C} - \overline{R}_{t+1,t+h}^{A/C}\right)\left(R_{t+i}^{B/C} - \overline{R}_{t+1,t+h}^{B/C}\right)}{\sqrt{\frac{1}{h}\sum_{i=1}^{h}\left(R_{t+i}^{A/C} - \overline{R}_{t+1,t+h}^{A/C}\right)^2}\sqrt{\frac{1}{h}\sum_{i=1}^{h}\left(R_{t+i}^{B/C} - \overline{R}_{t+1,t+h}^{B/C}\right)^2}}. \tag{2}$$

The plots of all correlation measures are illustrated in figures 2 to 9 in appendix (note that we have labeled the realized correlation as "historical correlation" as the latter is simply a lagged realized correlation as will be explained in more detail below). Both the figures and the table of descriptive statistics show that on daily basis, correlation changes over time. The most volatile measure appears to be the Historical Correlation. From the inspection of these plots, it seems that the correlations between the USD/EUR and JPY/EUR currency pairs, between the USD/EUR and GBP/EUR currency pairs, between the USD/GBP and JPY/GBP currency pairs, and between the USD/JPY and GBP/JPY currency pairs have fluctuated in the positive territory most of the time. Moreover, the positive correlation seems to be generally higher in the post-euro sub sample.

2.3 The Measures of Correlation

To forecast future realized correlation, four alternative correlation measures are applied. First, we calculate the implied correlation from options implied volatility. Being based on options data, implied correlation provides a forward-looking perspective to the analysis of co-movements between currency pairs. Because exchange rate options provide information on the currency options market's uncertainty about the price of one currency in terms of another, with *three* implied volatilities from currencies options we can derive an estimate of the market's expected future, or implied correlation. To put it in another way, implied correlation represents the degree of co-movement between two currencies using a third currency as a numeraire.

For the no-arbitrage condition to hold in the foreign currency market it must be that $S_t^{A/C}\, S_t^{C/B} = S_t^{A/B}$. This implies that $R_t^{A/B} = R_t^{A/C} - R_t^{B/C}$. It then follows that the variance of the exchange rate A/B at time t is

$$Var\left(R^{A/B}\right)_t = Var\left(R^{A/C}\right)_t + Var\left(R^{B/C}\right) - 2Cov\left(R^{A/C}, R^{B/C}\right)_t . \qquad (3)$$

It is then straightforward to derive the implied correlation (IC) between $R_t^{A/C}$ and $R_t^{B/C}$ using the squared implied volatilities as measures of the $Var(R^{A/B})_t$, $Var(R^{A/C})_t$, and $Var(R^{B/C})_t$.[8] The implied correlation is therefore defined as

$$\rho\left(R^{A/C}, R^{B/C}\right)_{t,h}^{IC} = \frac{1}{2}\frac{Var\left(R^{A/C}\right)_t + Var\left(R^{B/C}\right)_t - Var\left(R^{A/B}\right)_t}{\sqrt{Var\left(R^{A/C}\right)_t}\sqrt{Var\left(R^{B/C}\right)_t}} . \qquad (4)$$

In the formula of the implied correlation in (4), the under script *h* denotes the 1-month time horizon of the forecast that is object of this study.

[Place Table 3 here]

Bollerslev and Zhou (2005) point out that if the volatility risk is priced in the options markets then implied volatility is a biased predictor of realized volatility.[9] In fact, implied volatilities are often empirically found to be upward biased estimates of the objective volatility. In a standard stochastic volatility set up, it can be shown that if the price of volatility risk is zero, the process followed by the volatility is identical under the objective and the risk neutral measures. In such a case, there would be no bias. However, the volatility risk premium is generally estimated to be negative, which in turn implies that the volatility process under the risk neutral measure will have higher drift. These theoretical considerations do apply to the computation of implied correlation as well, with one major difference. Such a potential bias could affect implied volatilities both in the numerator, and in the denominator of the implied correlation in (4). Hence, whether bias is a problem for implied correlations is an empirical question.

The extant literature does not address this question. We use the largest cross section and longest time series of currency pairs appeared in the literature to our knowledge to fill this gap. As is shown below, one important finding of our study is that bias in correlations computed from options turns out to be often very small. On the other side, since our data includes only 1-month ATM options, we leave other potentially interesting issues related to the term structure of correlation forecast or the importance of return skewness in the sense of Carr and Wu (2007) and Bakshi, Kapadia and Madan (2003) for future research.

The other three volatility forecasts are derived from historical FX returns only. The simplest possible forecast is the historical *h*-day volatility, defined as

$$\rho\left(\mathrm{R}^{\mathrm{A/C}},\mathrm{R}^{\mathrm{B/C}}\right)_{t,h}^{HC}=\rho\left(\mathrm{R}^{\mathrm{A/C}},\mathrm{R}^{\mathrm{B/C}}\right)_{t-h,h}^{RC} \tag{5}$$

The historical correlation is simply the lagged realized correlation. Alternatively, we can consider second moments that apply an exponential weighting scheme putting progressively less weight on distant observations. The simplest measure using such a scheme is the Exponential Weighted Moving Average or RiskMetrics (RM) correlation. Daily variance and covariance then evolve as

$$Var\left(R^{A/C}\right)_{t+1}^{RM} = (1-\lambda)\sum_{i=1}^{\infty}\lambda^{i-1}\left(R_{t,t-i+1}^{A/C}\right)^2 = \lambda Var\left(R^{A/C}\right)_t^{RM} + (1-\lambda)\left(R_t^{A/C}\right)^2$$

$$Cov\left(R^{A/C}, R^{B/C}\right)_{t+1}^{RM} = (1-\lambda)\sum_{i=1}^{\infty}\lambda^{i-1}R_{t-i+1}^{A/C}\ R_{t-i+1}^{B/C} \qquad (6)$$

$$= \lambda Cov\left(R^{A/C}, R^{B/C}\right)_t^{RM} + (1-\lambda)\,R_t^{A/C}\ R_t^{B/C}.$$

Following JP Morgan we fix λ = 0.94 for all the daily FX returns. The forecast for *h*-day correlation is therefore

$$\rho\left(R^{A/C}, R^{B/C}\right)_{t+1}^{RM} = \frac{Cov\left(R^{A/C}, R^{B/C}\right)_{t+1}^{RM}}{\sqrt{Var\left(R^{A/C}\right)_{t+1}^{RM}}\sqrt{Var\left(R^{B/C}\right)_{t+1}^{RM}}}. \qquad (7)$$

The third estimate for correlation based on past exchange rate returns that is considered here is the GARCH correlation. The GARCH methodology permits the calculation of time-varying second moments for the universe of assets that are considered by the researcher. According to this approach, variances and correlations are conditional on a time-varying information set that allows one to update the estimated second moments at each point in time when new information becomes available. We have adopted a bivariate GARCH model where R_t is defined as the vector of returns

$$R_t = \left[R_t^{A/C}\ \ R_t^{B/C}\right]. \qquad (8)$$

We assume that R_t follows a GARCH process

$$R_t = H_t^{1/2}\varepsilon_t\,. \qquad (9)$$

In (9) ε_t is an identical and independently distributed vector sequence with mean zero and unit variance. The conditional covariance H_t evolves according to a diagonal BEKK GARCH process[10]

$$\begin{aligned}
&H_{t+1} = \Omega\Omega' + \mathrm{B}H_t\mathrm{B}' + \mathrm{A}R_t R_t'\mathrm{A}' \\
&\text{where} \\
&H_t = 2 \text{ x } 2, \qquad\qquad (10) \\
&\mathrm{A, B} = 2 \text{ x } 2 \text{ diagonal, and} \\
&\Omega = 2 \text{ x } 2 \text{ lower triangular.}
\end{aligned}$$

The next day GARCH correlation is thus defined as

$$\rho(R^{A/C}, R^{B/C})_{t+1}^{GARCH} = \frac{\sigma_{1,2,t+1}}{\sigma_{1,1,t+1}\sigma_{2,2,t+1}}. \qquad (11)$$

where $\sigma_{1,2,t+1}, \sigma^2_{1,1,t+1}$, and $\sigma^2_{2,2,t+1}$ are respectively the covariance and the variances contained in the matrix H_{t+1}. In contrast to the RiskMetrics model, which implies a flat term structure of the volatility process, to forecast the correlation over the next 21-days with GARCH it is necessary to consider the mean reversion of the model and iteratively forecast variances and covariance.[11] Appendix 2 shows the computations to obtain the GARCH correlation forecasts. The plots of the GARCH correlations for the various exchange rate pairs are in Figure 5 and 9 in Appendix 1. The plots are substantially smoother than those obtained from historical correlations.

3 CORRELATION FORECAST EVALUATION

3.1 Methodology

To compare the forecasting capability of the different correlation measures, the standard approach is to run linear predictability regressions for all the currency pair.

Through these regressions, we assess how various estimates of monthly exchange rate correlations have predicted realized monthly correlation one month ahead in time. More specifically, we first run the following univariate regressions for each correlation forecast

$$\rho_{t,h}^{RC} = a + b\rho_{t,h}^{j} + \varepsilon_{t,h}^{j} \tag{12}$$

for $j = IC, HC, GC,$

where IC, HC, GC stand for implied correlation, historical correlation, and GARCH correlation, respectively. These univariate regressions serve to assess the fit through the adjusted R^2 and to check how close the estimates of a are to 0 and how close the estimates of b are to 1.[12] In addition, we perform bivariate regressions which include the implied correlation and the two return-based forecasts in turn, as follows

$$\rho_{t,h}^{RC} = a + b\rho_{t,h}^{IC} + c\rho_{t,h}^{j} + \varepsilon_{t,h}^{IC,j} \tag{13}$$

for $j = HC, GC.$

These bivariate regressions shed some light into whether the return-based correlation forecasts add anything to the market-based forecast implied from currency options. Finally, we run an encompassing regression including the three correlation forecasts in the same equation, in order to asses the relative merits of the different correlation forecasts. It is worth noticing that the determinants of foreign exchange correlation, especially the macro economic ones, are possibly numerous and none of them appears in the forecasting regressions above. One glaring omission is for instance the role of central bank intervention in the foreign exchange market and its effects on correlations.[13] The omission is however due to the purpose of the forecasting exercise, which is to compare the forecasting power, bias and efficiency of different measures of correlations.

3.2 Results

Tables 4 and 5 report regression point estimates as well as robust t-stats. Since the forecasts are overlapping, to correct the standard errors for heteroskedasticity and autocorrelation, we use GMM with the robust Newey-West weighting matrix and a pre-specified bandwidth equal to 21 days. The regression fit is reported using the adjusted R^2. Table 5 includes the same regressions as table 4, but now using the sample period beginning from April 1999 and ending in November 2005. It also includes the results for the pairs USD/EUR-PLZ/EUR, and USD/EUR-CZK/EUR.

[Place Table 4 and 5 about here]

We find that correlation between foreign exchange pairs is predictable to a substantial extent. The adjusted R^2 of the GMM regressions for the entire sample ranges from 14 per cent for GBP/EUR-JPY/EUR to 33 per cent for USD/JPY-GBP/JPY. For the post 1999 sample, the adjusted R^2 ranges from 5 per cent for GBP/EUR-JPY/EUR to 52 per cent for USD/EUR-PLZ/EUR.[14] For the entire sample, implied correlation is not in all cases the best univariate forecast. Both GARCH and RiskMetrics correlation forecasts show considerable predictive power, too. The result for the entire sample is consistent with Lopez and Walter (2000). However, for the sample starting April 1999 and ending in November 2005 the adjusted R^2 of the implied correlation forecast univariate regressions is the highest in six out of eight currency pairs and in one case only marginally worse. The results for the sample starting April 1999 contrast with those for the entire sample and with those in Lopez and Walter (2000). That is, in the recent years for the eight currency pairs we consider, implied correlation is a good forecast and certainly worthwhile computing. The results of predictability regressions for volatility forecasts show that information from currency options prices seems to be as helpful in predicting correlation as it has been found in predicting volatility.

In both samples periods, the bivariate regressions show that return based correlations typically add something to the market-based forecast implied from currency options.

When we run the encompassing regressions, we find that the best forecasts obtain when return based measures are used jointly with market-based measures, as the highest adjusted R^2 is invariably obtained from the encompassing (multivariate) regressions. In other words, it seems that although implied correlation display good predictive power, it does not contain *all* the information about future correlation.

Overall, and especially in the more recent sample, our results are consistent with Campa and Chang (1998) in that implied correlation forecast seem to have remarkable forecasting properties along with the ability to incorporate information that cannot be found in past returns. In contrast with Campa and Chang (1998) however, we find that return based correlation measures provide non-negligible incremental explanatory power to explain realized correlation when used in addition to implied correlation.

3.2.1 Bias and Efficiency

If volatility risk is priced in the currency options markets, then the volatility under the risk neutral measure and the volatility under the objective measure diverge. Since implied correlation is computed using three different implied volatilities, if implied volatilities are biased estimates of objective volatilities, implied correlation estimates could also be biased. Whether the pricing of volatility or correlation risk in the three option markets induces bias in the implied correlation is then an empirical question. We provide some answers in what follows.

To study the merit of each correlation forecasts with regard to the relative bias and efficiency we perform a Mincer-Zarnowitz (1969) decomposition of the mean squared error (MSE) into bias squared, inefficiency, and random variation. First, define the MSE for forecast j as follows

$$MSE^{j} = T^{-1}(\rho_{t,h}^{RC} - \rho_{t,h}^{j})^{2}, \quad j = IC, HC, RM, GC, \quad h = 21. \tag{14}$$

This is simply the squared forecasting error normalized by the number of observations T. Mincer and Zarnowitz (1969) show that the MSE can be decomposed into bias squared, inefficiency, and random variation as follows

$$\text{MSE} = [\text{E}[\rho_{t,h}^{RC}] - \text{E}[\rho_{t,h}^{j}]]^{2} + (1 - ß)^{2}\ \text{Var}(\rho_{t,h}^{j}) + (1 - \text{R}^{2})\ \text{Var}(\rho_{t,h}^{RC}), \tag{15}$$

where one can obtain the slope coefficient β and the regression fit R^2 from the regression of the realized correlation on each correlation forecast and a constant in turn. The Mincer-Zarnowitz regressions are run for each of the currency pairs and for each of the currency forecasts.

[Place Table 6 and 7 here]

Table 6 for the entire sample and table 7 for the post 1999 sample report the MSE's in absolute value and their decomposition into bias squared, inefficiency, and residual variation, in *percentage* of the total MSE for each correlation forecast. The lower the MSE the better the forecast is. For a given MSE value, the higher the residual variation the more efficient and unbiased the forecast is. In the entire sample, GARCH correlations show the lowest MSE, followed by implied correlations. In the post 1999 sample, however implied correlation is the forecast with lowest MSE in most cases, followed by GARCH correlations.

The *direct* purpose of the paper is to investigate the bias and efficiency properties of correlation forecasts from option implied and return based correlation measures. The methodology used can however provide some *indirect* evidence regarding the correlation risk premium. In fact, while bias is measured without reference to an economic theory, the notion of correlation risk premium is built on a causality relationship established in an economic model.[15] Nevertheless, the link between bias in forecasts from option-implied correlation and the correlation risk premium deserves some discussion.

Diversification opportunities are valuable to investors. However, correlations increase in bear market (see e.g. Odier and Solnik (1993), and Longin and Solnik (2001)). In other words, the benefits of diversification may not be available when investors need them the most. The theory developed in Merton's (1973) ICAPM entails that investors will pay more for assets with high payoff when market-wide correlation is above expectation because such assets provide a hedge against correlation risk. In other words, diversification opportunities can be seen as one of the state variables describing the investment opportunity set. As well, market-wide correlation can bee seen as a suitable proxy for such state variable. The economic reason for the existence of a correlation risk premium for stocks is thus that, conditional on individual assets' volatility, an increase in correlation reduces investor's diversification opportunities.

This economic reasoning does not immediately translates to foreign currency exchanges. For foreign exchange rates, the appreciation of one currency corresponds to the decline of the other. The notion of "bear market" is thus less clear-cut. In addition, if there are no arbitrage opportunities, the correlation among currencies is entirely determined by the foreign exchange rate volatilities. Therefore, exchange rate interdependence rules out the existence of a premium for correlation risk of the type studied in Driessen et al. (2006) and Krisnan et al. (2007), i.e. a premium which is independent from the premium for volatility risk.

Furthermore, it is worth emphasizing, the correlation risk premium is not the only possible cause of bias. For instance, in tables 6 and 7, the relatively high measures of bias for the RiskMetrics forecasts for USD-EUR/GBP-EUR and USD/EUR-JPY/EUR rather suggest model misspecification. In this case, both the object of forecasts and the forecasting variable are under the objective measure, and hence the correlation risk premium cannot be the cause of bias. Likewise, in the few cases in which the bias of implied correlation forecasts is relatively high (e.g. for USD/GBP-JPY/GBP and USD/JPY-GBP/JPY) central banks intervention may have played a role.[16] As well, liquidity may affect the market price of currency options, which can in turn cause bias in implied volatilities and implied correlations.

In summary, in the case of implied correlations, bias results provide only indirect evidence about the correlation risk premium. Precisely, if the drift of the correlation process under the objective measure differs from the drift of the correlation process under the risk-adjusted measure only because of the risk adjustment, using implied correlation to forecast realized correlation yields biased forecasts. However, while bias does not imply the existence of a risk premium, the absence of bias would in general suggest on the contrary that correlation risk might not be priced in the foreign exchange markets. [17]

From the discussion above follows that, since the main economic reasons to expect a correlation premium in equity markets that motivate Driessen, et al. (2006) and Krisnan, et al. (2007) do not translate to foreign exchange markets, the presence of little bias in table 6 and 7 should not come as a surprise. The bias results in the tables hence indirectly suggest that there is no correlation risk premium in the foreign exchange markets**.** After the introduction of the single currency, bias becomes negligible in the case of USD/EUR-JPY/EUR, USD/EUR-GBP/EUR, GBP/EUR-JPY/EUR, and USD/EUR-PLZ/EUR. For these pairs of currencies the squared bias ranges in percentage of the total MSE ranges from 0.09 per cent for USD/EUR-GBP/EUR to 2.72 per cent for the JPY/USD-GBP/USD. A notable exception to this pattern is the USD/JPY-GBP/JPY implied correlation bias, which sizably increases to 42 per cent in the recent sample. It is noteworthy that when implied correlation is not the best predictor its bias is relatively high. In both sample periods, implied correlation appears to be most often the most efficient forecast. RiskMetrics correlation is more efficient for USD/EUR-JPY/EUR and USD/EUR-GBP/EUR, but is also quite biased.

In the post 1999 sample, the historical correlation is shown to be rather inefficient but substantially unbiased. RiskMetrics correlation appears to be somewhat inefficient for some currency pair and rather biased for others. GARCH often perform better than the other forecasts only under one measure.

In summary, implied correlations are generally efficient and only slightly biased forecasts. Return based measures are often less biased but also less efficient. These results are novel and suggest that there is no correlation risk premium in the foreign exchange market. The ranking however does not strictly hold for all the currency pairs in both sample periods.

4 CONCLUDING REMARKS

The different estimates of correlation between the major bilateral exchange rates show distinctive fluctuations over time. Implied correlation shows remarkable forecasting power, predicting up to 51 per cent of future realized correlation. After 1999, implied correlation is the best forecast in the six out of eight currency pair as measured by both the adjusted R^2, and the MSE. Nevertheless, GARCH correlations show often very good predictive power. When used together, implied correlation, GARCH correlation and RiskMetrics correlation are particularly powerful in predicting future correlation between the major euro currency pairs at the one-month horizon. The predictive power has strengthened after the introduction of the euro.

We extend upon the results in the literature by using a more diverse set of currencies, longer samples, and by looking at the *bias* and *efficiency* properties of correlation forecasts for eight currency pairs. We find that, with the exception of USD/EUR-CZK/EUR and USD/JPY-GBP/JPY pairs, the bias is very low for the implied correlation forecasts, especially after the introduction of the single currency. Such a low bias indirectly suggests that there is no correlation risk premium in the foreign exchange market. Implied correlations also score favorably in term of efficiency when compared with return based correlations. Why certain implied correlation forecasts perform better than others is an open question that is left for future research.

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Appendix 1

Figure 1. Foreign Exchange Spot Rates.

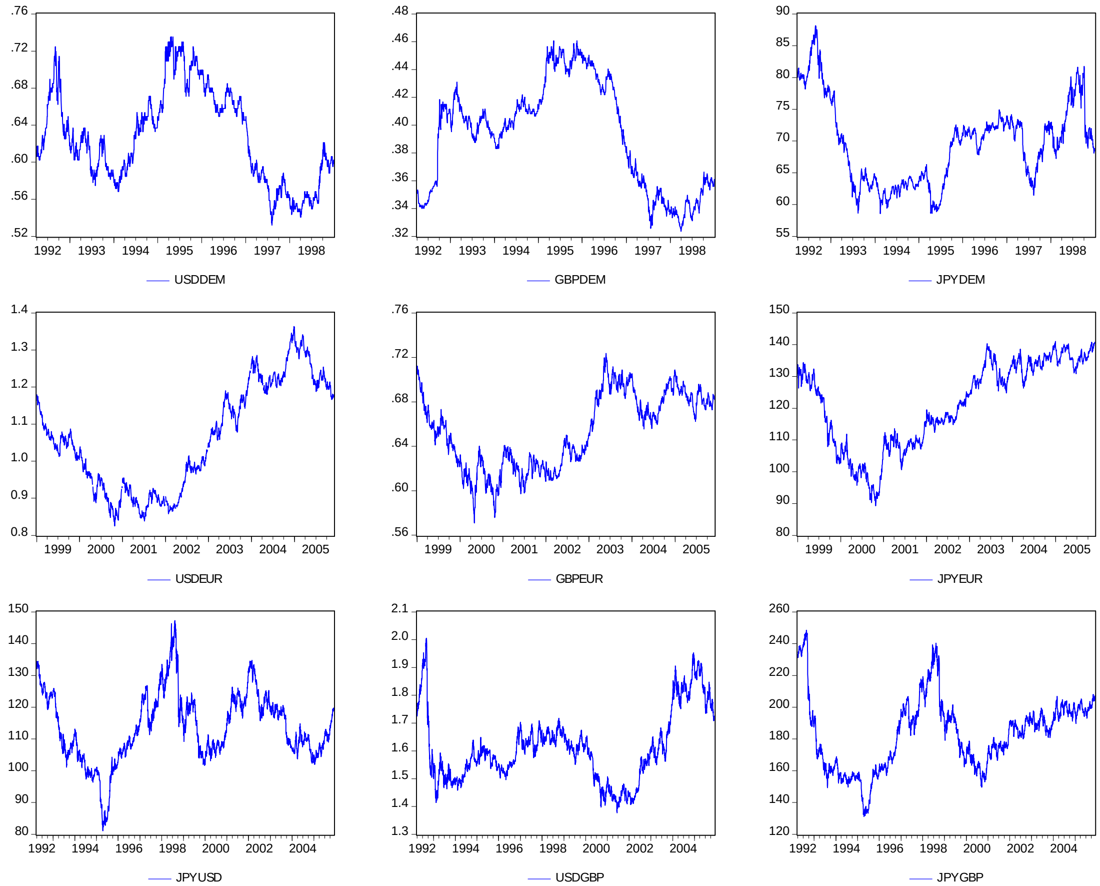


Figure 2. Foreign Exchange Spot Rates - Acceding Countries.

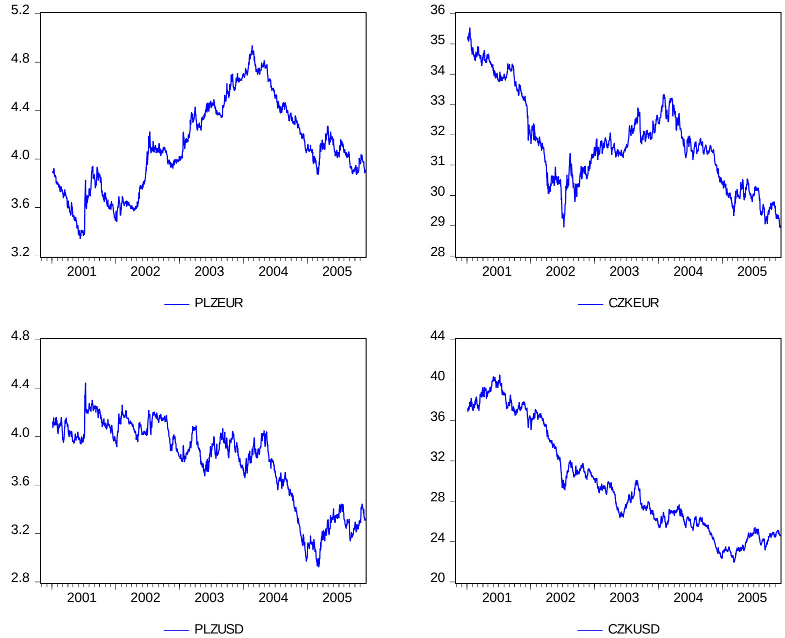


Figure 3. Implied Correlations - January 92 - December 98.

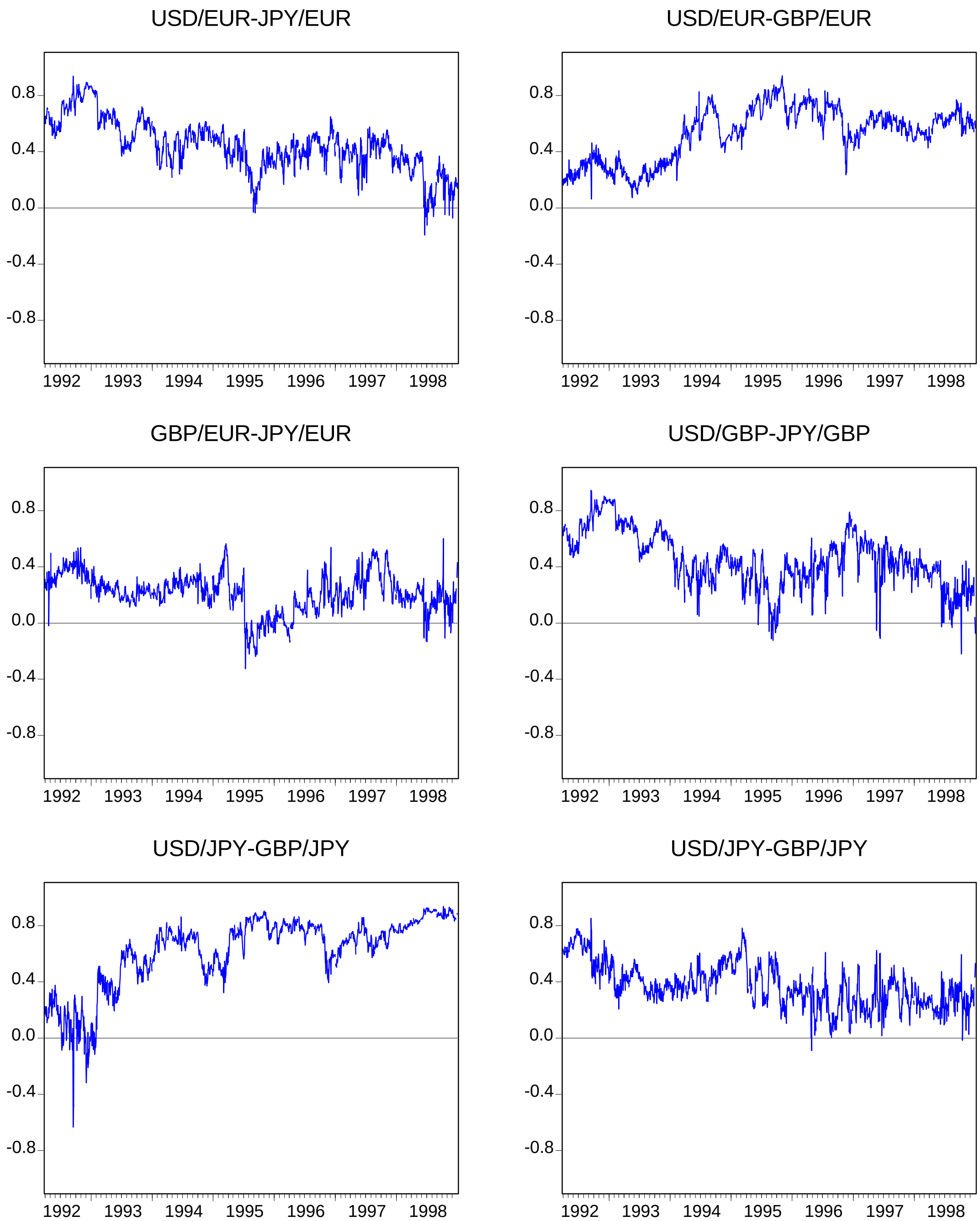


Figure 4. Historical Correlations - January 92 - December 98.

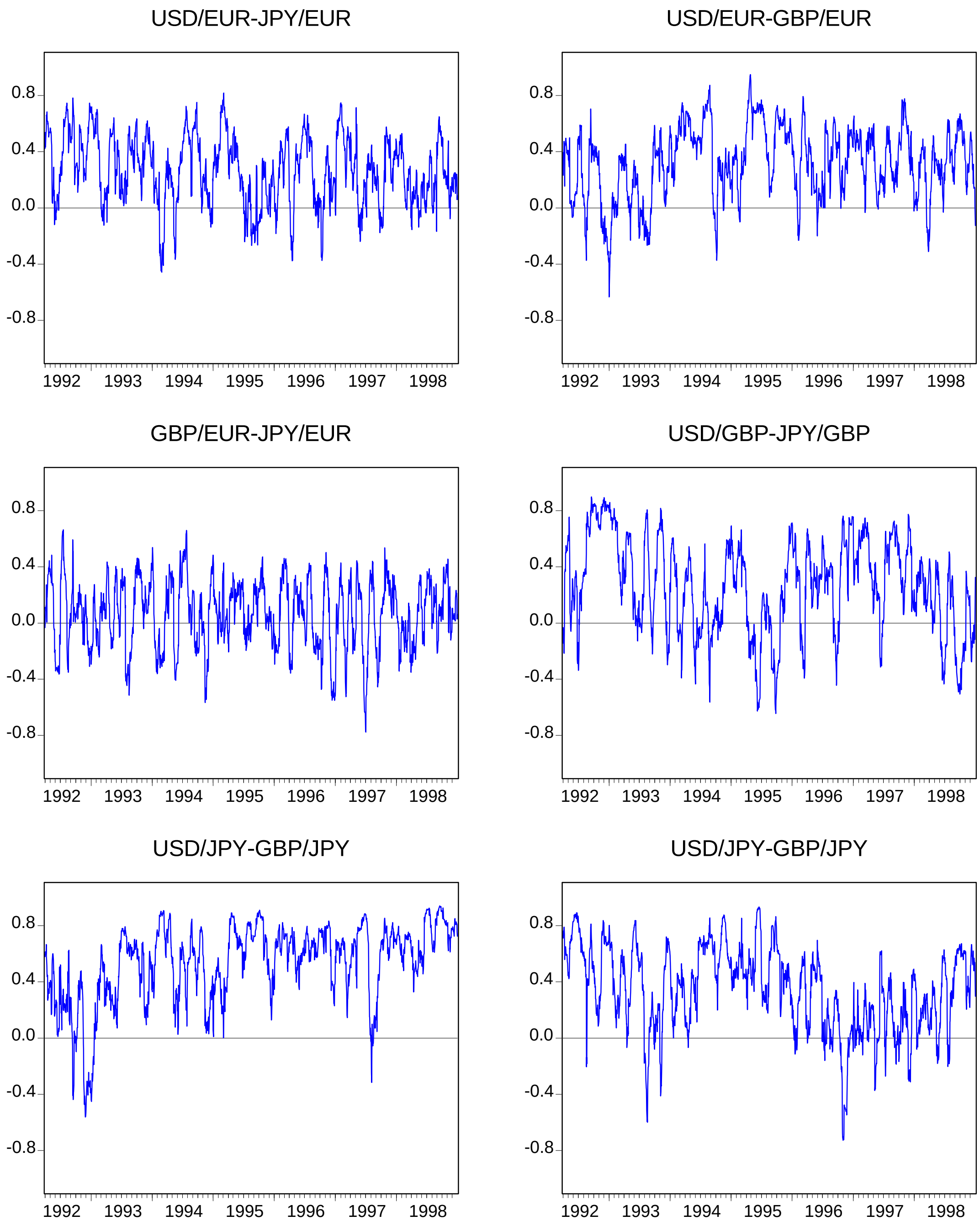


Figure 5. RiskMetrics Correlations - January 92 - December 98.

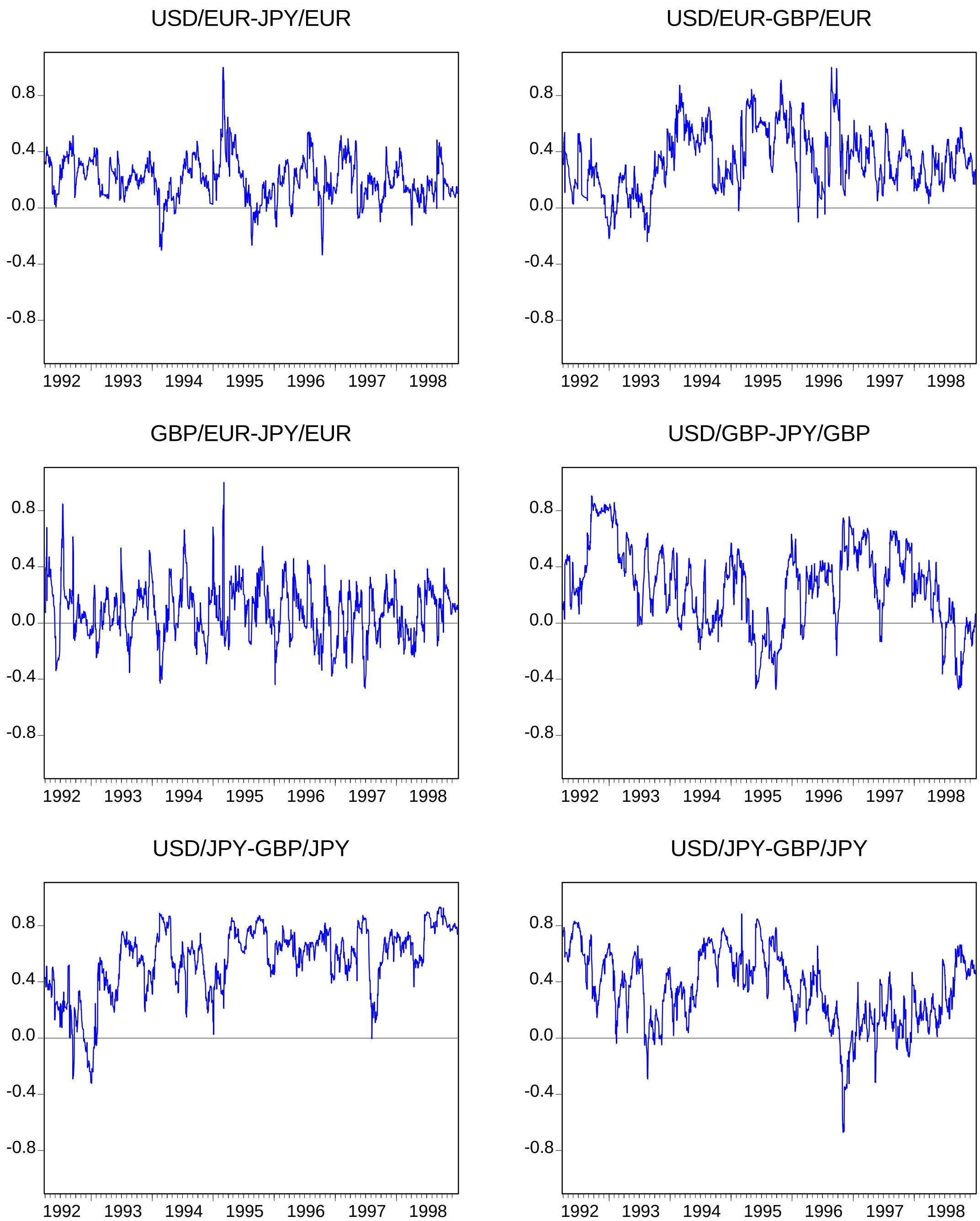


Figure 6. GARCH Correlations - January 92 - December 98.

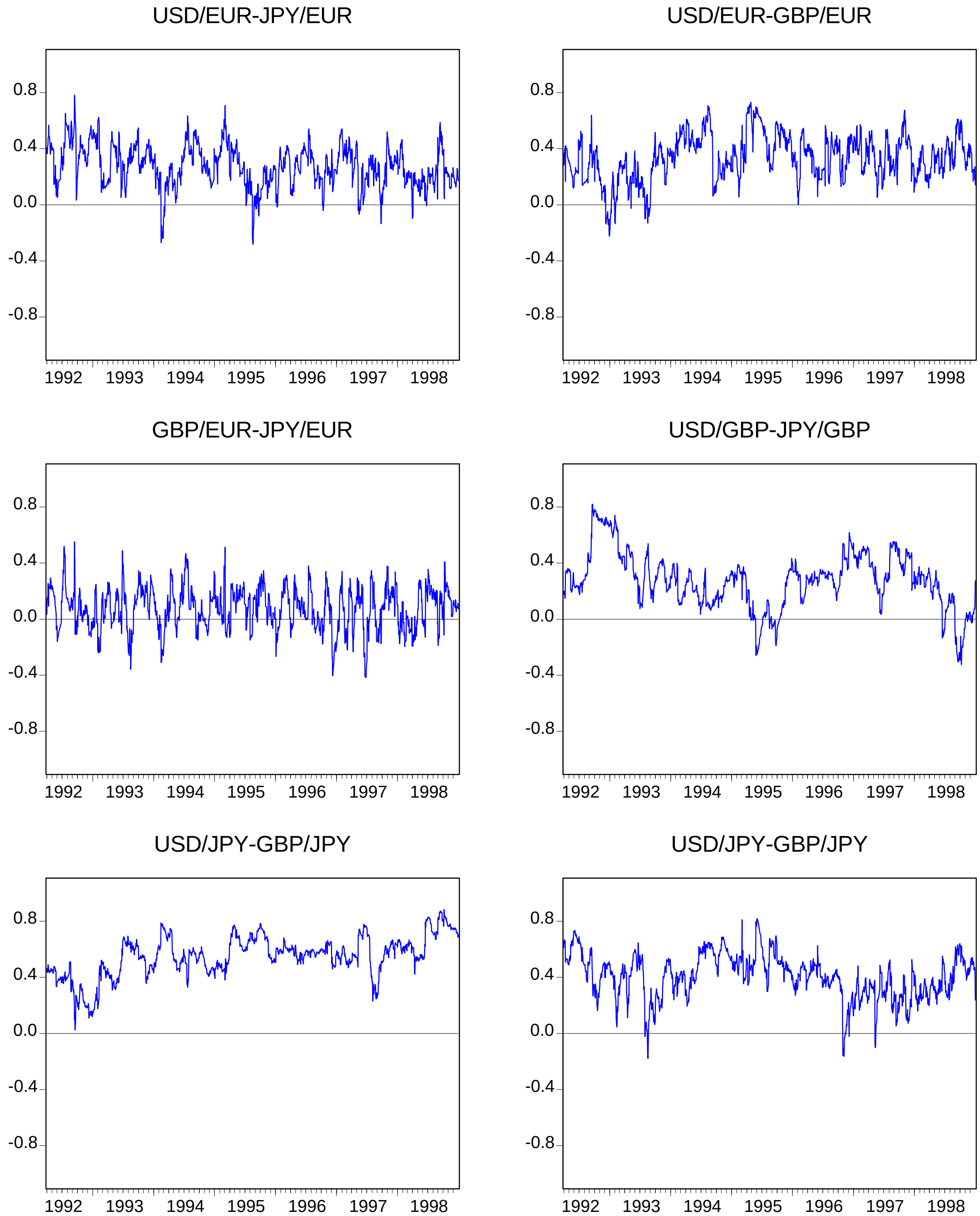


Figure 7. Implied Correlations. January 99 - November 05.

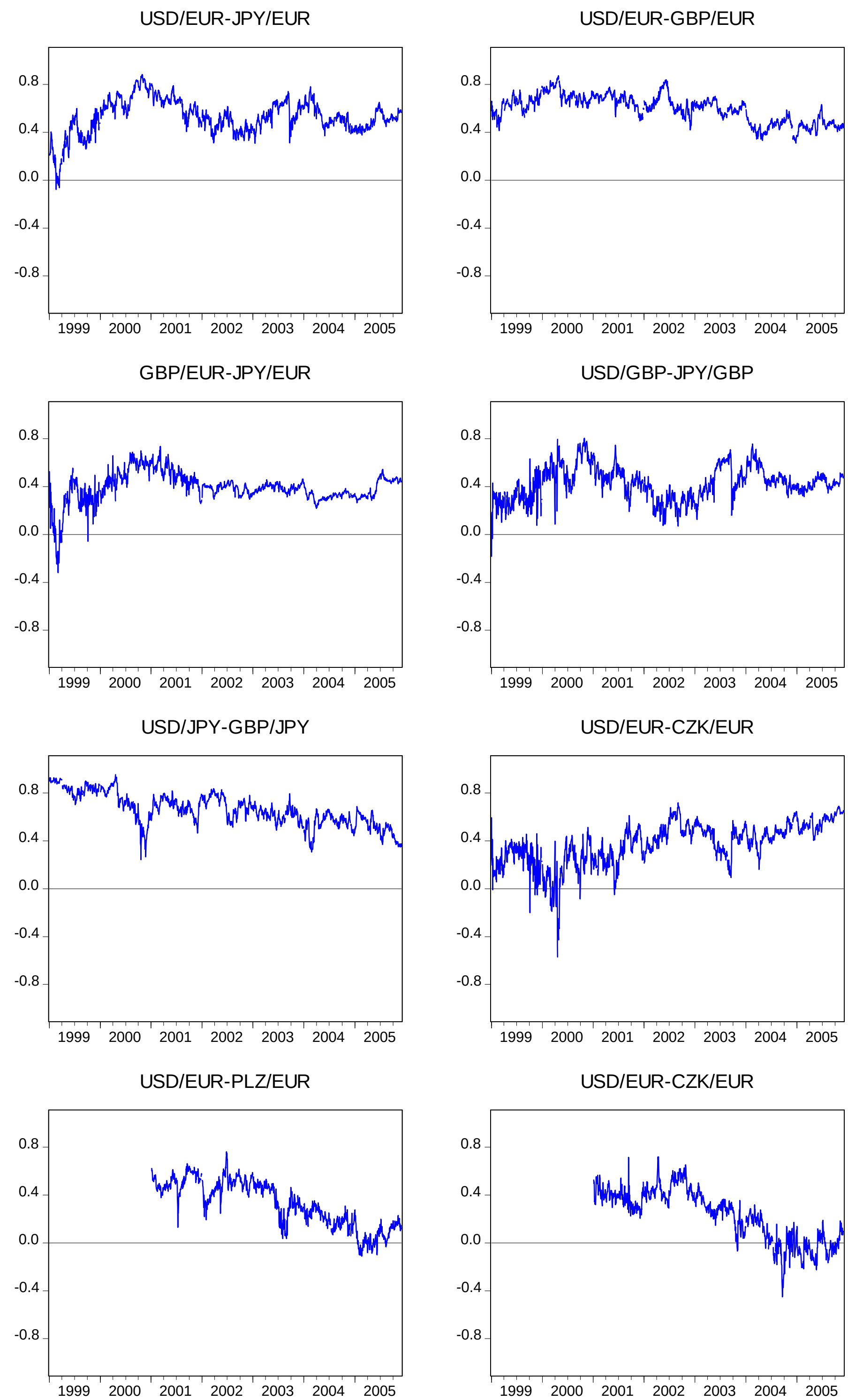


Figure 8. Historical Correlations. January 99 - November 05.

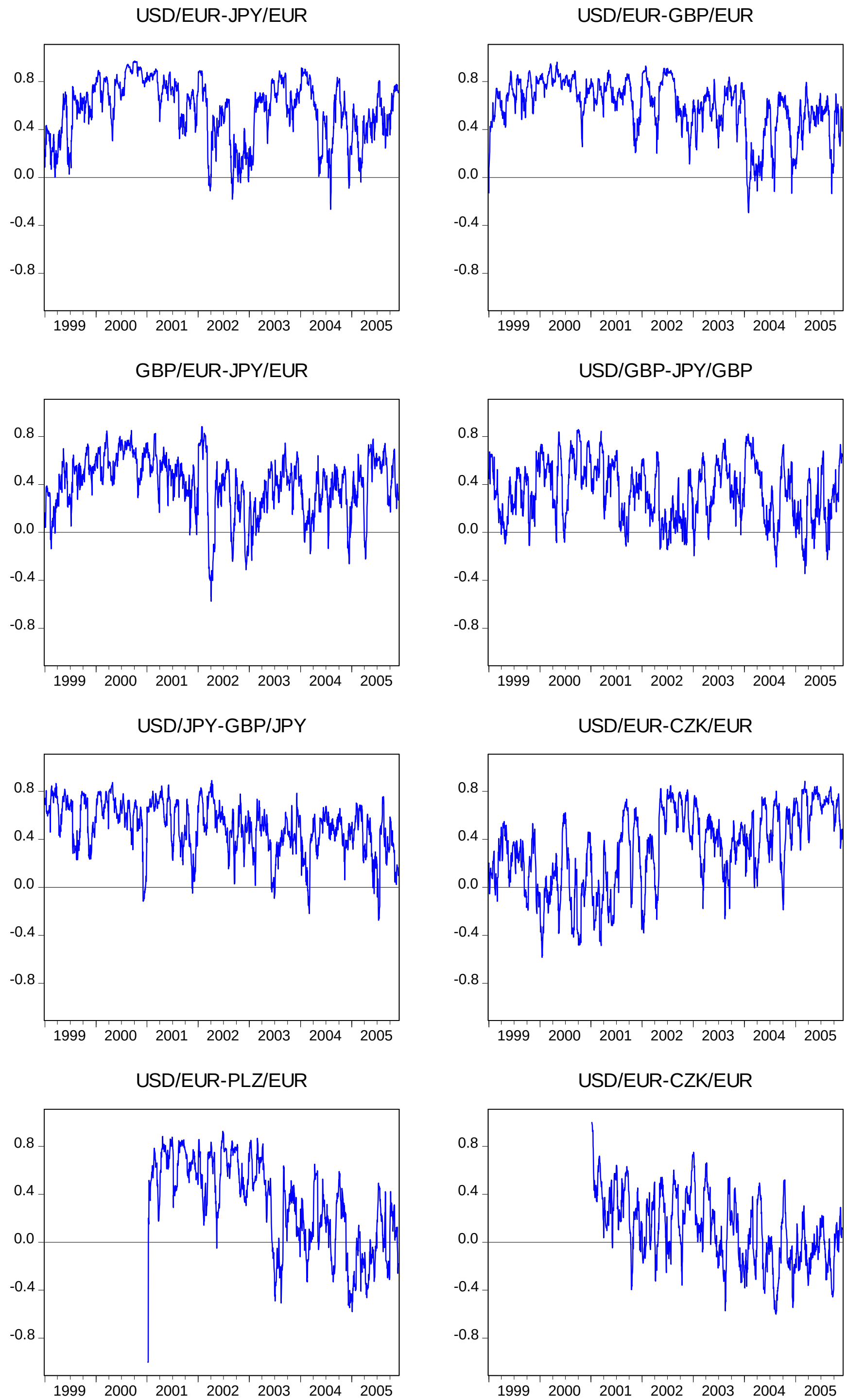


Figure 9. RiskMetrics Correlations. January 99 - November 05.

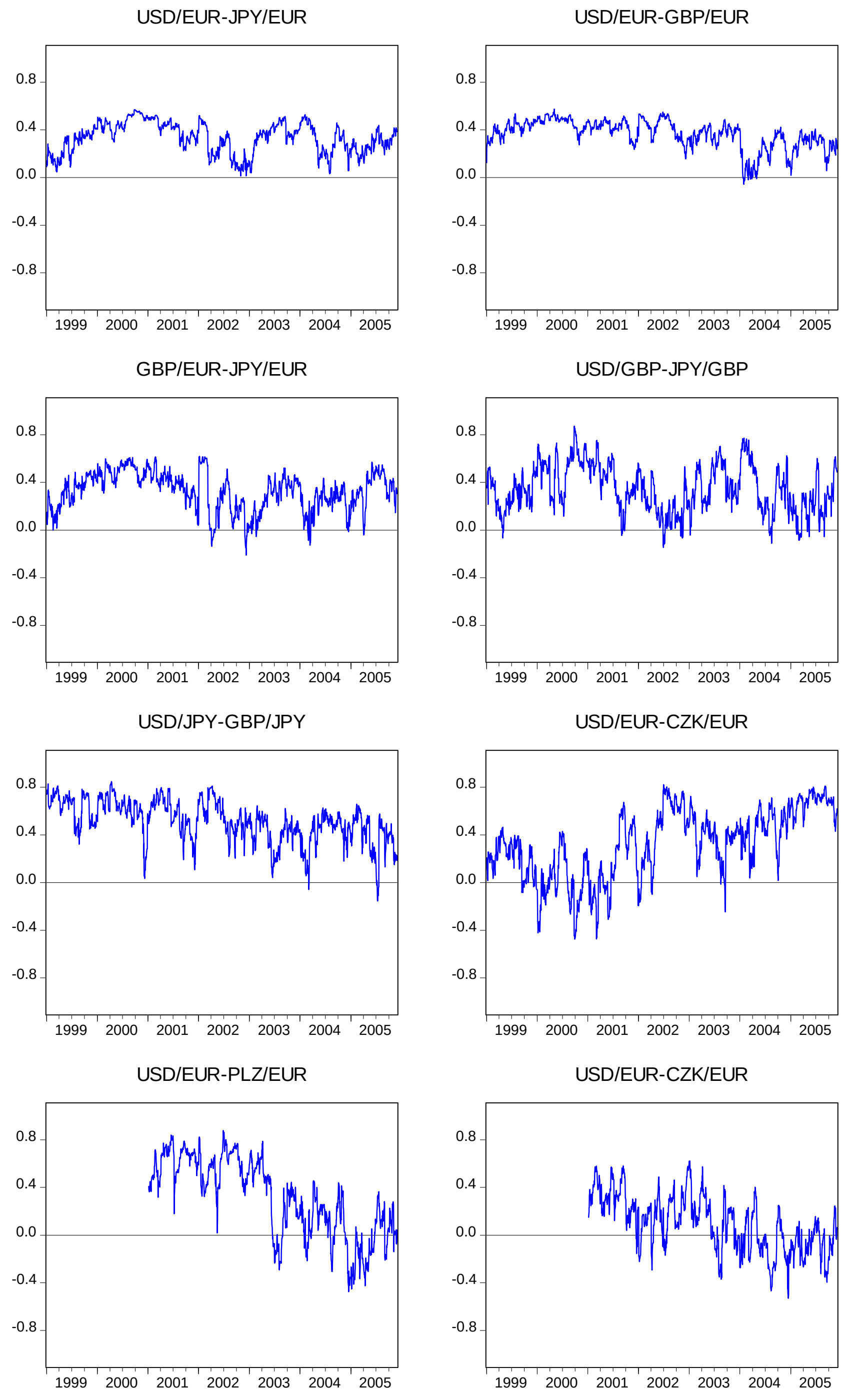
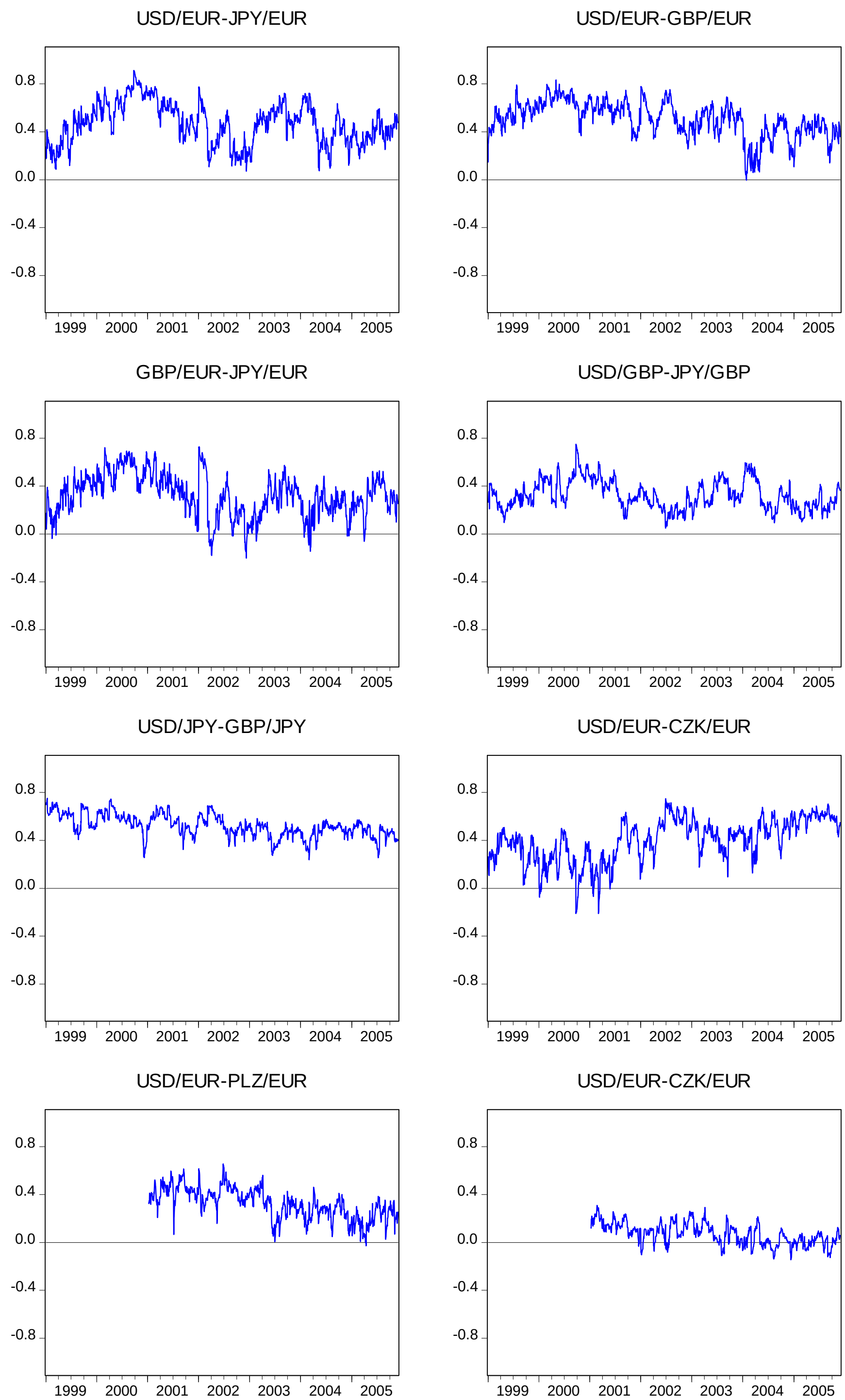


Figure 10. GARCH Correlations. January 99 - November 05.



Appendix 2

The GARCH model implies a non-constant term structure of variance and covariance. To compute the GARCH forecast it is necessary to take into account the mean reverting nature of the process. If the conditional covariance H_t evolves according to a diagonal BEKK-GARCH process specified as

$$H_{t+1} = \Omega\Omega' + \mathrm{B} H_t \mathrm{B}' + \mathrm{A} R_t^{A/C} R_t^{B/C} \mathrm{A}'$$

where

$R_t^{A/C}$ and $R_t^{B/C}$ are log returns on the currencies,

$$H_t \equiv \begin{bmatrix} \sigma_{1,1,t}^2 & \sigma_{1,2,t} \\ \sigma_{1,2,t} & \sigma_{2,2,t}^2 \end{bmatrix},$$

$$A \equiv \begin{bmatrix} a_{1,1} & 0 \\ 0 & a_{2,2} \end{bmatrix},$$

$$B \equiv \begin{bmatrix} b_{1,1} & 0 \\ 0 & b_{2,2} \end{bmatrix},$$

$$\Omega \equiv \begin{bmatrix} \omega_{1,1} & 0 \\ \omega_{1,2} & \omega_{2,2} \end{bmatrix}$$ and, for notational convenience

$$\Omega\Omega', \equiv \begin{bmatrix} \omega_{1,1}^* & \omega_{1,2}^* \\ \omega_{1,2}^* & \omega_{2,2}^* \end{bmatrix},$$

then the next day GARCH correlation is thus defined as

$$\rho\left(R^{A/C}, R^{B/C}\right)_{t+1}^{GARCH} = \frac{\sigma_{1,2,t+1}}{\sigma_{1,1,t+1}\sigma_{2,2,t+1}}.$$

Since we are interested in the correlation over the next h = 21 days, we must take into account the mean reverting nature of the GARCH process over the next 21 days.

The persistence of the variance $P_{A/C}$, $P_{B/C,}$ and persistence of the covariance $P_{A/C,B/C}$ are defined as

$$P_{A/C} = a_{1,1}^2 + b_{1,1}^2$$
$$P_{B/C} = a_{2,2}^2 + b_{2,2}^2, \text{and}$$
$$P_{A/C,B/C} = a_{1,1}a_{2,2} + b_{1,1}b_{2,2}$$

The unconditional variance $V_{A/C}$, $V_{B/C,}$ and the covariance $V_{A/C,B/C}$ can be computed as

$$V_{A/C} = \omega_{1,1}^{*}/\left(1\text{-}\ P_{A/C}\right)$$
$$V_{B/C} = \omega_{2,2}^{*}/\left(1\text{-}\ P_{B/C}\right), \text{ and}$$
$$V_{A/C,B/C} = \omega_{1,2}^{*}/\left(1\text{-}\ P_{A/C,B/C}\right).$$

The GARCH variance over the next h days is then

$$\sigma_{1,1,t+1,h}^2 \;=\; hV_{A/C} \;-\; V_{A/C}\sum_{i=1}^{h} P_{A/C}^{i-1} \;+\; \sigma_{1,1,t+1}^2 \sum_{i=1}^{h} P_{A/C}^{i-1}$$
$$\sigma_{2,2,t+1,h}^2 \;=\; hV_{B/C} \;-\; V_{B/C}\sum_{i=1}^{h} P_{B/C}^{i-1} \;+\; \sigma_{2,2,t+1}^2 \sum_{i=1}^{h} P_{B/C}^{i-1}\ ;$$
$$\sigma_{1,2,t+1,h} \;=\; hV_{A/C,B/C} \;-\; V_{A/C,B/C}\sum_{i=1}^{h} P_{A/C,B/C}^{i-1} \;+\; \sigma_{1,2,t+1} \sum_{i=1}^{h} P_{A/C,B/C}^{i-1}.$$

We estimate the parameters of the model by Quasi-Maximum Likelihood (QML), and obtain the 21-day GARCH correlation forecast as

$$\rho\left(R_{A/C}, R_{B/C}\right)_{t+1,21}^{GARCH} \;=\; \frac{\sigma_{1,2,t+21}}{\sigma_{1,1,t+21}\sigma_{2,2,t+21}} \quad .$$

Table 1. Foreign Exchange Descriptive statistics. April 92 - December 98.

Daily Returns

	USD/DEM	JPY/DEM	GBP/DEM	USD/GBP	JPY/USD	JPY/GBP
Mean	-7.87E-05	-4.18E-05	1.56E-05	-6.89E-05	-3.94E-05	-0.000131
Std. Dev.	0.00712	0.00732	0.00538	0.00581	0.008	0.008
Skewness	-0.111	-0.875	0.654	-0.609	-0.854	-0.478
Kurtosis	5.078	10.335	9.260	7.852	10.957	6.848
Jarque-Bera	304.31	3771.39	2830.16	1743.35	4614.08	1087.94
Observations	1672	1592	1661	1672	1672	1661

Daily Returns Pairwise Correlations

	USD/DEM	JPY/DEM	GBP/DEM	USD/GBP	JPY/USD
JPY/DEM	0.302				
GBP/DEM	0.348	0.087			
USD/GBP	0.622	0.223	-0.250		
JPY/USD	-0.467	0.228	-0.203	-0.310	
JPY/GBP	-0.030	0.271	-0.463	0.314	0.626

Table 2. Foreign Exchange Descriptive statistics. January 99 - November 05.

Daily Returns

	USD/EUR	JPY/EUR	GBP/EUR	USD/GBP	JPY/USD	JPY/GBP	CZK/EUR	PLZ/EUR	CZK/USD	PLZ/USD
Mean	-0.000021	-1.03E-05	-3.57E-05	1.47E-05	1.09E-05	1.11E-05	-1.35E-04	-1.88E-05	-0.000277	-0.00016
Std. Dev.	0.00656	0.00737	0.00459	0.005	0.006	0.007	0.00352	0.006	0.007	0.007
Skewness	0.247	-0.003	0.311	0.167	-0.121	-0.057	-0.083	0.685	-0.045	0.431
Kurtosis	4.284	6.569	4.369	4.129	4.741	5.037	5.362	7.364	3.576	5.308
Jarque-Bera	138.38	931.67	165.23	101.33	226.07	296.09	290.27	1,082.89	17.56	313.96
Observations	1755	1755	1755	1755	1755	1708	1242	1242	1242	1242

Daily Returns Pairwise Correlations

	USD/EUR	JPY/EUR	GBP/EUR	USD/GBP	JPY/USD	JPY/GBP	CZK/EUR	PLZ/EUR	CZK/USD
JPY/EUR	0.613								
GBP/EUR	0.622	0.452							
USD/GBP	0.717	0.377	-0.097						
JPY/USD	-0.331	0.543	-0.121	-0.312					
JPY/GBP	0.186	0.617	-0.146	0.369	0.540				
CZK/EUR	0.110	0.079	0.138	0.026	-0.033	-0.026			
PLZ/EUR	0.349	0.221	0.272	0.210	-0.137	0.034	0.261		
CZK/USD	-0.856	-0.475	-0.435	-0.692	0.407	-0.139	0.420	-0.183	
PLZ/USD	-0.568	-0.300	-0.246	-0.491	0.286	-0.090	0.133	0.574	0.587

Foreign Exchange Correlations Measures: Descriptive statistics.

Apr 92 -Nov 05

USD/EUR - JPY/EUR

Implied	Historical	RiskMetrics	GARCH
0.488	0.414	0.372	0.268
0.170	0.288	0.188	0.156
-0.359	-0.254	0.096	-0.185
3.516	2.358	2.835	3.325
113.22	99.67	9.52	36.01
3479	3567	3566	3567

USD/EUR - GBP/EUR

Implied	Historical	RiskMetrics	GARCH
0.562	0.457	0.421	0.352
0.159	0.276	0.176	0.177
-0.666	-0.583	-0.371	-0.101
2.914	2.773	2.831	3.447
257.68	209.61	86.00	35.75
3475	3567	3566	3567

GBP/EUR - JPY/EUR

Implied	Historical	RiskMetrics	GARCH
0.304	0.236	0.201	0.206
0.160	0.297	0.205	0.217
-0.506	-0.338	0.089	-0.248
3.618	2.537	2.635	2.605
204.11	99.89	24.50	59.86
3483	3567	3566	3567

USD/JPY - GBP/JPY

Implied	Historical	RiskMetrics	GARCH
0.638	0.518	0.536	0.534
0.201	0.246	0.123	0.209
-1.432	-1.029	-0.337	-0.867
5.924	4.393	3.761	3.990
2430.01	917.79	153.35	592.72
3481	3567	3566	3567

Apr 99 - Nov 05

USD/EUR - JPY/EUR

	Implied	Historical	RiskMetrics	GARCH
Mean	0.544	0.570	0.477	0.339
Std. Dev.	0.125	0.247	0.168	0.125
Skewness	0.125	-0.721	-0.037	-0.417
Kurtosis	2.869	2.845	2.400	2.374
Jarque-Bera	5.71	152.62	26.50	78.94
Observations	1710	1741	1740	1741

USD/EUR - GBP/EUR

	Implied	Historical	RiskMetrics	GARCH
Mean	0.597	0.593	0.506	0.364
Std. Dev.	0.114	0.226	0.149	0.122
Skewness	-0.262	-1.047	-0.618	-1.016
Kurtosis	2.301	3.851	3.216	3.723
Jarque-Bera	54.39	370.34	114.04	337.59
Observations	1709	1741	1740	1741

GBP/EUR - JPY/EUR

	Implied	Historical	RiskMetrics	GARCH
Mean	0.404	0.412	0.328	0.331
Std. Dev.	0.103	0.241	0.176	0.161
Skewness	0.365	-0.934	-0.223	-0.598
Kurtosis	3.749	3.928	2.741	2.875
Jarque-Bera	78.25	315.88	19.33	104.93
Observations	1715	1741	1740	1741

USD/JPY - GBP/JPY

	Implied	Historical	RiskMetrics	GARCH
Mean	0.648	0.494	0.516	0.507
Std. Dev.	0.128	0.209	0.090	0.171
Skewness	-0.174	-0.735	-0.141	-0.632
Kurtosis	2.557	3.432	2.952	3.376
Jarque-Bera	22.71	170.46	5.95	125.97
Observations	1716	1741	1740	1741

USD/GBP - JPY/GBP

Implied	Historical	RiskMetrics	GARCH
0.439	0.300	0.299	0.305
0.170	0.296	0.167	0.252
0.064	-0.367	-0.174	-0.295
3.247	2.668	4.128	2.972
11.25	96.56	207.10	51.93
3481	3567	3566	3567

JPY/USD - GBP/USD

Implied	Historical	RiskMetrics	GARCH
0.374	0.334	0.403	0.336
0.164	0.313	0.168	0.274
-0.370	-0.476	-0.501	-0.409
3.777	2.678	3.141	2.736
166.74	150.12	151.95	109.67
3481	3567	3566	3567

USD/GBP - JPY/GBP

	Implied	Historical	RiskMetrics	GARCH
Mean	0.434	0.335	0.324	0.343
Std. Dev.	0.135	0.247	0.122	0.202
Skewness	0.076	-0.102	0.437	0.073
Kurtosis	2.853	2.181	2.661	2.250
Jarque-Bera	3.20	51.65	63.79	42.35
Observations	1716	1741	1740	1741

JPY/USD - GBP/USD

	Implied	Historical	RiskMetrics	GARCH
Mean	0.381	0.330	0.404	0.330
Std. Dev.	0.173	0.329	0.178	0.293
Skewness	-0.934	-0.507	-0.614	-0.442
Kurtosis	4.616	2.383	2.928	2.400
Jarque-Bera	436.26	102.30	109.77	82.76
Observations	1716	1741	1740	1741

USD/EUR - PLZ/EUR

	Implied	Historical	RiskMetrics	GARCH
Mean	0.327	0.316	0.323	0.312
Std. Dev.	0.192	0.372	0.126	0.328
Skewness	-0.281	-0.445	-0.193	-0.370
Kurtosis	1.962	2.265	2.560	2.056
Jarque-Bera	73.44	71.00	18.24	76.70
Observations	1264	1278	1276	1279

USD/EUR - CZK/EUR

	Implied	Historical	RiskMetrics	GARCH
Mean	0.231	0.110	0.069	0.094
Std. Dev.	0.216	0.287	0.089	0.232
Skewness	-0.356	0.080	0.095	-0.030
Kurtosis	2.246	2.535	2.416	2.385
Jarque-Bera	56.60	12.87	20.06	20.35
Observations	1264	1278	1276	1279

Table 4. Correlation Predicatability Regressions. April 92 - November 05. (t-stats below).

USD/EUR-JPY/EUR

Intercept	IC	HC	RM	GARCH	Adj-rbar2
0.042	0.761				0.202
0.830	**7.424**				
0.192		0.536			0.285
7.502		**10.628**			
0.186			0.850		0.212
6.285			**7.991**		
0.098				0.847	0.306
3.441				**13.060**	
0.054	0.388	0.411			0.321
1.344	**4.113**	**7.355**			
0.029	0.477		0.565		0.267
0.682	**4.644**		**5.100**		
0.018	0.289			0.681	0.323
0.489	**3.007**			**8.103**	
0.041	0.311	0.310	-0.555	0.646	0.342
1.096	**3.297**	**3.147**	**-3.352**	**3.966**	

USD/EUR-GBP/EUR

Intercept	IC	HC	RM	GARCH	Adj-rbar2
0.032	0.760				0.192
0.466	**6.340**				
0.219		0.522			0.272
7.070		**9.478**			
0.264			0.551		0.125
6.302			**4.978**		
0.093				0.866	0.307
2.485				**11.385**	
0.061	0.378	0.404			0.306
1.016	**3.258**	**7.143**			
0.038	0.613		0.218		0.204
0.575	**4.903**		**2.006**		
-0.004	0.292			0.710	0.326
-0.077	**2.480**			**8.392**	
-0.015	0.413	0.133	-0.487	0.834	0.359
-0.255	**3.438**	1.259	**-3.560**	**5.075**	

GBP/EUR-JPY/EUR

Intercept	IC	HC	RM	GARCH	Adj-rbar2
0.027	0.692				0.140
0.712	**6.230**				
0.154		0.348			0.121
7.613		**6.108**			
0.145			0.444		0.105
6.731			**5.364**		
0.118				0.588	0.165
5.690				**8.225**	
0.033	0.500	0.224			0.179
0.946	**4.547**	**3.841**			
0.023	0.525		0.266		0.169
0.665	**4.683**		**3.166**		
0.026	0.415			0.422	0.203
0.809	**3.917**			**5.279**	
0.031	0.410	-0.033	-0.410	0.866	0.218
0.962	**4.027**	-0.372	**-3.250**	**5.387**	

USD/GBP-JPY/GBP

Intercept	IC	HC	RM	GARCH	Adj-rbar2
-0.032	0.757				0.192
-0.664	**7.608**				
0.210		0.302			0.091
7.517		**4.741**			
0.166			0.443		0.143
5.495			**6.249**		
0.085				0.720	0.166
2.266				**7.193**	
-0.022	0.671	0.092			0.198
-0.458	**6.060**	1.489			
-0.010	0.575		0.190		0.207
-0.211	**4.880**		**2.399**		
-0.034	0.519			0.357	0.214
-0.736	**4.130**			**2.841**	
-0.057	0.487	-0.165	-0.029	0.676	0.220
-1.208	**3.812**	-1.751	-0.226	**2.591**	

USD/JPY-GBP/JPY

Intercept	IC	HC	RM	GARCH	Adj-rbar2
0.067	0.703				0.326
1.094	**8.027**				
0.316		0.389			0.149
7.370		**5.542**			
0.229			0.539		0.207
4.496			**6.430**		
0.014				0.940	0.220
0.171				**6.596**	
0.067	0.695	0.010			0.326
1.095	**6.518**	0.136			
0.065	0.653		0.064		0.327
1.056	**5.674**		0.665		
0.027	0.624			0.169	0.329
0.365	**5.781**			1.079	
-0.036	0.618	-0.142	-0.003	0.435	0.332
-0.375	**5.474**	-1.176	-0.021	1.586	

JPY/USD-GBP/USD

Intercept	IC	HC	RM	GARCH	Adj-rbar2
0.009	0.871				0.208
0.217	**9.231**				
0.197		0.406			0.165
7.256		**7.764**			
0.144			0.564		0.244
5.313			**10.198**		
-0.002				0.830	0.200
-0.048				**9.641**	
0.028	0.636	0.206			0.235
0.704	**6.026**	**3.294**			
0.037	0.450		0.384		0.273
0.995	**4.395**		**5.588**		
-0.076	0.567			0.493	0.253
-1.764	**5.542**			**4.999**	
0.038	0.439	-0.313	0.730	-0.020	0.287
0.601	**4.405**	**-3.252**	**4.101**	-0.070	

Table 4 reports regression point estimates as well as robust t-stats. Since the forecasts are overlapping, to correct the standard errors for heteroskedasticity and autocorrelation, the GMM with the robust Newey-West weighting matrix and a pre-specified bandwidth equal to 21 days is used. The regression fit is reported using the adjusted R2.

Table 5. Correlation Predicatability Regressions. April 99 - November 05. (t-stats below).

USD/EUR-JPY/EUR

Intercept	IC	HC	RM	GARCH	Adj-rbar2
0.007	1.041				0.281
0.073	**6.986**				
0.290		0.500			0.252
6.423		**7.283**			
0.221			1.041		0.283
4.125			**7.383**		
0.200				0.784	0.288
3.824				**8.604**	
0.045	0.699	0.260			0.319
0.544	**3.803**	**2.871**			
0.045	0.605		0.586		0.322
0.554	**2.819**		**2.560**		
0.043	0.573			0.457	0.322
0.549	**2.877**			**3.136**	
0.048	0.574	0.067	0.213	0.216	0.324
0.598	**2.778**	0.587	0.461	0.763	

USD/EUR-GBP/EUR

Intercept	IC	HC	RM	GARCH	Adj-rbar2
-0.085	1.133				0.321
-0.826	**7.303**				
0.290		0.509			0.257
5.550		**6.762**			
0.226			1.004		0.290
3.923			**7.330**		
0.179				0.816	0.287
2.786				**7.542**	
-0.031	0.821	0.224			0.347
-0.346	**5.061**	**2.467**			
-0.018	0.739		0.463		0.344
-0.205	**4.138**		**2.273**		
-0.049	0.744			0.388	0.349
-0.546	**4.299**			**2.641**	
-0.039	0.745	0.097	-0.007	0.258	0.350
-0.436	**4.196**	0.604	-0.019	0.853	

GBP/EUR-JPY/EUR

Intercept	IC	HC	RM	GARCH	Adj-rbar2
0.194	0.539				0.052
2.441	**3.158**				
0.293		0.290			0.084
7.745		**3.649**			
0.255			0.477		0.101
6.089			**3.966**		
0.248				0.500	0.132
6.275				**5.151**	
0.183	0.328	0.234			0.099
2.633	1.983	**2.768**			
0.180	0.241		0.405		0.109
2.671	1.362		**2.911**		
0.195	0.167			0.454	0.137
3.073	1.076			**4.167**	
0.207	0.158	-0.127	-0.157	0.745	0.142
3.430	1.009	-0.940	-0.415	**2.446**	

USD/GBP-JPY/GBP

Intercept	IC	HC	RM	GARCH	Adj-rbar2
0.044	0.675				0.139
0.766	**5.417**				
0.259		0.237			0.056
7.699		**2.987**			
0.216			0.358		0.086
5.588			**3.774**		
0.139				0.615	0.093
2.570				**4.042**	
0.049	0.618	0.060			0.141
0.862	**4.524**	0.746			
0.052	0.563		0.119		0.144
0.926	**3.766**		1.098		
0.032	0.550			0.206	0.144
0.532	**3.419**			1.079	
0.035	0.536	-0.070	0.104	0.176	0.145
0.526	**3.277**	-0.618	0.659	0.454	

USD/JPY-GBP/JPY

Intercept	IC	HC	RM	GARCH	Adj-rbar2
-0.011	0.770				0.209
-0.118	**5.699**				
0.404		0.178			0.030
7.744		1.944			
0.319			0.340		0.075
5.108			**3.063**		
0.135				0.691	0.086
1.293				**3.599**	
-0.022	0.873	-0.113			0.217
-0.238	**5.597**	-1.156			
-0.018	0.843		-0.080		0.211
-0.196	**4.788**		-0.574		
0.014	0.834			-0.129	0.210
0.126	**4.479**			-0.482	
-0.100	0.773	-0.259	0.054	0.363	0.222
-0.764	**4.088**	**-2.009**	0.279	0.762	

JPY/USD-GBP/USD

Intercept	IC	HC	RM	GARCH	Adj-rbar2
-0.065	1.050				0.298
-0.975	**7.253**				
0.169		0.488			0.237
4.283		**6.854**			
0.123			0.629		0.310
3.123			**8.450**		
-0.072				0.996	0.290
-1.257				**8.494**	
-0.030	0.779	0.204			0.319
-0.499	**4.949**	**2.423**			
-0.005	0.571		0.367		0.342
-0.088	**3.605**		**3.802**		
-0.126	0.637			0.539	0.337
-2.014	**4.376**			**4.030**	
-0.064	0.534	-0.286	0.512	0.297	0.350
-0.774	**3.515**	-1.777	**2.610**	0.958	

USD/EUR-PLZ/EUR

Intercept	IC	HC	RM	GARCH	Adj-rbar2
-0.130	1.365				0.506
-2.406	**11.262**				
0.121		0.608			0.373
2.805		**8.935**			
0.079			0.751		0.445
1.858			**9.976**		
-0.277				1.826	0.387
-3.856				**9.920**	
-0.105	1.144	0.149			0.514
-2.183	**6.011**	1.270			
-0.092	1.022		0.234		0.517
-1.817	**4.055**		1.397		
-0.214	1.100			0.525	0.518
-2.578	**5.309**			1.494	
-0.172	1.026	-0.015	0.119	0.369	0.519
-1.829	**4.183**	-0.125	0.629	0.852	

USD/EUR-CZK/EUR

Intercept	IC	HC	RM	GARCH	Adj-rbar2
-0.051	0.666				0.270
-1.206	**5.064**				
0.076		0.230			0.056
2.327		**2.615**			
0.073			0.309		0.066
2.231			**2.751**		
0.050				0.735	0.055
1.340				**2.527**	
-0.051	0.675	-0.017			0.268
-1.210	**4.988**	-0.239			
-0.054	0.704		-0.064		0.271
-1.265	**5.192**		-0.697		
-0.047	0.698			-0.161	0.269
-1.109	**5.070**			-0.694	
-0.046	0.704	0.098	-0.084	-0.243	0.270
-1.117	**5.259**	0.946	-0.388	-0.466	

Table 5 reports regression point estimates and robust t-stats. Since the forecasts are overlapping, to correct the standard errors for heteroskedasticity and autocorrelation, the GMM with the robust Newey-West weighting matrix and a pre-specified bandwidth equal to 21 days is used. The regression fit is reported using the adjusted R2. It reports the same regressions as table 4, but now using the sample period beginning from April 1999 and ending in November 2005. It also includes the results for the pairs USD/EUR-PLZ/EUR, and USD/EUR-CZK/EUR.

Table 6. Mincer Zarnowitz Decomposition of MSE in Percentage.

Sample: Apr 92 - Nov 05

USD/EUR-JPY/EUR	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.073	7.521	2.259	90.221
Historical	0.077	0.001	23.073	76.926
RiskMetrics	0.087	24.287	0.627	75.086
GARCH	0.060	2.793	1.376	95.831

USD/EUR-GBP/EUR	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.073	14.588	1.980	83.433
Historical	0.073	0.000	23.912	76.088
RiskMetrics	0.084	13.190	7.510	79.301
GARCH	0.055	2.419	1.034	96.548

GBP/EUR-JPY/EUR	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.082	5.289	2.945	91.766
Historical	0.115	0.001	32.570	67.429
RiskMetrics	0.095	1.041	15.407	83.552
GARCH	0.082	1.545	8.732	89.723

USD/GBP-JPY/GBP	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.091	21.096	1.896	77.008
Historical	0.122	0.003	34.882	65.115
RiskMetrics	0.095	0.013	20.872	79.115
GARCH	0.075	0.004	2.910	97.087

USD/JPY-GBP/JPY	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.060	25.209	5.965	68.826
Historical	0.074	0.005	30.241	69.754
RiskMetrics	0.058	0.535	15.949	83.517
GARCH	0.048	0.700	0.115	99.185

JPY/USD - GBP/USD	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.079	1.889	0.560	97.551
Historical	0.116	0.001	29.793	70.206
RiskMetrics	0.088	0.006	16.166	83.827
GARCH	0.084	5.856	0.970	93.174

Table 6 reports the MSE's in absolute value and their decomposition into bias squared, inefficiency, and residual variation, in percentage of the total MSE for each correlation forecast for the entire sample period. The lower the MSE, the better the forecast is. For a given MSE value, the higher the residual variation, the more efficient and unbiased the forecast is.

Table 7. Mincer Zarnowitz Decomposition of MSE in Percentage.

Sample: Apr 99 - Nov 05

USD/EUR-JPY/EUR	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.044	1.908	0.060	98.033
Historical	0.061	0.052	25.335	74.612
RiskMetrics	0.099	55.941	0.027	44.032
GARCH	0.054	17.349	2.484	80.167

USD/EUR-GBP/EUR	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.035	0.090	0.649	99.259
Historical	0.051	0.003	24.323	75.673
RiskMetrics	0.089	58.690	0.000	41.310
GARCH	0.045	16.526	1.676	81.798

GBP/EUR-JPY/EUR	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.058	0.120	3.932	95.947
Historical	0.083	0.000	35.456	64.544
RiskMetrics	0.067	10.025	10.773	79.202
GARCH	0.066	10.687	11.850	77.463

USD/GBP-JPY/GBP	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.063	14.890	3.089	82.021
Historical	0.093	0.042	38.158	61.801
RiskMetrics	0.072	0.018	23.370	76.613
GARCH	0.057	0.361	3.879	95.761

USD/JPY-GBP/JPY	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.061	42.114	1.338	56.546
Historical	0.071	0.041	40.240	59.720
RiskMetrics	0.053	0.578	23.395	76.029
GARCH	0.042	1.498	1.837	96.663

JPY/USD - GBP/USD	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.078	2.712	0.095	97.193
Historical	0.112	0.001	25.510	74.489
RiskMetrics	0.087	0.003	13.577	86.421
GARCH	0.083	6.518	0.001	93.481

USD/EUR-PLZ/EUR	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.072	0.145	6.807	93.048
Historical	0.107	0.019	19.901	80.079
RiskMetrics	0.083	0.000	8.104	91.896
GARCH	0.095	0.079	11.437	88.483

USD/EUR-CZK/EUR	MSE	$Bias^2$	Inefficiency	Residual
Implied	0.078	21.112	6.723	72.165
Historical	0.123	0.060	40.166	59.774
RiskMetrics	0.099	0.063	26.398	73.540
GARCH	0.075	1.356	0.750	97.896

Table 7 reports the MSE's in absolute value and their decomposition into bias squared, inefficiency, and residual variation, in percentage of the total MSE for each correlation forecast for the post 1999 sample. The lower the MSE, the better the forecast is. For a given MSE value, the higher the residual variation, the more efficient and unbiased the forecast is.

Footnotes

[1] Statistics from the BIS indicate that as of December 2004 the daily average transaction volume of OTC foreign exchange options denominated in PLZ was 260 millions of US dollars. The similar figure for CKZ was 98 millions of US dollars. Data from the National Bank of Poland indicate that In the Polish domestic market, 43% of the contracts were EUR/PLZ denominated and 43% were USD/PLZ denominated contracts. These figures compare with 92,276, 51,085, 37,430, and 11,645 millions of US dollars for the USD, EUR, JPY, and GBP OTC option markets respectively.

[2] See e.g. Driessen, et al (2006)

[3] See e.g. Beckers (1981), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Jorion (1995) Christensen and Prabhala (1998), Fleming (1998), Blair et al. (2001), Neely (2003), Pong, et al. (2004), Covrig and Low (2003), and Christoffersen and Mazzotta (2004).

[4] However, there exists a more generous literature in correlations among stock and bond markets. Good reviews of such studies are provided Kroner and Ng (1998) and Cappiello et al. (2006).

[5] For the institutional features of the OTC currency option market see e.g. Malz (1997).

[6] See e.g. Carr and Wu (2006).

[7] The Bank for International Settlements (2005) estimates that on December 31, 2004, the notional amount outstanding in the global OTC derivatives market was US$248,288 billion. This compares to the notional amount outstanding in globally exchange-traded contracts, which was US$46,592 billion on the same date.

[8] See e.g. Malz (1997), Butler and Cooper (1997) and Brandt and Diebold (2003) for further details.

[9] See also Bandi and Perron (2007), Chernov (2007), and Bates (2002).

[10] See Engle and Kroner (1995) for further details.

[11] The GARCH model contains parameters that must be estimated. We do this using the quasi-maximum-likelihood estimator (QMLE) over the entire sample.

[12] See e.g. Fleming et al (1995)

[13] The repeated attempts by Japanese authorities in the 1990s and in the early 2000s to prevent excessive yen appreciation against the US dollar were often quoted by market participants as a potential factor affecting the entire G3 exchange rate dynamics. See e.g. Castrén and Mazzotta (2005), Castrén (2004), and Ito (2002) for a thorough analysis of the Japanese central bank intervention and related issues.

[14] For the technicalities regarding the GMM implementation, we refer the reader to Christoffersen and Mazzotta (2005).

[15] See for instance Driessen et al. (2006) and Krisnan et al. (2007) who show that for stocks, correlation risk can command a risk premium on its own right.

[16] See e.g. Castrén (2004) and Ito (2002).

[17] The low bias shown in table 6 and 7 also indirectly suggest the absence of a sizable liquidity premium.