Human Propulsion

Introduction
Movement is very important for humans. From our earliest days, we seem eager to get up and move around. The most celebrated moments of a child's life seem to be geared around the ability to move: the first crawl, the first walk, the first bike ride, and (every parent's nightmare) the first driver's license. The ability to get around is one of the most cherished abilities and freedoms. For many Americans, our ability to get around when and how we want is almost considered a sacred right.

Physics textbooks, which have many chapters devoted to motion, build a great many examples and problems around our means of propulsion. We model displacement problems on people walking in different directions along city blocks. We work constant acceleration problems that deal with dragsters speeding down a track. We study the issue of relative velocities with problems of airplanes flying in crosswinds. Even when we discuss relativity, we start it by considering motion on a train.

However, rarely do we ever investigate the motion of these objects very closely. In most problems and examples, we assume that the motion occurs either as constant velocity or constant acceleration. This is hardly ever the case. Due to factors like friction, wind resistance, or even the manner in which the propulsion occurs, motion often involves complicated accelerations both in the positive and negative directions that cause the velocity to change on a constant basis.

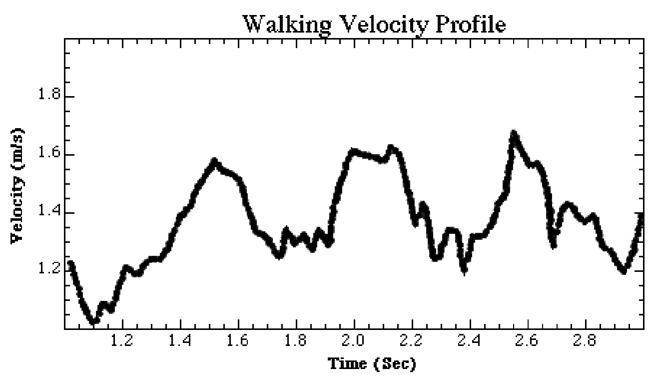

Fig. 1: Velocity vs. time plot for someone walking

This is especially true when we study the various methods of self-propulsion. Most forms of this, such as walking, crawling, or swimming, involve motion that is fraught with wild swings in the acceleration. Figure 1 is an example velocity profile of a person walking. This plot shows that the velocity of the person has a cyclical nature that has a period of about one-half second. Even though the changes in the velocity are not very much (about .2 m/s), such rapid changes correspond to accelerations that are quite large.

For someone who is walking, these accelerations are occurring mostly because of the manner of bipedal propulsion. Most of the time that a person is walking, they are actually pivoting upon one foot. As the foot that is in contact with the ground passes underneath the body on its way behind the body, there is a pushing forward motion that provides the body with a positive acceleration. However, as the other foot comes forward and hits the ground, there is the beginning of a braking motion that slows the body. This continues until the body begins to pass over this foot and the pushing off motion begins. At the same time, the person is also bobbing up and down and swaying side to side as they pivot on their feet. These motions also involve accelerations which will have the same periodicity as the accelerations in the path of travel.

Energy Transformation
These series of positive and negative accelerations comes at a price. The legs have to do work, as they are required to apply forces to perform both of these actions. This means that energy has to be expended by the body. What seems somewhat counterintuitive is that some of this energy is being

expended to slow down the motion, as when the front foot hits the ground. If this part of the motion did not occur, then additional energy would not be needed to accelerate the body back to speed. This expenditure of energy to slow down the body and then accelerate it back decreases the efficiency of the body, leading to more energy loss.

All forms of self-propulsion have some form of this phenomenon, although to different degrees. For instance, walking on crutches involves a much greater difference between highest and lowest velocities reached, as the motion more resembles a series of falling-forwards accelerations followed by crutches hitting the ground decelerations. Movement on a bike or in a wheelchair is much smoother, as the main impediments that must be overcome are wind resistance and friction in the wheels.

Model and Theory

For this system, we must go back to the basic definitions for kinematics. In doing so, let us look at the motion of an object in just one dimension. Later, we will be able to generalize this result to all three dimensions. The average velocity and acceleration of an object that is moving in 1-dimension are given by

$$v_{avg} = \Delta x/\Delta t$$
$$a_{avg} = \Delta v/\Delta t$$

It should be noted that the velocity used in the calculation of the average acceleration is the instantaneous velocity, whereas the first formula is for the average velocity. This creates a problem for us, as one cannot just simply calculate the acceleration of an object from the information about its position over time. However, we can get a pretty good estimate for the instantaneous velocity of an object with a judicious choice of Δt. If we make our time intervals very small, we can approximate the acceleration as being constant over the interval, which means that the instantaneous velocity can be approximated by the average velocity repositioned at a new time, which then will allow us to measure the acceleration of the object over the time interval.

With this in mind, it becomes possible to measure the amount of work that an individual does by moving in a non-constant manner using only position information and a simple spreadsheet. If we have a set of position and time values (x_n,t_n with n=1, 2, 3, …N) where $t_{n+1} - t_n = \Delta t$ is small, then we can approximate the instantaneous velocity values v_n by

$$v_n = (x_{n+1} - x_n)/ \Delta t, \; n = 1, 2, 3, \ldots N-1$$

This instantaneous velocity can then be used to calculate the acceleration values a_n as

$$a_n = (v_{n+1} - v_n)/ \Delta t, \; n = 1, 2, 3, \ldots N-2$$

With this acceleration, we can now calculate the average force that is required over this time interval to create this motion by multiplying the mass times this acceleration

$$F_n = ma_n , \; n = 1, 2, 3, \ldots N-2$$

To find the work that is performed, we need to simply multiply each of these forces and the distance over which each of these forces operates. This is where we encounter a small problem. The n^{th} acceleration is calculated using v_{n+1} and v_n. These two velocity terms, in turn, are calculated using the position data x_{n+2}, x_{n+1}, and x_n. At first glance, this would seem to indicate that the acceleration an occurred over the distance x_{n+2} - x_n. However, this is only the span of position data that went into the calculation; it covers much more than the distance over which the acceleration is supposed to have occurred. To see this, look at the range of the data that went into calculating the acceleration a_{n+1}. It covers the span of data from x_{n+1} to x_{n+3}, which overlaps with the data that went into calculating the acceleration a_n. If we were to take this data span as the distance over which the accelerations occurred, we would have two different accelerations for the span x_{n+1} to x_{n+2}.

To find out the true distance span for each acceleration, we need to go back to our derivation of the velocity. We defined the n[th] velocity as $v_n = (x_{n+1} - x_n)/ (t_{n+1} - t_n)$. Since this velocity really comes from the average velocity over the time interval, we really should "reposition" it if we are going to allow it to stand in for the instantaneous velocity. We can approximate this as the instantaneous velocity if we reposition it at the average time and distance, i.e. $(x'_n, t'_n) = ((x_{n+1} + x_n)/2, (t_{n+1} + t_n)/2)$. Doing so makes it much more evident over which distance span the acceleration occurred. That is, it is over the distance interval $x'_{n+1} - x'_n = (x_{n+2} + x_{n+1})/2 - (x_{n+1} + x_n)/2 = (x_{n+2} - x_n)/2$. Using this, we get that the amount of work done over the each time interval is given by

$$W_n = F_n \Delta x_n = m\, a_n\, (x_{n+2} - x_n)/2 \quad , n = 1, 2, 3, \ldots N-2$$

One thing that we should note is that the work done will be either positive or negative depending upon if the acceleration is positive or negative. In cases where we are discussing an outside agent operating on an object, the sign of the work is very important, as it denotes whether the external agent is doing work on the object (positive work) or the object is doing work on the external agent (negative work). However, for this system, the external agent and the object are one in the same: the human body. Therefore, the sign of the work is irrelevant, as the body is going to be the one that is doing the work. Therefore, it is proper for us to take the absolute value of the work for the purposes of summing to find the total work, i.e.

$$W_{total} = \sum_{n=1}^{N-2} |W_n| = \sum_{n=1}^{N-2} \left| \frac{m a_n (x_{n+2} - x_n)}{2} \right|$$

If this approximation is valid, then this sum is the total amount of work done to accelerate/decelerate the body while moving the distance traveled in the dimension under consideration. If we perform this operation for all three dimension (back-forward, up-down, and side-to-side), we will have the total energy expended by the body to propel itself a certain distance. We can then figure out how much energy we expend to this process in moving 1,600 meters (close to a mile) by multiplying this quantity by 1,600 meters and dividing by the total distance traveled during our motion.

Apparatus and Experimental Procedures

To perform the activity correctly, we will need to be able to measure the instantaneous acceleration in all three dimensions while a person propels themselves over some distance. This will be very hard to do without the aid of a three axis accelerometer. One such device that is commercially available is made by Pasco, which also sells the software that will sum the data as in the model above if it is programmed correctly. Using such an accelerometer, the procedure is quite simple.

Experimental setup:

1. Measure 10 meters along an unobstructed floor space. This will be the distance through which each student will propel themselves
2. Attach the Pasco accelerometer to a student.
3. Start the Pasco software and have the student walk through the 10 meters. Stop the software and record the total work if it is available. If only the time-position-acceleration data is available, copy the data into a spreadsheet and analyze it to calculate the total work.
4. Repeat the procedure twice.
5. Repeat for all lab partners.

ESA21: Environmental Science Activities

Activity Sheet
Walking

Name:

Instructor:

To process this large quantity of data, it is best to use either the Pasco or a spreadsheet program to calculate velocities, accelerations and forces from the position data. Once you have used this data to calculate the total work, fill in the data below from your three trials. To convert from joules to food calories, multiply by .000239. Since you went 10 meters, divide the total work by 10 to get W/meter

Trial	$W_{forward/back}$	$W_{up/down}$	$W_{side/side}$	Total Work	Work/meter
1					
2					
3					

A. Average work/meter = ________ J/m
B. Work per mile = W/m * 1609m/mile = A * 1609 = ______ J/mile
C. Work in food calories/mile = B * .000239 food calories/J = _______ food calories/mile

1. What are the possible random errors in our measurements?

2. What are the possible systematic errors in our experiment?

3. It is estimated that the total amount food calories used to walk a mile is about 1.3 fcal/kg. Given your mass, how does this estimate fit to what you have estimated above just for the acceleration/deceleration part of your motion?

4. Where is other energy used in your body while you are moving?