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The Consequences of Information Revealed in Auctions

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Abstract

This paper considers the ramifications of post-auction competition on bidding behavior under different bid announcement policies. In equilibrium, the auctioneer's announcement policy has two distinct effects. First, announcement entices players to signal information to their post-auction competitors through their bids. Second, announcement can lead to greater bidder participation in certain instances while limiting participation in others. Specifically, the participation effect works against the signalling effect, thus reducing the impact of signalling found in other papers. Revenue, efficiency, and surplus implications of various announcement policies are examined.

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1 Introduction

Auctions are often a precursor to market competition. Examples include auctions of timber tracts, oil leases, and PCS spectrum rights and the corresponding lumber, petroleum, and telecommunications markets.[1] In these situations, a number of authors have pointed out that information contained in bids can be used in subsequent competition, should those bids be announced. For instance, Rothkopf et al. (1990) and Engelbrecht-Wiggans and Kahn (1991) argue that third parties may be able to use bid information to extract surplus from the winning bidder. They point out that surplus extraction is only possible if the winner's bid is available, thus helping explain why so many English auctions are often observed while there are relatively few second price auctions. However, only recently has auction theory considered that bidders may try to alter their bids to manipulate the information therein contained. This paper examines the link between post-auction competition and bids at auction with an eye towards how announcements affect bids, bidder participation, revenues, and post-auction welfare.

The actual "commodity" being auctioned can take on a number of interpretations. Most simply, it may be the right to participate in a post-auction duopoly with an incumbent monopoly (i.e. only the winner of the auction moves on to the post-auction game). For example, entrant firms bid for the right to compete against incumbent firms in the recent Turkish spectrum auctions. Alternatively, it could be viewed as in Das Varma (2003) where it is access to a process innovation that enhances the winner's competitive position in the post-auction market (i.e. all bidders move on, but only the winner obtains the innovation). No matter what the interpretation, bids are based on their expectation of post-auction payoffs, conditional on a bidder's private type c.[2] When the auctioneer announces some subset β of all bids submitted, the winner's post-auction profits will depend on rival beliefs concerning the winner's type, $G(\beta)$, which are based on the auctioneer's announcement. In general, this implies a payoff function of $\pi(G(\beta), c)$ in the post-auction game for the winner. Since bids are based on post-auction profits, and these profits depend on the auctioneer's announcement, it

[1] See Baldwin et al. (1997), Hendricks and Porter (1988), and Cramton et al. (1997) respectively for discussions concerning auctions of these commodities.

[2] We interpret a bidder's type as a marginal cost draw. For example, when bidding on a product innovation, the draw represents the bidder's new marginal cost if they win the auction. In all of our applications a higher type is worse than a lower type.

follows that the auctioneer can influence bidding through its choice of which bids to announce.

We show that the impact of the auctioneer's announcement can be broken down into separate bid *signalling* and bidder *participation* effects. This differs from Goeree (2003) and Das Varma(2003) who also identify a signalling effect,[3] but do not endogenize bidder participation and extends Goeree (2003) to the case where bidder beliefs are positively related to post-auction profits. Like their papers, we show that bid signalling stems from the fact that in a pure strategy separating equilibrium, bids are a monotone function of a bidder's private type. Thus, an announcement of a one's bid is tantamount to announcing their type. Realizing that their type can be inferred from their bid, bidders will try to manipulate the information conveyed to post-auction rivals by skewing their bids. Hence, the auctioneer can elicit (or nullify) the signalling effect by announcing (or not announcing) the winner's bid. On the other hand, the participation effect results from imposing individual rationality on potential bidders. That is, of all bidders eligible to compete in an auction, only those with non-negative expected payoffs in the post-auction game will submit bids. For a given announcement policy, the cut-off type bidder (just indifferent between entering and not entering the auction) expects zero profit in the post-auction game. The fact that different announcement policies impart different information structures on the post-auction game leads to distinctly different cut-off types and, hence; varied levels of competition at auction.

The specific impacts of bid signalling and bidder participation on auction revenues depend on the relation between profits and rival beliefs in the post-auction game. When profits and beliefs are positively related ($\pi_1(G(\beta), c) > 0$), bidders earn higher profits if they can convey a weaker type than they are in actuality.[4] Thus, when signalling is possible, systematically lower bids are submitted than if signalling is not possible. The result is a negative effect on revenues relative to a policy of not announcing the winner's bid. When profits and beliefs are inversely related ($\pi_1(G(\beta), c) < 0$), the effect is opposite and announcing the winner's bid is revenue enhancing. In fact, for certain distributions of type (see Section 2: Condition 1), the temptation to under-bid is so strong that bids are degenerately low and a separating

[3] It should be pointed out that a number of other authors have examined the possibility of bid signalling and highlighted the fact that it may help explain revenue dominance of one type of auction over another. They include Biglaiser and Mezzetti (2000), Haile (2001, 2003), Laffont and Tirole (1988), and Ortega-Reichert (1968).

[4] Of course in equilibrium, types can be inferred by inverting bids and post-auction competitors are not deceived by the skewing of bids.

equilibrium in monotone bid strategies ceases to exist. This phenomenon is similar to that derived by Jehiel and Moldavanu (1995a, 1995b) in bargaining models where externalities exist between bidder valuations and Haile (2003) where resale is possible following the auction.

Adding to the findings of Goeree (2003) and Das Varma (2003), we show that the revenue enhancing policies based on the signalling effect are counteracted by the participation effect. When $\pi_1(G(\beta), c) < 0$ ($\pi_1(G(\beta), c) > 0$), fewer types participate in the auction when the winner's bid is (is not) announced. Intuitively, when the winning bidder is of the cut-off type and their bid is not announced, rival firms infer that the winner's type is better (e.g. lower marginal cost) than it actually is. If it is more desirable that the rival believes that the winner has a low type (as when $\pi_1(G(\beta), c) < 0$) it follows that more types will participate when the winner's bid is not announced. If it is more desirable that rivals believe that the winner has a high type (as when $\pi_1(G(\beta), c) > 0$), fewer types participate under a policy of announcing the winner's bid. Clearly, lower levels of competition an the auction result in less expected revenue.

Thus far we have focused on policies of announcing or not announcing the winner's bid, independent of the specific auction form used. Fortunately, the number of auction forms requiring examination is greatly simplified by noticing that Myerson's (1981) seminal work on revenue equivalent auctions can be extended to any auction where the seller's expected revenue is wholly determined by the allocation, the expected payoff of the worst type bidder, and the information released by the auction. That is, any auctions satisfying Myerson's original conditions will generate identical expected revenues *only* when followed by the same announcement policy. For example, if the auctioneer releases the highest of all bids, then the second price and English auctions will generate different expected revenue. However, if the auctioneer only announces the price paid then the second price and English auction will generate the same expected revenue. Because of this, we can restrict focus to two policies, one in which the winner's bid is announced, the other in which the winner's bid is not announced, but the second highest bid is.[5] Since the actual auction form is irrelevant, we restrict attention to the commonly studied first price, second price, and English (button) auctions.

The main difference between first and second price auctions and an English auction lies in

[5] Of course, there are policies that release other bid information. However, in our model, the winner's type is the information relevant in the post-auction game, thus the highest bid announced provides a sufficient statistic for the winner's type and announcement of all lower bids is superfluous.

the amount of information collected during the auction process. Since bidding in our English (button) auction stops when the second to last bidder drops out, the winner's maximum bid is not observed. The auctioneer thereby limits the information available for announcement by holding an English auction. Clearly, when the auctioneer wishes to take advantage of the signalling effect by announcing the winner's bid, this makes English auctions sub-optimal. When a policy of not announcing the winner's bid is optimal, the Revenue Equivalence Theorem tells us that by withholding the winner's bid, the auctioneer earns the same expected revenue using first and second price auctions as they do using English auctions. However, we point out that English auctions have the added benefit that they give the auctioneer a way of credibly committing to the non-announcement policy.[6] Thus, providing yet another explanation of why so many English auctions and so few second price auctions are observed in the real world.

The conflicting impacts of the signalling and participation effects combined with the two-stage nature of our model limit general revenue comparisons. Instead, we offer a comparison to the standard IPV case presented in Jehiel, et al. (1996) where nature reveals all private information after the auction, but before the post-auction competition. Since nature reveals the winner's type, there is no link between the auctioneer's announcement and post-auction profits and attempts to signal are fruitless. Also, since the winner's type is known with certainty in the post-auction game, the cut-off type bidder will be the same regardless of the auction form used. We show that the benchmark model consistently over- (under-) estimates expected revenue relative to the cases where $\pi_1(G(\beta), c) > 0$ ($\pi_1(G(\beta), c) < 0$).

We believe that the case where $\pi_1(G(\beta), c) > 0$ provides interesting insight into the overwhelming success of the recent PCS bandwidth auctions. Since the market for mobile telecommunications is best characterized by pricing competition, imagine that competitors in the auction are bidding for the right to enter into a post-auction Bertrand duopoly with an incumbent monopolist. The nature of Bertrand competition dictates that the incumbent will set a higher price if they believe that the entrant has a relatively high marginal cost. Hence, bids will be less aggressive if the winning bid will be announced in an effort to signal that the bidder expects low post-auction profits because of a high marginal cost draw. The

[6] Even a non-strategic auctioneer may operate in an environment where credible commitment to a policy is not guaranteed. Such a case exists in auctions held by the U.S. government where results may be subject to Freedom of Information Act petitions.

auctioneer may want to nullify signalling by not announcing the winner's bid, especially if it is anticipated that the participation effect will be relatively small.[7] Although an English auction format was chosen because of winner's curse concerns (see McMillan (1994)), a nice by-product of that choice was the assurance to bidders that the winners' bids would not be announced.

The paper proceeds as follows. Section 2 introduces the general model and a regularity condition. Section 3 presents Bertrand and Cournot examples (corresponding to the cases of $\pi_1(G(\beta), c) > 0$ and $\pi_1(G(\beta), c) < 0$ respectively) that will be used to convey intuition throughout the paper. Bidding behavior based on a general second stage profit function is derived in Section 4. Section 5 provides revenue rankings and returns to the specific cases of Bertrand and Cournot competition to examine the impact of various announcement policies on welfare. Conclusions are offered in Section 6 and all proofs are contained in the appendix.

2 The Model

Stage one of the game considers a set of firms, N $(i = 1, ..., n)$, competing in an auction, the results of which impact a second stage game. Entering the auction, each firm has expected payoff $\Pi(x_i, c_i)$ for the entire game where c_i is its type and x_i is the type reported at the auction. The specific form of this profit function depends on the auctioneer's bid announcement policy and the equilibrium payoff function in the post-auction game. Types are drawn independently from a C^2 function $F(c), c \in [\underline{c}, \overline{c}]$ with corresponding density $f(c)$. We interpret types as marginal cost draws, therefore; a higher type is less desirable. $\overline{c}$ is set so that those who draw a relatively high marginal cost choose not to participate in the auction, thus; allowing us to endogenize participation in the auction by identifying the cut-off type firm that is just indifferent between placing and not placing a bid. The firm with that cut-off type in auction form a $(= 1st, 2nd, Eng)$ is denoted c^*_a.

The auctioneer's credible bid announcement policy will affect the bidder's post-market profit in that it reveals private information to post-auction competitors. Of the set of bids submitted, B, the auctioneer announces some subset, β, before the second stage game begins. We focus on instances where the auctioneer announces either the first or second highest bid.

[7] As discussed above, the participation effect tends to offset this maneuver. However, competition in the PCS auctions was strong and the impact of relatively fewer weak typed firms participating was likely minimal.

Following the bid announcement, the winning bidder's post-auction competitors update their beliefs concerning the winner's type, giving an expectation of that type, $G(\beta)$. The winning bidder's second stage profits, $\pi(G(\beta), c)$, depend on this expectation, as well as, their true type c.[8]

The first partial derivatives of $\pi(G(\beta), c)$ are assumed to be monotonic in each argument. Both announcement policies are compared to the benchmark (BM) IPV case that results from assuming that nature reveals the winner's true type, independent of the information revealed by the auctioneer. In that case, $G(\beta) = c$ in all situations which is equivalent to maintaining $\pi_1(G(\beta), c) = 0$. When nature does not reveal the winner's true type, the sign of $\pi_1(G(\beta), c)$ dictates the relation between rival beliefs and the winner's post-auction profits. The marginal cost interpretation of type indicates that $\pi_2(G(\beta), c) < 0$. It is further assumed that the second partial derivatives $\pi_{11}(G(\beta), c)$, $\pi_{22}(G(\beta), c)$, and $\pi_{12}(G(\beta), c)$ exist and are continuous. Finally, the following regularity condition ensures existence of equilibrium in all of the auction forms considered.

Condition 1.

$$\frac{\pi_2(x, c)}{\pi_{12}(x, c)} < \frac{[1 - F(x)]}{(n-1)f(x)}, \ \forall x, c \in [\underline{c}, c^*].$$

Remark. This regularity condition ensures that the tendency to under-report types at auction is not so great as to produce degenerately low (zero) bids. When $\pi_{12}(x, c) < 0$, the condition is satisfied for all distribution functions. However, when $\pi_{12}(x, c) > 0$, it may be violated for certain distribution functions. Condition 1 is similar to that found in Das Varma (2003) but differs in two respects. First, higher types are better in his model but worse in ours; thus, changing the interpretation of the sign of $\pi_2(x, c)$. Second, our condition allows for any finite number of bidders, producing his result when the number of bidders is restricted to two. In addition, Condition 1 nests Goeree's restriction that $\pi_{12}(x, c) > 0$ and therefore allows us to consider a larger class of profit functions. For example, his assumption precludes Bertrand competition in the post-auction since $\pi_{12}(x, c) < 0$ in that case.

[8] It is important to note that the assumed form of the dependence of profits on beliefs is fairly restrictive. However, it does not affect our equilibrium bidding results and is necessary for obtaining tractable solutions for comparing auction revenues and subsequent post-auction market welfare.

3 Stage Two: Post-Auction Competition

Since backward induction is used to derive the equilibrium of the entire game, we begin by examining the second stage. In our general examination of the auctions, we simply assume that the second stage game has a unique Bayes-Nash equilibrium and that the profits in that game, $\pi(G(\beta), c)$, possess the properties discussed in Section 3. For the remainder of the paper we assume that firms losing the auction receive zero profit in the post-auction market.

In order to assess the impact of various announcements on this post-auction market we must first determine how the set of possible participants in the post-auction market is impacted by the announcement policy. The following theorem is driven by the relation between rival beliefs and the winner's post-auction profits.

Theorem 1 (Endogenous Bidder Participation) *A policy of announcing (not announcing) the winner's bid leads more types to participate in the auction if* $\pi_1(G(\beta), c) > 0$ *(*$\pi_1(G(\beta), c) < 0$*).*

This Theorem applies to many forms of post auction game. Examples include mergers, competition for new technologies, rights to bandwidth or other resources necessary for production. Most simply it can be viewed as bidding for the right to compete against an incumbent monopoly. This last example provides a well known framework for conveying the intuition behind our results. The remainder of this section will use this example to explore the dependence of auction participation and post-auction profits on the inferences made from bid announcements, $G(\beta)$, by appealing to the Bertrand and Cournot duopoly games.

In these contexts the winning bidder may be thought of as an <u>E</u>ntrant firm competing against an <u>I</u>ncumbent monopoly. In each case considered, the entrant knows the incumbent's marginal cost with certainty. However, the entrant's marginal cost is only revealed when the winner's bid is announced. When their bid is not announced, the incumbent optimizes using the expected value of the entrant's cost given the second lowest marginal cost (obtained by inverting the second highest bid).

3.1 Bertrand Competition[9]

The specific game considered is based on Hotelling's "linear city" model of differentiated Bertrand competition. In that game, consumers with unitary demands are located uniformly along a line of length one. The incumbent firm locates on the left end of the line and the entrant locates on the right end. A consumer's distance to either end is multiplied by a scaling parameter $t > 0$, representing the lack of desirability of that service, stemming from exogenous differentiation of the goods (a short distance represents desirability of a product).[10]

A consumer located at point y receives utility u_0 from consuming the good and their corresponding surplus is $u_0 - ty - p_I$ if they buy from the incumbent and $u_0 - t(1-y) - p_E$ if they buy from the entrant. Surplus is assumed large enough (t small enough) that there exists some central consumer that is indifferent between buying from the incumbent and entrant. By equalizing the utilities, it follows that the central consumer is located at $\widetilde{y} = \frac{p_E - p_I + t}{2t}$.

In the Hotelling game, entrant and incumbent profits are

$$\pi_E = (p_E - c_E)\frac{p_I - p_E + t}{2t} \tag{1}$$

$$\pi_I = (p_I - c_I)\frac{p_E - p_I + t}{2t}. \tag{2}$$

Differentiating the profit functions with respect to the relevant price and solving the resulting system of equations yields the equilibrium prices

$$p_E = \frac{1}{3}(3t + c_I + \frac{3}{2}c_E + \frac{1}{2}G(\beta)) \tag{3}$$

$$p_I = \frac{1}{3}(3t + G(\beta) + 2c_I). \tag{4}$$

Thus, giving the equilibrium quantities of

$$q_E = \frac{1}{6t}(3t + c_I - \frac{3}{2}c_E + \frac{1}{2}G(\beta)) \tag{5}$$

$$q_I = \frac{1}{6t}(3t + \frac{3}{2}c_E - \frac{1}{2}G(\beta) - c_I). \tag{6}$$

[9] Moldovanu and Sela (2003) provide a related examination based on Bertrand Competition in Patent Licensing.

[10] Linearity of "transportation costs" is assumed solely for simplicity. All calculations are the same for quadratic costs with the exception of consumer surplus, which is less.

The resulting equilibrium profits are

$$\pi_E = \frac{1}{18t}(3t + c_I - \frac{3}{2}c_E + \frac{1}{2}G(\beta))^2 \tag{7}$$

$$\pi_I = \frac{1}{18t}(3t + \frac{3}{2}c_E - \frac{1}{2}G(\beta) - c_I)(3t + G(\beta) - c_I). \tag{8}$$

Note the dependence of profits on $G(\beta)$, the incumbent's expectation of the entrant's type based on the auctioneer's announcement. The positive relation between incumbent beliefs and entrant profits can be seen through the partial derivative of Eq. (7) with respect to $G(\beta)$. The relation between this derivative and pricing decisions is that if the incumbent believes the entrant has a high marginal cost, they anticipate that the entrant will set a high price. Bertrand logic then dictates that the incumbent will follow suit, setting a high price. But, because the incumbent sets a high price, the entrant obtains more consumers and hence makes more profit.

In the benchmark case, $G(\beta)$ is independent of the auctioneer's announcement and hence equals c_E. Thus, the benchmark profit function is

$$\pi_E(c_E, c_E) = \frac{1}{18t}(3t + c_I - c_E)^2. \tag{9}$$

Equating this profit to zero gives us the cut-off type, $c^*_{BM} = 3t + c_I$, that is just indifferent between competing and not competing in the auction.

When nature does not reveal the entrant's type and the auctioneer announces the winner's bid, inversion reveals the winner's reported type, x, resulting in a payoff of

$$\pi_E(x, c_E) = \frac{1}{18t}(3t + c_I - \frac{3}{2}c_E + \frac{1}{2}x)^2. \tag{10}$$

In equilibrium, bidders truthfully report their types, $x = c$. Setting Eq. (10) equal to zero and imposing truthful revelation once again gives the cut-off type $c^*_{1st} = 3t + c_I$.

If only the second highest bid ($b_{(2)}$) is announced, inversion yields the second lowest marginal cost ($c_{(2)}$), giving an expectation of the lowest marginal cost of $G(\beta) = E[z|z \leq \min(c^*_{Eng}, c_{(2)})]$. The profit function in this case is

$$\pi_E(E[z|z \leq \min(c^*_{Eng}, c_{(2)})], c_E) \tag{11}$$

$$= \frac{1}{18t}(3t + c_I - \frac{3}{2}c_E + \frac{1}{2}E[z|z \leq \min(c^*_{Eng}, c_{(2)})])^2. \tag{12}$$

A bidder with the cut-off type (c^*_{Eng}) in this case will only win the auction if $c_{(2)} \geq c^*_{Eng}$. Therefore, when the cut-off bidder wins the auction, $\min(c^*_{Eng}, c_{(2)}) = c^*_{Eng}$ giving the implicit equation for the cut-of type

$$c^*_{Eng} = 3t + c_I - \frac{\int_{\underline{c}}^{c^*_{Eng}} F(x)dx}{2F(c^*_{Eng})}. \tag{13}$$

Clearly, $c^*_{Eng} < c^*_{BM}$ in the Bertrand case, a consequence of Theorem 1 since $\pi_1(G(\beta), c) > 0$.

In each of the three cases above (Eqs. (9), (10), and (11)), profits are negatively related to the entrant's marginal cost. However, only in the case where the winner's bid is announced (Eq. (10)) does the bidder's choice of type at the auction, x, affect profits. This dependence is central to the presence of the bid signalling effect. Eq. (10) yields the partial derivative

$$\pi_1(x, c_E) = \frac{1}{18t}(3t + c_I - \frac{3}{2}c_E + \frac{1}{2}x). \tag{14}$$

The sign of Eq. (14) gives the positive relation between the type reported at auction (x) and profits. In other words, potential entrants will tend to understate their expected profits (pretend to be of weak type) in an attempt to deceive the incumbent. The cross partial resulting from Eq. (10) is

$$\pi_{12}(x, c_E) = -\frac{1}{12t}. \tag{15}$$

Since the signs of Eqs. (14) and (15) work in opposite directions, Condition 1 may be violated. This is explored further below in Sections 5.1 and 5.2.

3.2 Cournot Competition

The specific Cournot game considered uses the normalized demand function $P = 1 - Q$.[11] The incumbent's marginal cost, c_I, is assumed to be less than $\frac{1+\underline{c}}{2}$, guaranteeing that the winning bidder will not supplant the incumbent as the monopolist.

[11] All results hold for the more general case of $P = a - bQ$. However, the notation becomes excessive without adding insight.

Expected profits for the two firms in the Cournot market are

$$\pi_E = (1 - q_E - q_I)q_E - c_E q_E \tag{16}$$

$$\pi_I = (1 - E[q_E] - q_I)q_I - c_I q_I. \tag{17}$$

Maximizing each firm's profit with respect to its choice of quantity and rearranging gives

$$q_E = \frac{1 + c_I - \frac{3}{2}c_E - \frac{1}{2}G(\beta)}{3} \tag{18}$$

$$q_I = \frac{1 + G(\beta) - 2c_I}{3}. \tag{19}$$

The profits associated with these equilibrium quantities are

$$\pi_E = \left[\frac{1 + c_I - \frac{3}{2}c_E - \frac{1}{2}G(\beta)}{3}\right]^2 \tag{20}$$

$$\pi_I = \left[\frac{1 + c_E - 2c_I}{3}\right]^2. \tag{21}$$

As in the Bertrand environment, profits depend on $G(\beta)$. The negative relation between incumbent beliefs and entrant profits is seen through the partial derivative of Eq. (20) with respect to $G(\beta)$. The intuitive relation between this derivative and output decisions is that if the incumbent believes the entrant has a high marginal cost, they anticipate that the entrant will produce a relatively small quantity. Cournot logic then dictates that the incumbent will take advantage of the lack of supply and produce a relatively large amount. Because the incumbent produces a high level of output, the price is lowered and the entrant makes lower profit.

In the benchmark case, $G(\beta) = c_E$, as nature reveals c_E after the auction and the auctioneer's announcement policy is irrelevant. The profit function in this case is

$$\pi_E(c_E, c_E) = \left[\frac{1 + c_I - 2c_E}{3}\right]^2. \tag{22}$$

Equating this profit to zero gives us the cut-off type, $c^* = (1 + c_I)/2$, that is just indifferent between competing and not competing in the auction.

When a bidder announces their type as x and their bid is invertible, their payoff becomes

$$\pi_E(x, c_E) = \left[\frac{1 + c_I - \frac{3}{2}c_E - \frac{1}{2}x}{3}\right]^2. \tag{23}$$

In equilibrium, bidders truthfully report their types, $x = c$. Setting Eq. (10) equal to zero and imposing truthful revelation once again gives the cut-off type $c^*_{1st} = (1 + c_I)/2$.

If only the second highest bid ($b_{(2)}$) is announced, inversion yields the second lowest marginal cost ($c_{(2)}$), giving an expectation of the entrant's marginal cost of $G(\beta) = E[z|z < \min(c^*_{Eng}, c_{(2)})]$. The profit function in this case is

$$\pi_E(E[z|z \;<\; \min(c^*_{Eng}, c_{(2)})], c_E) \tag{24}$$

$$= \left[\frac{1 + c_I - \frac{3}{2}c_E - \frac{1}{2}E[z|z < \min(c^*_{Eng}, c_{(2)})]}{3}\right]^2 \tag{25}$$

A bidder with the cut-off type (c^*_{Eng}) in this case will only win the auction if $c_{(2)} \geq c^*_{Eng}$. Therefore, when the cut-off type wins the auction, $\min(c^*_{Eng}, c_{(2)}) = c^*_{Eng}$ giving the implicit equation for the cut-of type

$$c^*_{Eng} = \frac{1 + c_I}{2} + \frac{\int_{\underline{c}}^{c^*_{Eng}} F(s)ds}{4F(c^*_{Eng})}. \tag{26}$$

Clearly, $c^*_{Eng} > c^*_{BM}$ in the Cournot case, a consequence of Theorem 1 since $\pi_1(G(\beta), c) < 0$.

As in the Bertrand case, profits are negatively related to the entrant's marginal cost in each of the three cases above (Eqs. (22), (23), and (24)). Once again, only in the case where the winner's bid is announced (Eq. (23)) does the bidder's choice of type at the auction, x, affect profits. This dependence is central to the presence of the bid signalling effect. Eq. (23) yields the partial derivative

$$\pi_1(x, c_E) = \frac{-1}{9}(1 + c_I - \frac{3}{2}c_E - \frac{1}{2}x). \tag{27}$$

The sign of Eq. (27) gives the negative relation between the type reported at auction (x) and profits. In other words, potential entrants will tend to overstate their expected profits (pretend to be of strong type) in an attempt to deceive the incumbent. The cross partial resulting from Eq. (23) is

$$\pi_{12}(x, c_E) = \frac{1}{6}. \tag{28}$$

Since the signs of Eqs. (27) and (28) work in the same direction, Condition 1 will not be violated.

In summary, the Bertrand and Cournot models provide examples where firm types are negatively related to profits, but depend on reports at auction in opposing fashions (see Eqs. (14) and (27)). These examples give not only an intuitive background in which our results can be framed, but also provide analytic solutions that will be necessary in our analysis of post-auction welfare (Section 6).

4 Stage One: The Auction

This is the most general section in the paper in that derivation of equilibrium bid strategies is based on the general profit function $\pi(G(\beta), c)$. Three different auction forms are considered: $1st$ price, $2nd$ Price, and English. The first and second price formats are analyzed under a policy of announcing the winner's bid, while only the second highest bid is announced following an English auction. For each auction form, the symmetric equilibrium bid functions are derived and then compared to those resulting in the benchmark case. The benchmark equilibrium bid functions are

$$b_{1st}(c) = \frac{\int_c^{c^*_{BM}} \pi(z,z)(n-1)[1-f(z)]^{n-2} f(z)dz}{[1-F(c)]^{n-1}} \tag{29}$$

$$b_{2nd}(c) = \pi(c,c) \tag{30}$$

$$b_{Eng}(c) = \pi(c,c). \tag{31}$$

The first price bid function simply requires that an individual's bid equal the expected profit of the next most competitive bidder, given that the individual has the best type. The second price and English auction bid functions represent the famed Vickrey (1961) result concerning the dominance of bidding one's valuation.

4.1 First price auctions (with winning bid announced)

Given that its opponents are all using the strictly increasing differentiable bid function $b(c)$, a firm's expected payoff from entering a first price auction is

$$\Pi(x,c) = [\pi(x,c) - b(x)]\,[1 - F(x)]^{n-1}. \quad (32)$$

The first term is simply the firm's surplus if it wins the auction with a report of x. The second term is the probability that the report wins the auction.

Theorem 2 *The symmetric equilibrium bid function in a first price auction is given by*

$$b_{1st}(c) = \frac{\int_c^{c^*_{1st}} \left[\pi(y,y) - \pi_1(y,y)\frac{[1-F(y)]}{(n-1)f(y)}\right](n-1)[1-F(y)]^{n-2} f(y)dy}{[1-F(c)]^{n-1}} \quad (33)$$

if Condition 1 holds.

Examination of Eq. (33) reveals that the benchmark model is nested in this model. That is, when nature reveals the winner's type, $\pi_1(c,c) = 0$, the fact that $c^*_{1st} = c^*_{BM}$ results in Eq. (29). If signalling is possible, $\pi_1(c,c) < 0$ leads to bids above the benchmark level and $\pi_1(c,c) > 0$ leads to bids below the benchmark level. The tendency to lower bids below expected profit is an attempt to convince the incumbent that a worse type was drawn than actually was. It is this tendency that necessitates Condition 1. The condition ensures that the tendency is not so strong as to produce degenerately low bids (e.g. zero) that result in non-monotonically increasing bid functions. If Condition 1 is violated, non-existence of a separating equilibrium follows.

4.2 Second price auctions (with winning bid announced)

This section considers a second price auction where the winner's bid is announced. The Theorem of Revenue Equivalent Classes tells us that the expected revenue generated in this setting will equal that generated by the first price auction just discussed. However, the properties of the equilibrium bid function provide interesting intuition and a meaningful comparison to established results, particularly in the special case where post-auction profits are negatively related to rival beliefs.

A bidder acting as type x, whose opponents are using the strictly increasing bid function $b(c)$ has expected payoff entering a second price auction of:

$$\begin{aligned}\Pi_{2nd}(x,c) &= \pi(x,c)\left(1-F(x)\right)^{n-1} \\ &\quad - \int_x^{c^*_{2nd}} b(y)(n-1)\left(1-F(y)\right)^{n-2} f(y)dy.\end{aligned} \tag{34}$$

The first term is the second stage profit times the probability of winning with a report of x. The second term is the expected payment if at least one other bidder has marginal cost below c^*_{2nd}, which equals c^*_{BM} because the winning bid is announced. If no other bidder has marginal cost below c^*_{2nd}, the bidder wins and pays zero in the absence of a reserve price.

Theorem 3 *The symmetric equilibrium bid function in a second price auction is given by*

$$b_{2nd}(c) = \pi(c,c) - \frac{\pi_1(c,c)\left(1-F(c)\right)}{(n-1)f(c)}. \tag{35}$$

if Condition 1 holds.

As in the first price equilibrium, Eq. (35) nests the benchmark bid function. That is, when $\pi_1(c,c) = 0$, Eq. (30) results. If $\pi_1(c,c) < 0$, bids are above the benchmark level and if $\pi_1(c,c) > 0$, bids are below the benchmark level. The case where is $\pi_1(c,c) < 0$ is interesting since bids are actually above the value obtained from winning the auction, $\pi(c,c)$. The upside to this strategy is the deception imparted on the incumbent. The downside is running the risk that the price paid will fall between profit and the inflated bid, generating a loss. When there are relatively few bidders, an individual is willing to take the risk of bidding more than their expected profit. However, as the number of opponents grows, the downside becomes more likely, leading to lower bids. Bids approach post-auction profits, $\pi(c,c)$, as the number of bidders approaches infinity.[12]

The negative relation between bids and number of bidders leads to an interesting policy implication for auctioneers choosing between first and second price auctions. A common method of disrupting collusive rings in procurements is to invite a subset of the pool of

[12] Although the underlying intuition is different, this prediction is similar to Rosenthal (1980) where a greater number of sellers leads to a higher price.

potential bidders.[13] Consider a seller interested in inviting a subset of the best type firms without letting them know the selection criterion (i.e. beliefs about opponents types went unchanged, but beliefs concerning number of opponents did change). He would expect higher revenue from a second price auction than if he invited all bidders, and lower revenue if he used a first price auction since the number of bidders is small and bids are deceasing (increasing) in the number of bidders in second (first) price auctions.

Next, consider an the auctioneer that invites the top bidders, but those bidders recognized that only the top bidders had been invited (i.e. changing a bidder's beliefs about his opponents' distribution over types). This auctioneer would do better to use a first price auction as the downside of the second price auction is more likely and thus second price bids will tend to be lower. Since bidders are commonly aware of who their closest competitors are, it is our conjecture that it is more likely that firms will recognize when the top bidders have been invited. The anti-collusive strategy explains why subsets of bidders are invited and our conjecture helps explain why first price auctions are commonly used in such instances.[14]

4.3 English auctions (with price paid announced)

Firms participating in an English auction must choose a level at which they will drop-out of the bidding. Bidding stops when one firm remains and the current price is paid.[15] Since the winner's drop-out level is never reached, it is not observed and their type cannot be obtained through inversion. Hence, signalling is not possible. However, there is uncertainty regarding their type in the second stage game and thus second stage payoffs differ from cases where their type is known. In addition, the inability to signal alleviates the problems with non-existence found in the case where post-auction profits are positively related to rival beliefs.

The equilibrium drop-out rule is obtained by determining the level of bidding at which a firm expects zero profit. Using a second price auction as a proxy, this is obtained by maximizing a firm's expected profit given that their opponents are using the strictly increasing

[13] Those interested in collusion in auctions are referred to Graham and Marshall (1987).

[14] Engelbrecht-Wiggans, and Kahn (1991), Rothkopf and Harstad (1995), and Rothkopf et al. (1990) for other explanations.

[15] This form of the English auction has been referred to as a "button" model of the English auction. It is decisively different than the English auction considered by Goeree (2003) and Avery (1998) where a bidder is allowed to keep bidding, even after all other competition has dropped out.

drop-out rule, $b(c)$. A firm's expected payoff is

$$E_{c_{(2)}}\left[\Pi(E[y|y \leq \min(c_{(2)}, c^*_{Eng})], c)\right] \tag{36}$$
$$= \pi(E[z|z \leq c^*_{Eng}], c)\left(1 - F(c^*_{Eng})\right)^{n-1}$$
$$+ \int_x^{c^*_{Eng}} \left[\pi(E[z|z \leq y, c]) - b(y)\right](n-1)\left(1 - F(y)\right)^{n-2} f(y)dy.$$

The first term is the firm's expected payoff if no other firms participate in the auction. The second term is the firm's expected profit over possible values of the lowest opponents' type, given that the opponent competes in the auction.

Theorem 4 *The symmetric equilibrium bid function in an English auction is given by*

$$b_{Eng}(c) = \pi(E[z|z \leq c], c). \tag{37}$$

Although not as immediate as in the preceding auctions, this equilibrium also nests the benchmark equilibrium, in a different manner however. Since reports do not affect profits, the sign of $\pi_1(c, c)$ is immaterial. The nesting lies in the nature of the announcement in the benchmark model. When nature reveals the winner's type, $E[z|z < c]$ simply degenerates to c and Eq. (31) results.

In this section we have derived the symmetric equilibrium bid functions for three different auction formats: first price, second price, and English. Each of the bid functions nest those found in the independent private values paradigm, but also account for the endogenization of valuations caused by the presence of a post-auction market. It should not be surprising that deviations from IPV bid functions leads to a deviation from revenue equivalence. The following section examines the how the revenue generating properties of the various auctions are affected by post-auction market concerns.

5 Expected Revenue and Welfare

This section examines the impact of the auctioneer's announcement on all parties involved. From the auctioneer's perspective, we examine both revenue generating and efficiency properties of the different auctions. All of the auctions examined are allocatively efficient in that the firm with the best type wins the auction. However, announcing the winner's bid generates a game of complete information in the post-auction market whereas, a game of incomplete information results when the winner's bid is not announced. These informational differences lead to varied expectations of consumer and producer surpluses.

The signalling and participation effects established in the previous sections have opposite impacts on the revenue generating properties of the various auctions. The participation effect leads to relatively more (less) competition when post-auction profits are positively (negatively) related to rival beliefs and the winner's bid is announced and, hence; tends to increase (decrease) revenues. Signalling has the opposite effect, it leads to less (more) aggressive bidding when post-auction profits and rival beliefs are positively (negatively) related and the winner's bid is announce and, hence; tends to decrease (increase) revenues. Unfortunately, the net of these effects cannot be generalized. However, the conclusion is clear, the dominance of certain auctions over others found when only the signaling effect is considered is at least tempered (and can be overturned) by the participation effect. The following theorem assesses the impacts of the signalling and participation effects on auction revenues relative to the benchmark case.

Theorem 5 *The benchmark model over- (under-) predicts the level of bids for a given type and hence over- (under-) estimates the expected revenues generated by the auction when* $\pi_1(G(\beta), c) > 0$ ($\pi_1(G(\beta), c) < 0$)*, regardless of the announcement policy.*

The lack of a general revenue comparison based on the various announcement policies is best seen graphically. Figures 1 and 2 show representative bid functions for the second price and English auctions for the cases of $\pi_1(G(\beta), c) > 0$ and $\pi_1(G(\beta), c) < 0$ respectively.[16] Region A in Figure 1 (Figure 2) represents the fact that low types tend to bid more aggressively in the English (first price) auctions when $\pi_1(G(\beta), c) > 0$ ($\pi_1(G(\beta), c) < 0$). The vertical intercepts are obtained by evaluating each bid function at $\underline{c}$. A bidder with that type bids the

[16] We choose to compare English and second price auctions here since expectations of revenue are taken over the distribution of the second lowest order statistic for each.

following in the three situations:

$$b_{BM} = \pi(\underline{c}, \underline{c}) \tag{38}$$

$$b_{2nd} = \pi(\underline{c}, \underline{c}) - \frac{\pi_1(\underline{c}, \underline{c})}{(n-1)f(\underline{c})} \tag{39}$$

$$b_{Eng} = \pi(\underline{c}, \underline{c}) \tag{40}$$

Eq. (39) is obtained using Eq. (30) and recognizing that $F(\underline{c}) = 0$. Eq. (40) results from Eq. (31) and the fact that $E[w|w \leq \underline{c}] = \underline{c}$. The implication is that when $\pi_1 > 0$ the second price bid function has a lower vertical intercept than the benchmark and English auction bid functions (which have the same intercept). When $\pi_1 < 0$, the vertical intercept of second price bid function is above that of the benchmark and English bid functions. These properties provide insight into the success of the PCS auctions. As the post-auction market for mobile telecommunications is best characterized by pricing competition ($\pi_1 > 0$), and several highly competitive firms participated in the auction, the prices paid were determined in region A of Figure 1. Hence, the decision to use English auctions resulted in systematically higher bids than auctions where the bidder's type could be inferred.

The difference in the horizontal intercepts of the bid functions represents the endogenous participation effect and highlights the lack of a general revenue comparison. Both the benchmark and second price auctions have the same intercept as the information available to post-auction market competitors is the same. In Figure 1 (Figure 2), the intercept for the English auction is lower (higher) than that in the other auctions. Combining the properties of the vertical and horizontal intercepts provides the intuition behind Theorem 5. When $\pi_1 < 0$ ($\pi_1 > 0$), the benchmark case has both the lowest (highest) vertical and horizontal intercepts and hence predicts lower (higher) expected revenues than the other auctions. The intercepts also provide insight into the lack of a general revenue comparison of the second price and English auctions. As bid functions are monotonic in each auction, one with a higher vertical intercept, the other with a higher horizontal intercept, one does not decisively dominate the other.

We now turn to the specific cases of Bertrand and Cournot competition in the post-auction market to examine the welfare effects of the various announcement policies.

5.1 Bertrand Competition

Predictions concerning market prices and surplus hinge on two consequences of the announcement policies. First, announcing versus not announcing the winner's bid leads to distinctly different post-auction games. When the bid is announced, the game is one of complete information. Alternatively, not announcing the winner's bid produces a game of incomplete information. Second, more types tend to participate when the winner's bid is going to be announced. The following theorem assesses the impact of these two consequences.

Theorem 6 *The Incumbent's expected price and the average market price are lower in the differentiated Bertrand market when the winner's bid is announced than when it is not.*

The intuition behind the theorem is as follows. The nature of the Bertrand model produces a linear relation between entrant type and prices. Hence, prices are, on average, the same regardless of the announcement policy whenever the auction produces an entrant. However, the fact that more types participate when the winner's bid is announced makes it more likely that the auction will produce a competitor for the incumbent monopoly. Because it is less likely that the incumbent retains their monopoly power, their price tends to be lower following an announcement of the winner's bid as are average market prices. Also notice that there is no statement regarding the entrant's price. Whenever an entrant emerges as a result of the auction, their expected price is the same under each policy. However, in the region above c^*_{Eng} but below c^*_{1st}, there is no entrant price if an English auction were held and thus comparisons are meaningless.

The conflicting participation and signalling effects lead to an indeterminate ranking of surpluses. The following theorem addresses surplus comparisons in the case where a monopoly result is not possible.[17]

Theorem 7 *Differentiated Bertrand competition subsequent to a first price auction leads to lower expected consumer surplus and incumbent profits and higher expected entrant profits than subsequent to an English auction, when there is no possibility of monopoly.*

[17] Once again, the FCC spectrum auctions provide an excellent example of this situation. Most economists agreed that there would certainly be an entrant in every market. The Mexican spectrum auction provides an example where such an assumption is inappropriate. One wavelength actually went unsold, potentially indicating that no one was below the cut-off type (see Chakraborty (2002) for a discussion).

The above theorem provides insight into the preferences of the parties involved. Consumers in general prefer English auctions to first price auctions. However, these preferences are split between those buying from the incumbent and those buying from the entrant. Consumers buying from the incumbent have higher expected surplus following a first price auction while those buying from the entrant prefer an English auction. The incumbent prefers an English auction for two reasons. First, they have higher expected profit when facing an entrant and second, there is a greater chance that they will retain their monopoly position. Entrants have opposite preferences. They prefer first price auctions because they expected higher profits due to the certainty introduced by the announcement, and they are more likely to enter the post-auction market.

5.2 Cournot Competition

Predictions concerning production levels depend on informational structure of the Cournot game resulting from the announcement policy. The following theorem assesses the impact of participation decisions and that informational structure.

Theorem 8 *Expected entrant output, q_E, is higher following first price auctions than they are following English auctions. Expected incumbent output, q_I, and total production, $q_E + q_I$, are lower following first price auctions than following English auctions.*

Unlike the corresponding theorem for the Bertrand market (Theorem 6), the entrant's decision is included in Theorem 8. In Bertrand competition, a definitive conclusion regarding the entrant's price required specifying the price charged by the entrant when, in fact, they never entered the market (i.e. a monopoly situation). The Cournot case is more straight forward. When an entrant does not enter the market, their output is clearly zero.

Expected quantities are linear in the entrant's type, as were expected prices in the Bertrand case. Therefore, the quantity will be the same under each announcement policy as long as the incumbent does not retain its monopoly power. Clearly this leads to greater entrant output since more types participate in post-auction competition and there is less chance of reversion to a monopolistic market. However, a higher likelihood of post-auction competition actually leads to less incumbent output, on average. There is also an opposite effect on incumbent output caused by the fact that they will face, on average, more high cost entrants and thus will tend to produce more. In fact, this latter effect dominates leading to the conclusion that the incumbent produces more after an English auction. The totality of these effects results in less expected market output following an English auction. Finally, higher quantities clearly lead to lower prices in this market.

Another interesting aspect of this market is that following an announcement of the winner's bid, as the entrant's marginal cost approaches the highest possible type (c^*_{1st}), the incumbent's output approaches the monopoly output. However, following an English auction, as the entrant's marginal cost approaches c^*_{Eng}, the incumbent's output approaches a level strictly less than the monopoly output. Or, there is a discontinuity in the incumbent's output at c^*_{Eng}. The problem is that the incumbent tends to over-estimate the strength of weak type entrants in the game of incomplete information. Thus, even when entrant is of the worst possible type and produces zero output, the incumbent still does not produce the monopoly output because they expected the entrant to provide a positive level of output. Therefore, when a high type entrant emerges from the auction, the market is actually less efficient under competition than it would be under a monopoly.

As in the Bertrand case, general comparisons are not possible because of the implicit definition of cut-off types. The following theorem assesses the surplus impacts of the announcement policies in a situation where monopoly is impossible.

Theorem 9 *When a monopoly outcome is not possible, Cournot competition subsequent to a first price auction leads to higher expected consumer surplus and entrant profit than after an English auction. Expected incumbent profits are independent of the auction form.*

In the end, the four different parties involved (auctioneer, consumers, entrant, and incumbent) have different preferences over the auction form employed, likely leading to varied lobbying efforts. Unlike the Bertrand case, consumers in general prefer that a first price auction be held. The incumbent firm is indifferent between auction forms in situations where they face an entrant. However, they are more likely to retain their monopoly power after a first price auction and hence prefer it to an English auction. Finally, potential entrants face an interesting dilemma in the case of Cournot competition. They expect higher profit in the post-auction market following a first price auction. However, at the auction, they must bid more aggressively in a first price auction to signal that they are of strong type and thus, pay more on average to enter the market.

6 Conclusion

It has been shown that when an auction is held for the right to engage in post-auction competition, the auctioneer can affect auction revenues and efficiency through the announcement of bids. In the separating equilibrium where a bid is a monotonic function of the bidder's type, announcing the winner's bid is tantamount to announcing their type. Realizing that their type can be inferred from their bid under such a policy, bidders attempt to manipulate their bids in an attempt to signal false information regarding their type to their post-auction rivals. Thus, in terms of mechanism design, auctioneer's have an additional dimension (announcement choice) that they can consider when maximizing expected auction revenues.

When post-auction profits are inversely related to rival beliefs, the signalling effect leads to more aggressive bidding in an effort to signal a strong type. In this case, the auctioneer can exploit the signalling effect by announcing the winner's bid, thereby enhancing revenues. However, the revenue enhancing effects of such a policy are tempered as fewer firms choose to participate in that auction. Alternatively, when post-auction profits are positively related to rival beliefs, the signalling and participation effects are reversed. Bids are less aggressive as bidders attempt to signal a relatively poor type. In this case, the auctioneer prefers to withhold the winner's bid to nullify the signalling effect, but once again faces the counteracting effect that fewer bidders will participate than if the winner's bid were announced.

At this point, the two-stage nature of our model keeps us from making general statements regarding the welfare effects brought about by different announcement policies. However, for the Cournot and Bertrand models considered in this paper, we establish the impact that announcing/not announcing the winner's bid has on expected prices and output, as well as, consumer and producer surplus.

7 References

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A Appendix

A.1 Proof of Theorem 1

Let c^*_{1st} denote the cut-off type when the winner's bid is announced and c^*_{Eng} denote the cut-off type when only the second highest bid is announced. Consider the case where $\pi_1(G(\beta), c) < 0$. If the winner's bid is announced, $G(\beta) = c$, in equilibrium. By definition, $\pi(c^*_{1st}, c^*_{1st}) = 0$. When the winner's bid is not announced, $G(\beta) = E(x|x < c)$. Now assume that $c^*_{Eng} = c^*_{1st}$. It follows that $\pi(E(x|x < c^*_{1st}), c^*_{1st}) = 0$. However, since $\pi_1(G(\beta), c) < 0$, $\pi(E(x|x < c^*_{1st}), c^*_{1st}) > \pi(c^*_{1st}, c^*_{1st})$ and the assumption that $c^*_{Eng} = c^*_{1st}$ is contradicted. Finally, since

$\pi(E(x|x < c^*_{1st}), c^*_{1st}) > \pi(c^*_{1st}, c^*_{1st})$, a bidder with type epsilon above c^*_{1st} will remain active in the English auction indicating that $c^*_{Eng} > c^*_{1st}$which are clearly greater than zero since $\pi_1(G(\beta), c) < 0$ and $E(x|x < c) < c$. Hence, bidders with types epsilon above the first price cut-off type will earn positive profit in the English auction and $c^*_{Eng} > c^*_{1st}$. The argument for $\pi_1(G(\beta), c) > 0$ yields the opposite ranking by examining the relevant payoffs at c^*_{Eng}. ■

A.2 Proof of Theorem 2

Differentiating Eq.(32) with respect to x gives the following first order condition

$$[\pi_1(x, c) - b'(x)]\,[1 - F(x)]^{n-1} - [\pi(x, c) - b(x)]\,(n - 1)[1 - F(x)]^{n-2} f(x) = 0. \tag{A1}$$

Imposing symmetry gives

$$[\pi_1(c, c) - b'(c)]\,[1 - F(c)]^{n-1} - [\pi(c, c) - b(c)]\,(n - 1)[1 - F(c)]^{n-2} f(c) = 0. \tag{A2}$$

Differentiating both sides of Eq.(33) with respect to c gives Eq.(A2).

Using Eq.(A2) to obtain $b'(x)$ and substituting $b'(x)$ into the derivative of Π (the left hand side of Eq. (A1) gives

$$[\pi_1(x, c) - \pi_1(x, x)]\,[1 - F(x)] + [\pi(x, x) - \pi(x, c)]\,(n - 1)f(x). \tag{A3}$$

When $\pi_{12}(c, c) > 0$, both terms in Eq.(A3) work in the same direction and are positive (negative) for $x < c$ ($x > c$), indicating that the proposed bid function is the globally optimal response. When $\pi_{12}(c, c) > 0$, the two terms in Eq.(A3) work in opposite directions indicating a possible non-existence of a separating equilibrium. However, Condition 1 ensures that Eq.(A3) is positive (negative) for $x < c$ ($x > c$), indicating that the proposed bid function is the globally optimal response in this case as well. Finally, since Eq.(33) satisfies the boundary condition that the worst possible bidder (type c^*_{1st}) earns zero profit in equilibrium, the proof is complete. ■

A.3 Proof of Theorem 3

Differentiating Eq.(34) gives the following first order condition

$$\pi_1(x, c)[1 - F(x)]^{n-1} - [\pi(x, c) - b(x)]\,(n - 1)[1 - F(x)]^{n-2} f(x) = 0. \tag{A4}$$

Imposing symmetry and solving for $b(c)$ gives Eq.(35).

Substituting the equilibrium bid function $b(x)$ into the derivative of Π (the left hand side of Eq.(A4) gives

$$[\pi_1(x,c) - \pi_1(x,x)]\,[1 - F(x)] + [\pi(x,x) - \pi(x,c)]\,(n-1)f(x). \tag{A5}$$

As in the previous proof, both terms in Eq.(A5) work in the same direction when $\pi_{12}(c,c) > 0$ and are positive (negative) for $x < c$ $(x > c)$, indicating that the proposed bid function is the globally optimal response. When $\pi_{12}(c,c) > 0$, the two terms in Eq. (A3) work in opposite directions indicating a possible non-existence of a separating equilibrium. Once again, Condition 1 ensures that Eq. (A3) is positive (negative) for $x < c$ $(x > c)$, indicating that the proposed bid function is the globally optimal response in this case as well. Finally, since Eq. (35) satisfies the boundary condition that the worst possible bidder (type c^*_{2nd}) earns zero profit in equilibrium, the proof is complete. ■

A.4 Proof of Theorem 4

The first order condition obtained from maximizing Eq.(36) with respect to x is

$$-\left[\pi(E[z|z \leq x], c) - b(x)\right](n-1)\left(1 - F(x)\right)^{n-2} f(x) = 0. \tag{A6}$$

Imposing symmetry yields the equilibrium bid function $b(c) = \pi(E[z|z \leq c], c)$. Substituting $b(x) = \pi(E[z|z \leq x], x)$ into the left hand side of Eq.(A6) shows the first derivative of Π is,

$$-\left[\pi(E[z|z \leq x], c) - \pi(E[z|z \leq x], x)\right](n-1)\left(1 - F(x)\right)^{n-2} f(x). \tag{A7}$$

But, since π is decreasing in its second argument, Eq.(A7) is positive for $x < c$ and negative for $x > c$, indicating that the proposed bid function is the globally optimal response. Finally, since Eq.(37) satisfies the boundary condition that the worst possible bidder (type c^*_{Eng}) earns zero profit in equilibrium, the proof is complete. ■

A.5 Proof of Theorem 5

We prove the case where $\pi_1(G(\beta), c) > 0$. The truth of the opposite case follows easily.

When $\pi_1(G(\beta), c) > 0$, $E[x|x \leq c] \leq c$, implying that $\pi(c, c) \geq \pi(E[x|x \leq c], c), \forall c \in [\underline{c}, c^*_{Eng}]$. Since, $c^*_{BM} > c^*_{Eng}$, expected revenue in the English auction is less than predicted by the benchmark model.

Second price bids equal $\pi(c, c) - \frac{\pi_1(c,c)(1-F(c))}{(n-1)f(c)}$. Since $\pi_1(G(\beta), c) > 0$, this is clearly less than the benchmark bid of $\pi(c, c)$. Finally, $c^*_{BM} = c^*_{2nd}$ indicating that benchmark bids are higher than second price bids for all $c \in [\underline{c}, c^*_{2nd}]$. ■

A.6 Proof of Theorem 6

The proofs regarding the incumbent's price and average price are based on which of the following three ranges the lowest entrant marginal cost draw lies: $[\underline{c}, c^*_{Eng}]$, $[c^*_{Eng}, c^*_{1st}]$, or $[c^*_{1st}, \overline{c}]$. In addition, we will use the fact that the expectation of the entrant's marginal cost c_E, equals the expectation of the entrant's marginal cost, given that it is below the second lowest marginal cost of all bidders (i.e. the information released after the English auction). Or, $E[c_E] = E[E[c_E|c_E \leq c_{(2)}]] = E[E[\int_{\underline{c}}^{y} wf(w)dw/F(y)]]$.

Incumbent's Price: In the range $[\underline{c}, c^*_{Eng}]$, an entrant competes in the post-auction market, regardless of the announcement policy. The expected incumbent price is $E[p_I] = E[1/3(3t + 2c_I + G(\beta)]$ where the expectation is taken over $G(\beta)$. Following the first price auction $G(\beta) = c_E$ whereas after the English auction $G(\beta) = E[c_E|c_E \leq c_{(2)}]$ where the expectation is taken over possible values of the second highest bidder's type $c_{(2)} \in [\underline{c}, c^*_{Eng}]$. Since $E[p_I]$ is linear in $G(\beta)$ and the expectation of $G(\beta)$ equals c_E in the English auction, the expected price will be the same in this range of types. For potential types in the range $[c^*_{Eng}, c^*_{1st}]$, the expected price after the first price auction is still $E[1/3(3t + 2c_I + G(\beta)]$, however; the price following the English auction is simply the monopoly price $p_m = c_I - t$ which is clearly higher than the expected following the first price auction. Finally, once all types are above c^*_{1st}, the monopoly price persists in both situations and the expectation is the same over the range $[c^*_{1st}, \overline{c}]$. Since the expected price is the same in the upper and lower ranges, but higher in the intermediate range when the winner's bid is not announced, the expected incumbent price is lower under a policy of announcing the winner's bid.

Average Price: The average price in the market over the range $[\underline{c}, c^*_{Eng}]$ is $P_{Avg} = 1/6[6t +$

$3c_I + 3c_E]$ following an announcement of the winner's bid. An announcement of only the second highest bid produces an average price of $1/6[6t + 3c_I + 3/2c_E + 3/2E[c_E|c_E \leq c_{(2)}]]$. Once again, these expectations are linear in c_E and $E[c_E|c_E \leq c_{(2)}]$ indicating that the expected average price is the same over this range. In the range $[c^*_{Eng}, c^*_{1st}]$, both the entrant an incumbent prices following a first price auction are below the monopoly price of $p_m = c_I - t$ that arises after the English auction and thus the average price is lower in the former situation. Finally, both markets exist under the monopoly price and thus have the same expectation in the range $[c^*_{1st}, \overline{c}]$. Since the expected price is the same in the upper and lower ranges, but higher in the intermediate range when the winner's bid is not announced, the expected average price is lower under a policy of announcing the winner's bid. ■

A.7 Proof of Theorem 7

Each part is based on convexity/concavity of the relevant functions.

Consumer Surplus: Consumers buying from the entrant have surplus of

$$\begin{aligned}
&\int_{\widetilde{y}}^{1} [u_0 - t(1-y) - p_E]dy \\
= \; & (u_0 - p_E)(p_I - p_E + t)/2t - (p_I - p_E + t)^2/8t \\
= \; & (u_0 - 1/3(3t + c_I + 3/2c_E + 1/2G(\beta)))(3t + 1/2G(\beta) + c_I - 3/2c_E)/6t \\
& -1/9(3t + 1/2G(\beta) + c_I - 3/2c_E)^2/8t.
\end{aligned}$$

The second derivative, taken twice with respect to the incumbent's belief is negative. Hence, concavity dictates that consumers buying from the entrant have higher expected surplus following an English auction than following a first price auction.

Consumers buying from the incumbent have expected surplus of

$$\begin{aligned}
&\int_{0}^{\widetilde{y}} [u_0 - ty - p_I]dy \\
= \; & (u_0 - p_I)(p_E - p_I + t)/2t - (p_E - p_I + t)^2/8t \\
= \; & (u_0 - 1/3(3t + G(\beta) + 2c_I))(3t - 1/2G(\beta) - c_I + 3/2c_E)/6t \\
& -1/9(3t - 1/2G(\beta) - c_I + 3/2c_E)^2/8t.
\end{aligned}$$

The second derivative, taken twice with respect to the incumbent's belief, is positive. Hence, convexity dictates that consumers buying from the incumbent have higher expected surplus following a first price auction than an English auction.

Finally, summing the expected surplus of both types of consumers yields a second derivative, taken twice with respect to the incumbent's beliefs, of $-1/144t < 0$. Hence, consumers on average receive higher expected surplus following an English auction than following a first price auction.

Entrant Profit: Expected entrant profit in the post-auction market is $E_xE_y[\pi_E]$ where x is the lowest of the potential entrants' marginal cost draws, y is the second lowest, and π_E is given by Eq. (7). The second derivative of π_E with respect to the incumbent's belief, is greater than zero. Hence, the convexity of the profit function in the incumbent's belief dictates that expected entrant profits are higher following a first price auction than following an English auction.

Incumbent Profit: Expected incumbent profit in the post-auction market is $E_xE_y[\pi_I]$ where x is the lowest of the potential entrants' marginal cost draws, y is the second lowest, and π_I is given by Eq. (8). The second derivative of π_I with respect to the incumbent's belief is less than zero. Hence, the concavity of incumbent profits indicates that expected incumbent profit is greater following an English auction than following a first price auction. ■

A.8 Proof of Theorem 8

As in the proof of Theorem 6, we use the fact that the expectation of the entrant's marginal cost c_E, equals the expectation of the entrant's marginal cost, given that it is below the second lowest marginal cost of all bidders (i.e. the information released after the English auction). Or, $E[c_E] = E[E[c_E|c_E \leq c_{(2)}]] = E[E[\int_{\underline{c}}^{y} wf(w)dw/F(y)]]$. The relevant ranges of bidder types in the Cournot setting are $[\underline{c}, c^*_{1st}]$, $[c^*_{1st}, c^*_{Eng}]$, or $[c^*_{Eng}, \overline{c}]$.

Entrant's Quantity: When both types of announcement policy produce an entrant firm, the expected entrant quantity following an announcement of the winner's bid is $E[q_E] = E[1/3(1+c_I - 2c_E]$. Following the announcement of only the second highest bid, the expected entrant quantity is $1/3(1+c_I-3/2c_E-1/2E[c_E|c_E \leq c_{(2)}])$. Since both expectations are linear in c_E and $E[c_E|c_E \leq c_{(2)}]$, the expected quantities are the same. In the range $[c^*_{1st}, c^*_{Eng}]$, an entrant firm is only present in the market following the English auction. The other market is monopolistic and entrant quantity is therefore zero. Finally, both situations produce a monopoly and zero entrant output in the range $[c^*_{Eng}, \overline{c}]$. Since the entrant produces the same expected quantity in the upper and lower ranges, and they continue to produce in the intermediate range following the English auction, expected entrant production is higher following the English auction.

Incumbent's Quantity: In the range $[\underline{c}, c^*_{1st}]$, the expected incumbent's quantity following an announcement of the winner's bid is $E[1/3(1 - 2c_I + c_E)]$. Following the English auction, it is $E[1/3(1 - 2c_I + E[c_E|c_E \leq c_{(2)}])]$. Once again, the fact that these two expectations are linear in c_E and $E[c_E|c_E \leq c_{(2)}]$ indicates that the expected incumbent quantity is the same in this range. In the range $[c^*_{1st}, c^*_{Eng}]$, the incumbent still has competition following an English auction, but becomes a monopoly following a first price auction. The monopoly produces $(1 - c_I)/2$ which is strictly more than the incumbent produces following a first price auction. Finally, the incumbent produces the same monopoly output in the range $[c^*_{Eng}, \overline{c}]$. Since the incumbent's expected output is the same in the upper and lower ranges and they supply more following an announcement of the winner's bid in the intermediate range, expected incumbent output is less following an English auction.

Total Quantity: In the range $[\underline{c}, c^*_{1st}]$, the expected total output in the market following an announcement of the winner's bid is the expectation of the sum of the entrant and incumbent output, $E[1/3(2 - c_I - c_E)]$. Following an English auction, expected total output is $E[1/3(2 - c_I - 3/2c_E + 1/2E[c_E|c_E \leq c_{(2)}]]$. Since each of the expectations is linear in c_E and $E[c_E|c_E \leq c_{(2)}]$, they are equal. Over the range $[c^*_{1st}, c^*_{Eng}]$, it has been established that the entrant produces more and the incumbent produces less following the English auction. The question is whether or not the sum is greater or less than the monopoly output following the announcement of the winner's bid. The monopoly output is $\frac{1-c_I}{2}$. The expected total output after the English auction is $E[1/3(2 - c_I - 3/2c_E + 1/2E[c_E|c_E \leq c_{(2)}]]$ which is strictly less than $E[1/3(2 - c_I - 2c_E)]$ since the expectation is taken over the entire range of possible types. Hence , the difference between total output in the market and the monopoly output is at least $(1 + c_I - 2c_E)/6$. This is clearly negative in the relevant range since the lowest c_E can be is $c^*_{1st} = \frac{1+c_I}{2}$. Finally, both policies result in the monopoly output in the $[c^*_{Eng}, \overline{c}]$ range. Since the total output is the same in the upper and lower ranges and strictly less following the

English auction in the intermediate range, expected total output is less following an English auction. ■

A.9 Proof of Theorem 9

Each part is based on the convexity/concavity of the relevant functions.

Consumer Surplus: Expected consumer surplus is $E_x E_y[Q^2]$ where x is the lowest of the potential entrants' marginal cost draws, y is the second lowest, and $Q = (2 - c_I - \frac{3}{2}x + \frac{1}{2}G(\beta))/3$ is the sum of Eqs. (18) and (19). The second derivative taken twice with respect to the incumbent's belief is positive. Hence, convexity dictates that expected consumer surplus is higher following a first price auction than following an English auction.

Entrant Profit: Expected entrant profit in the post-auction market is $E_x E_y[\pi_E]$ where x is the lowest of the potential entrants' marginal cost draws, y is the second lowest, and π_E is given by Eq. (20). The second derivative of π_E, taken twice with respect to incumbent beliefs is $1/6 > 0$ indicating the convexity of entrant profits in the direction of interest. Hence, expected entrant profits are higher following a first price auction than following an English auction.

Incumbent Profit: Expected incumbent profit in the post-auction market is $E_x E_y[\pi_I]$ where x is the lowest of the potential entrants' marginal cost draws, y is the second lowest, and π_I is given by Eq. (21). Since expected incumbent output does not depend on the incumbent's beliefs, the incumbent expects the same level of profit independent of the auction form used. ■

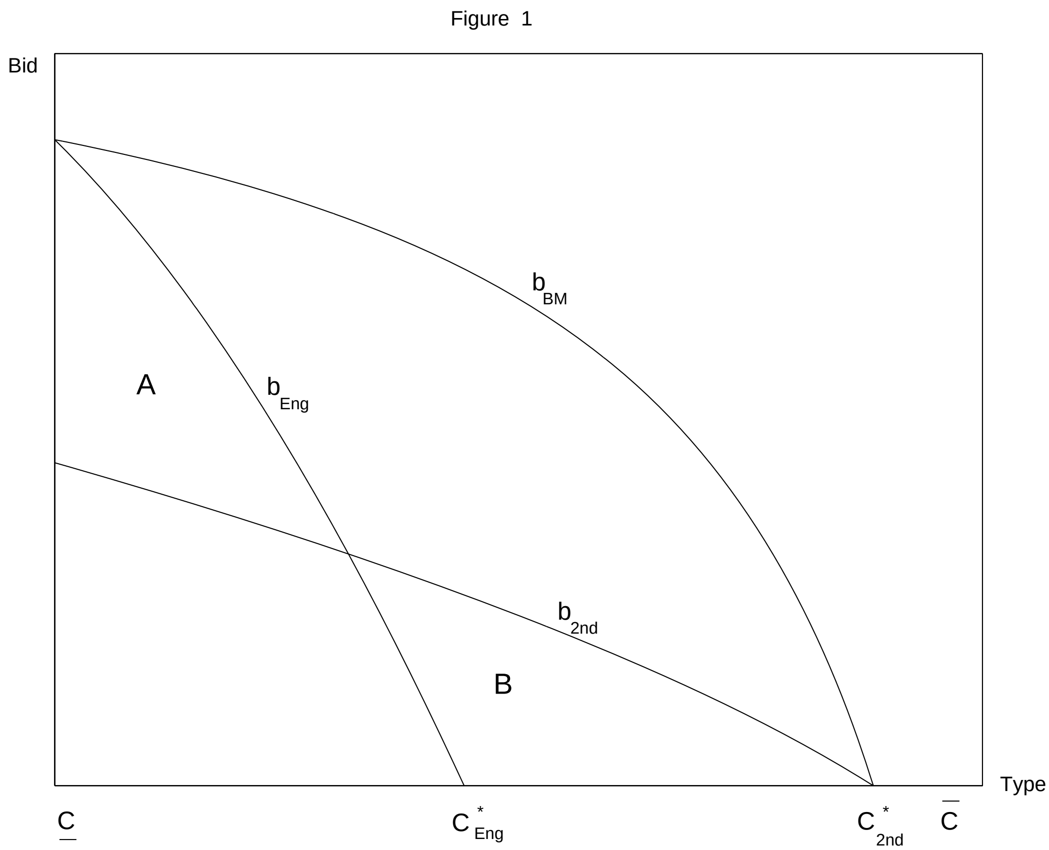

Figure 1: $\pi_1(G(\beta), c) > 0$

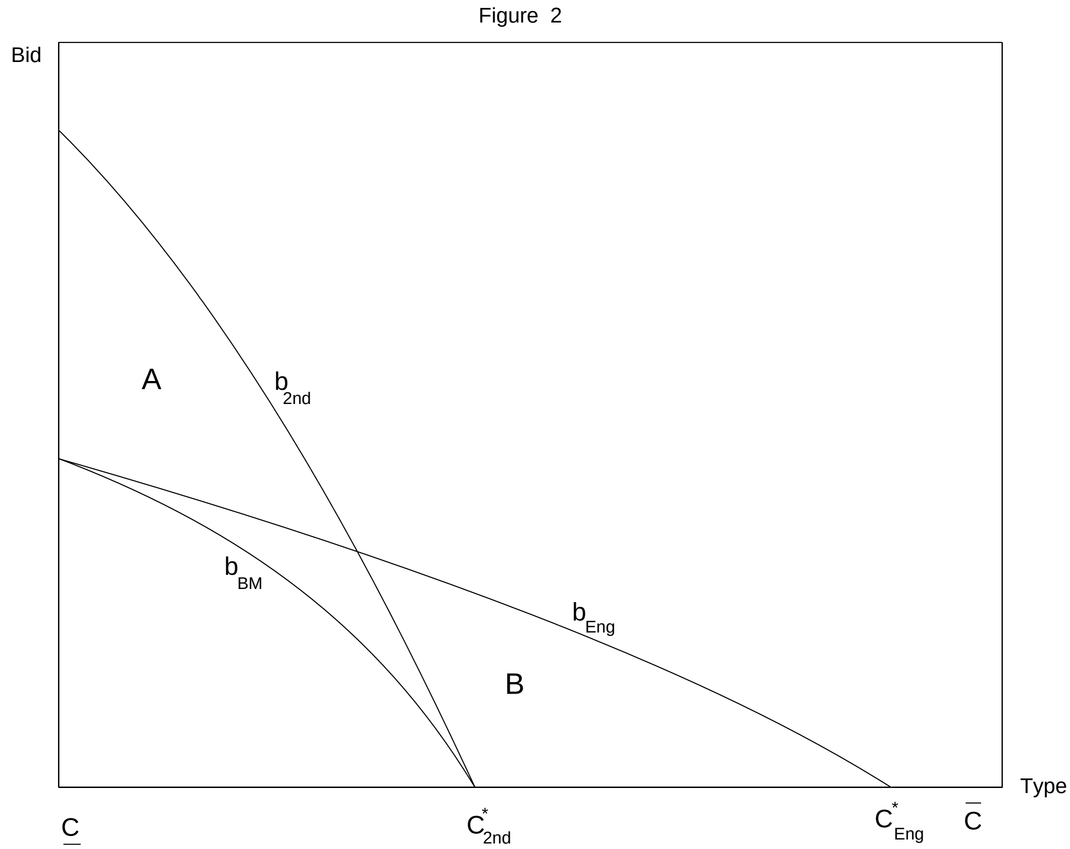

Figure 2: $\pi_1(G(\beta), c) < 0$