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Interferometry in FMCW Radars

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ABSTRACT

Interferometry is used in many fields using all frequencies of the electromagnetic spectrum and sound waves. In this study, data was collected from an FMCW radar is used at multiple heights above a flat surface on which sat a single retroreflector. These data were post-processed to discover the signal obtained from the target and then the phase form the radar at multiple locations was compared. Using the known geometry and measured phase, we find the interferometry is possible using a freerunning radar under certain geometric conditions.

Keywords: FMCW, radar, interferometry

Introduction

A Frequency Modulated Continuous Wave (FMCW) radar is typically utilized where low-cost, close-range, and all-weather distance measurement is desired. Because an FMCW radar transmits continuously, the peak to average ratio of power on the target is high, which tends to lower the cost of the transmitter as compared to pulse radars with the same relative performance. Short-range operation is achieved because the receiver is constantly listening as opposed to pulse radars where the receiver is quieted, or blanked, during transmit. FMCW radars are typically found in automotive, forward-looking collision avoidance systems for the reasons stated above. Radar interferometry has applications for remote sensing in low visibility environments. Fielding an FMCW interferometric radar would be possible for applications where size, weight, power, and cost prohibit most other sensors.

A typical FMCW radar block diagram is shown in Fig. 1 transmits a modulated radar signal and receives delayed versions of that signal from scattering objects, commonly called targets. The advantages of a linearly modulated FMCW radar utilize triangle or saw-tooth modulation, the radar transmits frequency should be a linear function of time [1] as shown in Fig.2. During any one segment of time corresponding to a linear segment of the modulation waveform, the transmit frequency is

$$(t) = mt + {}_o \text{ rad/s} \qquad (1)$$

where m is the modulation rate in Hz/s and ω_o is the low-frequency limit of the radar in rad/s. Since phase is the time integral of frequency:

$$(t) = \quad (t)dt = \frac{mt^2}{2} + {}_o t + {}_0 \text{ rad} \qquad (2)$$

where θ_o is a constant phase.

Fig. 1 FMCW Radar block diagram

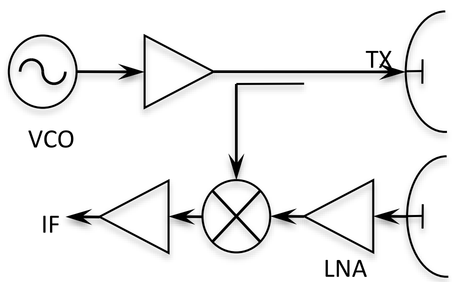
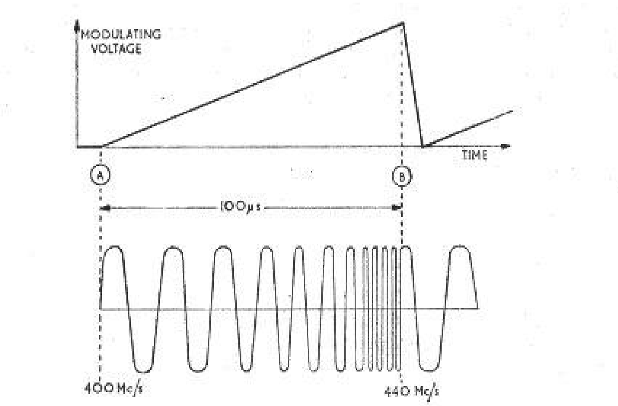


Fig. 2 Linear modulation waveform



Consider the case of a single point-target, meaning a target that occupies a single point in space. The receive signal would be a time-delayed version of the transmit signal of Equation 2. The phase of the receive signal is

$$\phi_r(t) = \frac{m(t+\tau)^2}{2} + \omega_o(t+\tau) + \theta_r \text{ rad} \qquad (3)$$

where τ is the time delay and θ_r is the constant receive phase offset. When the mixer in Fig. 1 is used as a phase detector, the mixer output is the phase difference between transmit and receive:

$$\phi_i(t) = \frac{m(t+\tau)^2}{2} - \frac{mt^2}{2} + \omega_o(t+\tau) - \omega_o t + \theta_r - \theta_o \text{ rad} \qquad (4)$$

$$\phi_i(t) = \frac{m(2t\tau + \tau^2)}{2} + \omega_o\tau + (\theta_r - \theta_o) \text{ rad} \qquad (5)$$

In general, the transmit and receive phase, θ_o and θ_r, is not explicitly known.

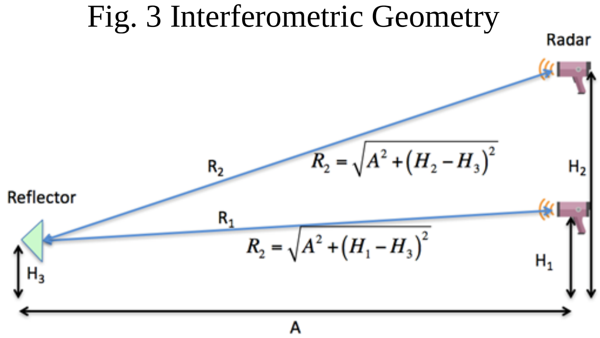
The process described so far gives phase as a function of time delay, τ, which is a one-dimensional measure of round-trip distance between the radar and the target. The FMCW radar described here has also been used in a Synthetic Aperture Radar (SAR) mode which yields a two-dimensional map in range and cross-range [2]. Radar interferometry is typically used stable, instrumentation radars to yield the third dimension of height by comparing the relative phases of two or more measurements at different heights [3]. An interferometric radar depends on measuring the phase difference between two radar measurements, θ_{i1} and θ_{i2}. The problem is that θ_o and θ_r are not known for any one measurement. If a low-cost radar such as the one described here can be used as an interferometer, there may be widespread applications such as remote monitoring of the water level in a drainage pond or irrigation system.

Many interferometers utilize a single coherent radiation source, such as a laser, split that beam and send one on a path that is different than the other. These two paths have different propagation times and the phase between the two is detected and compared. In radar interferometry, radio waves and are generated at different times. For example, data is collected at one location, the radar is moved, and the second data set is collected. The radar usually has a stable source such as a crystal oscillator or locked to GPS time. In the simple FMCW radar shown in Fig. 1, there is no stable source of radio waves. Furthermore, the modulation and data collection are not synchronized or coherent to any stable time base. To discover if a simple FMCW radar can be used for interferometry, the principle equations need to be derived and data collected with a candidate radar.

Experimental Setup

Fig. 3 shows the basic geometry using the same radar and at two heights, H_1 and H_2. A strong radar reflector is placed at a height H_3. The two propagation paths are represented R_1 and R_2. The goal in this case is the discover the height of the target.

Fig. 3 Interferometric Geometry



For each path, R_i, the instantaneous receive phase due to round trip propagation delay is

$$phase = \frac{(2\ \ )}{\ } 2R_i \text{ rad} \qquad (6)$$

where λ is the wavelength of the radio wave. The difference in phase between the two paths is

$$phase = \frac{(2\ \ )}{\ } 2\left(\sqrt{A^2+(H_1\ \ H_3)^2}\ \ \sqrt{A^2+(H_2\ \ H_3)^2}\right) \text{ rad} \qquad (7)$$

Equation 7 cannot be solved for H_3, but an approximation may be relevant. To solve for H_3, Equation 7 is first rearranged

$$phase = \frac{(2\ \ )}{\ } 2A\left(\sqrt{1+\frac{(H_1\ \ H_3)^2}{A^2}}\ \ \sqrt{1+\frac{(H_2\ \ H_3)^2}{A^2}}\right) \text{ rad} \qquad (8)$$

and expanded in a Taylor series

$$phase = \frac{(2\ \ )}{\ } 2A\left(\sum_0^\infty \frac{(\ \ 1)^2(2n)!}{(1\ \ 2n)(n!)4^n}\left(\frac{H_1\ \ H_3}{A}\right)^{2n}\ \ \sum_0^\infty \frac{(\ \ 1)^2(2n)!}{(1\ \ 2n)(n!)4^n}\left(\frac{H_2\ \ H_3}{A}\right)^{2n}\right) \text{ rad} \qquad (9)$$

If A is much greater than the (H1-H3) or (H2-H3) then the higher-order terms can be ignored, resulting in:

$$phase \approx \frac{(2\ \ )}{\ } 2A\left(1+\frac{1}{2}\frac{(H_1\ \ H_3)^2}{A^2}\ \ 1\ \ \frac{1}{2}\frac{(H_2\ \ H_3)^2}{A^2}\right) \text{ rad} \qquad (10)$$

Which can be solved for H_3:

$$H_3 = -\left(\frac{\lambda}{2\pi}\right)\frac{\Delta phase-(H_1^2-H_2^2)}{2(H_1-H_2)} \text{ rad} \qquad (11)$$

If $A >> H_i$, a careful measurement of phase difference may reveal target height.

Experimental Results

H_1: Height of the radar at low standing
H_2: Height of the radar at high standing
Expected H_3 value is -0.314325 m

Experiment 1 results: Target distance is 25 feet (7.62 m).

Radar Location	H (m)	Phase (rad)
A	1.266825	0.6147
B	0.9525	-0.4906
C	0.314325	0.2061

Table 1 Result data for a target distance of 7.62 meters

	∆Phase	H3 (m)
BC	-0.4086	-0.0508177
AC	0.6967	0.1081400

The resulting theoretical height differs greatly from the actual height of the target.

Experiment 2 results: Target distance is 12 feet (3.6576 m).

Radar Location	H (m)	Phase (rad)
A	1.266825	-2.016
B	0.9525	-1.962
C	0.314325	1.455

Table 2 Result data for a target distance of 3.6576 meters

	∆Phase	H3 (m)
BC	3.417	-0.538800
AC	3.471	-0.431689

The resulting theoretical height H3 is slightly different from the actual height of the target. In addition, when the radar at height H2 is placed at a higher elevation, the resulting theoretical height H3 is much closer to the actual height of the target.

Conclusion

The study described here was to calculate the height of a target using an FMCW radar at different heights. From our findings, we conclude that the distance between the target and the FMCW radar plays a big role in this study. It is possible to approximate an accurate height of the target if the distance between the radar and the target is small (i.e., less than 12 feet) but not too small or the approximation in Equation is no longer valid. However, as this distance grows, the measured height value begins to differ greatly from the true height.

References

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