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The Ptolemaic System: A Detailed Synopsis

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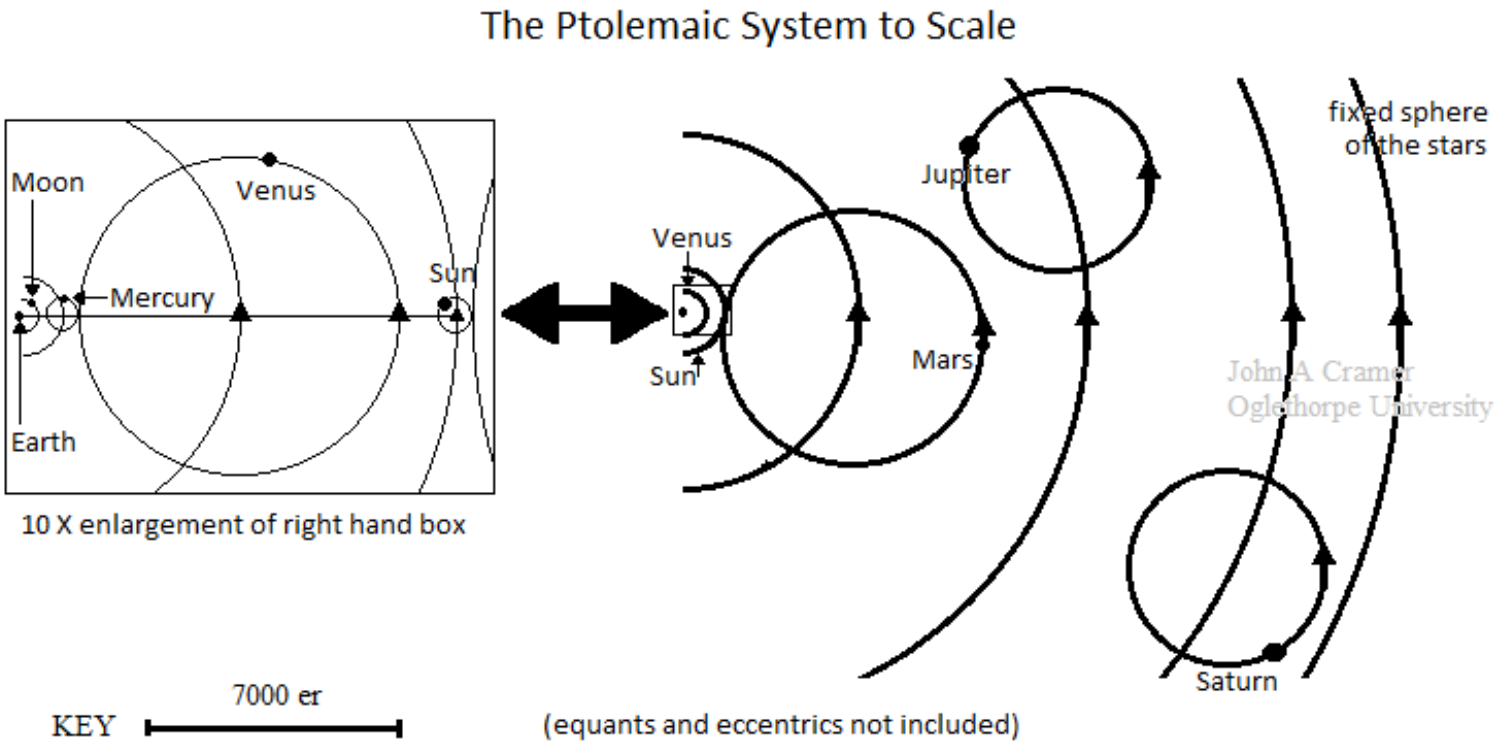
The Ptolemaic System, constructed by Claudius Ptolemeus (the Latin form of his name), was the most influential of all Earth centered cosmological systems. His ingenious and creative work is primarily recorded in his book The Mathematical Systematic Treatise which the Arabs characterized as "the greatest" and, in so doing, gave the book its most used name, Almagest.

Ptolemy lived in or near Alexandria, Egypt in the middle of the first century AD and had access, evidently, to the great library of the Museum of Alexandria because he made free use of what seems to have been an enormous supply of planetary positions extending back as far as perhaps 900 years. His science, thus, was experiment based in the sense that he worked with an extensive database of observations. He even made quite a number of his own observations with self-constructed instruments whose designs he passed down in his writings.

On the theoretical side, he relied most on geometry and argument in the style of Euclid's great work, The Elements of Geometry, which, even in Ptolemy's time, had set the record for time as the standard textbook, 500 years at that time (later extended to about 2200 years!). But his world view and metaphysics derived from Pythagoras, Plato, and Aristotle from whom he learned the heavenly bodies and the Earth are spheres and the heavenly bodies move around the Earth at uniform speeds on circles and combinations of circles. (The circles were also sometimes called spheres and then had to be imagined to be clear crystalline, all of which eventuated in a less than crystal clear understanding of the circles/spheres.) From Aristotle he also learned there is no empty space in the universe ("nature abhors a vacuum") which Ptolemy understood to entail that the planetary spheres fitted tightly together with no intervening space. Thus, the Ptolemaic



system is quite crowded as a look at the figure above reveals. The figure is artificial in that the seven "planets" are seldom on the same side of the Earth as in the figure. Each planet revolves on a small circle (an epicycle) that in turn revolves on a larger circle (the deferent). The epicycle of the Moon is too small to indicate on the scales of this figure.

Note especially, in the box on the left of the figure, that the centers of the epicycles of Venus and Mercury are fixed to the line between the Earth and the center of the Sun's epicycle. Nature forced this feature on the system because Ptolemy was trying to replicate the actually observed motions of these planets which always appear in the general direction of the Sun from Earth. For greater details on the motions, we must examine each planetary model individually. The scale of the main figure is in Earth radii (er).

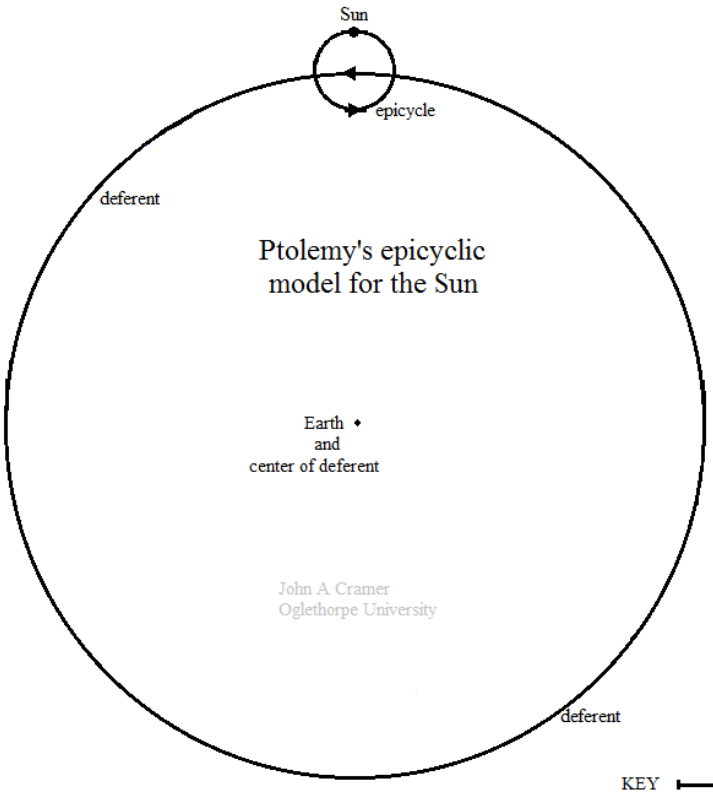
Ptolemy's numbers are listed in the following table. An er is an Earth radius and a yr is an Earth year (which equals a solar year). Ptolemy actually used an "Egyptian year," of just 365 days, in his calculations. Mercury alone has a third circle of radius 5.75 er which rotates in one year but the reverse (clockwise) direction, carrying the plane of the eccentric deferent with it. The stars are all at 20,000 er away.

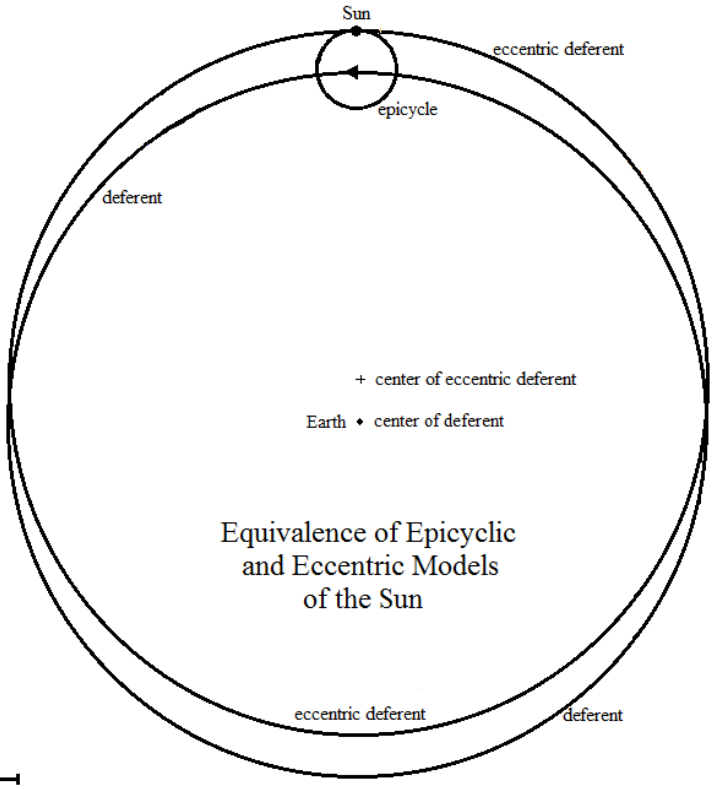
Planet Name	Deferent radius (er)	1st epicycle (eccentric) radius (er)	2nd epicycle radius (er)	Deferent Period (yr)	1st epicycle (eccentric) period (yr)	2nd epicycle period (yr)
Sun	1210	50.4	-	1	-	-
Moon	48.5	10.07	5.125	.081	-	-0.07448
Mercury	115	5.75	43.13	1	-	0.31765
Venus	622.5	12.95	447.85	1	-	0.6248906
Mars	5040	504.3	3318	1.882	-	2.1368
Jupiter	11503.5	527.2	2462	11.866	-	1.09284
Saturn	17026	971.2	1998	29.452	-	1.03587

Ptolemy's basic units are all wrong. His mean distance from the Earth to the Sun is 20 X too small. Then too, he uses the value 3250 "miles" for the radius of the Earth which is about 18% too low. The Egyptian year is just 365 days. Ptolemy uses it even though he is well aware the year is more like 365.24 days.

Starting with the simplest models first, Ptolemy thought the motion of the Sun required only two circle, an epicycle on a deferent. As such, it is a good model of the equivalence of an "eccentric" and deferent/epicycle combination where the epicycle center revolves around the earth but the planet has zero speed (does not revolve) about the epicycle center. Ptolemy takes pains to show that either an eccentric or an epicycle on a deferent can be made to give exactly the same results. I draw both models in the figure below. An eccentric is an epicycle and deferent but a special one where the planet does not revolve on the epicycle while the epicycle center revolves on the deferent. Using the Sun values, for example, the center of the epicycle revolves on the deferent in a year but the planet has zero rotation on the epicycle. The sum is that the planet merely travels on the off center (eccentric) circle.

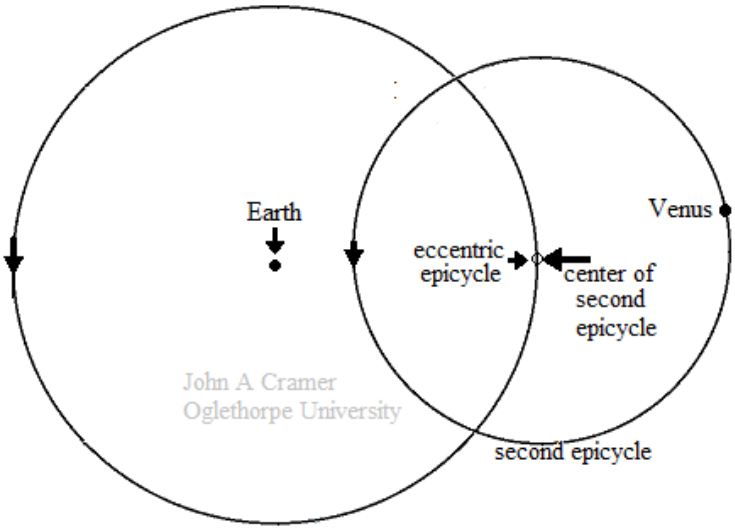
If we are counting circles, regardless of whether we use the eccentric or the epicycle models, Ptolemy used just two circles for the Sun. In addition, Ptolemy is aware of the "precession of the equinoxes" through the work, three hundred years earlier, of Hipparchus of Rhodes. He believed it amounted to 1^0 per century rotation of every planetary orbit. This is an overestimate of about 40%. Nonetheless, it implies a third circle for the Sun.

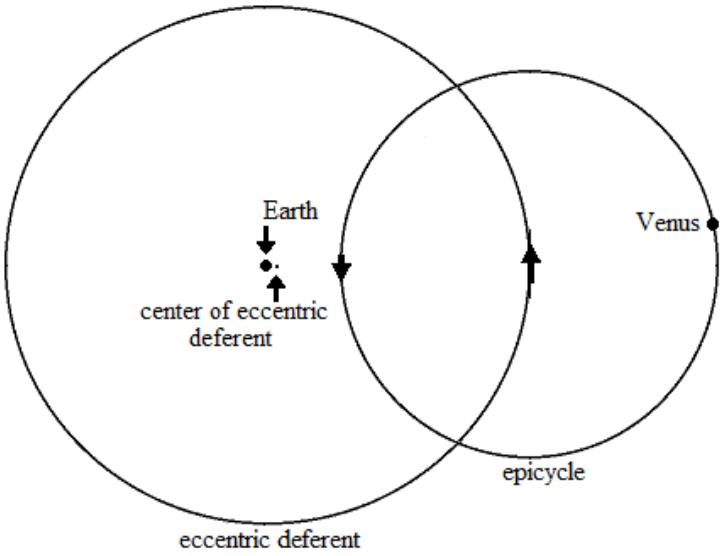




KEY 500 er

The next simplest model is for Venus. Ptolemy felt he needed only an epicycle on an





Epicycle model of Venus

KEY 526 er

Eccentric & epicycle
model of Venus

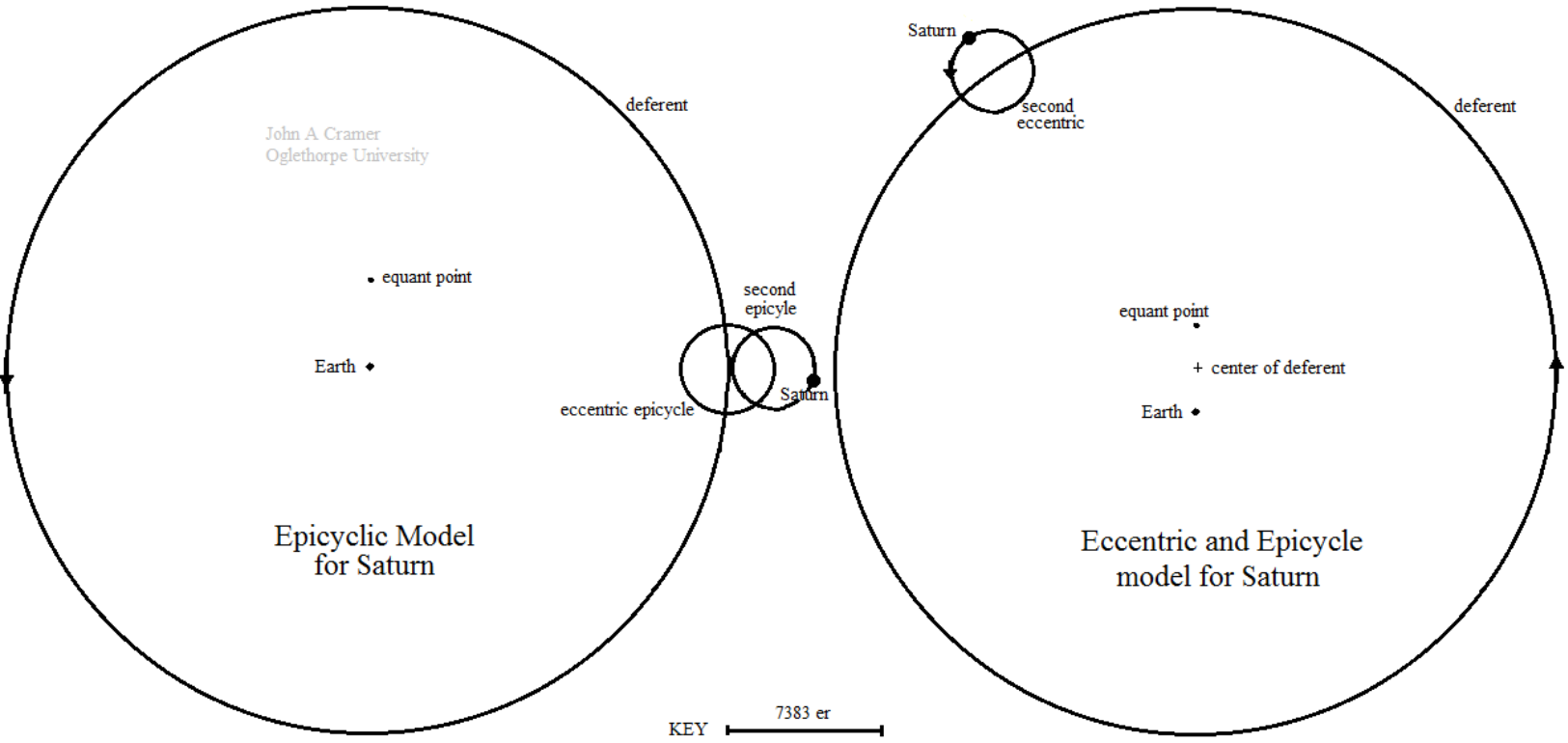
eccentric deferent to model the motions of Venus. In epicyclic terms, it is an epicycle on an epicycle.

Note especially a problem Ptolemy ignored. The second epicycle is so large that Venus will be 6 and a half times larger when close to Earth as opposed its size at its greatest distance from Earth. Perhaps Ptolemy thought Venus too small for the differences to be detected. The effect is quite real, however, and Ptolemy's figures are not far wrong.

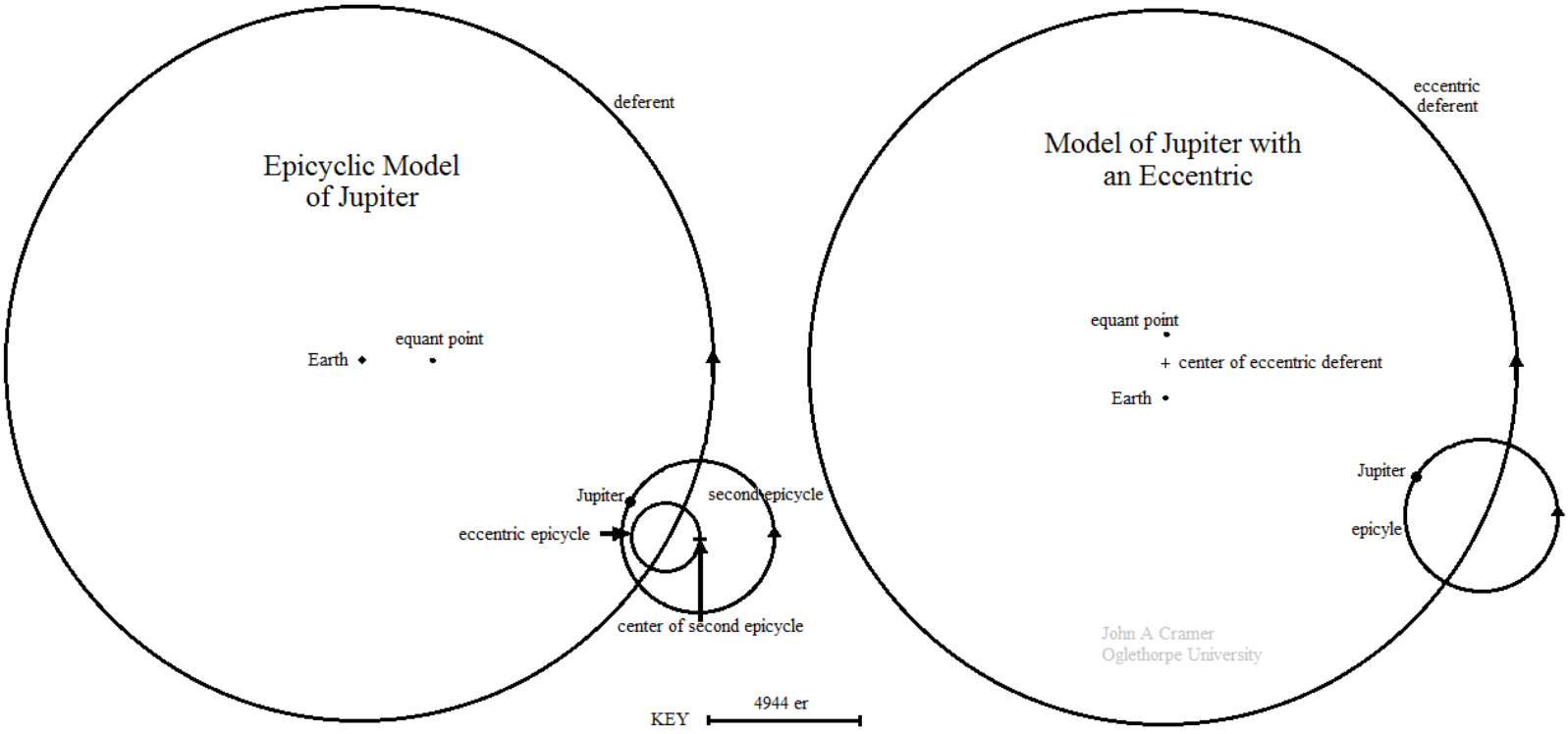
With an eccentric and an epicycle, Venus requires 3 + 1 (for precession of the equinoxes) circles for a total of 4 circles.

Next in simplicity are the three planets Saturn, Jupiter and Mars. Their models require an epicycle on an eccentric deferent but even that is not sufficient and Ptolemy was forced to note that the motion of the centers of the epicycles of these three planets move at a variable speed when viewed from either the center of the eccentric or the Earth. To regain the lost uniform (constant speed) motion, he was forced to invent a point, the "equant point," from which the motion of the center of the epicycle would be seen to be uniform (although no viewer of the time had the slightest chance of ever being there to see it). The equant point position was always such, for Ptolemy, that the center of the eccentric was midway between it and the center of the Earth and the diameter of the epicycle on which the planet sat pointed at the equant point.

Starting with the motion of Saturn and working inward then, we have the following models

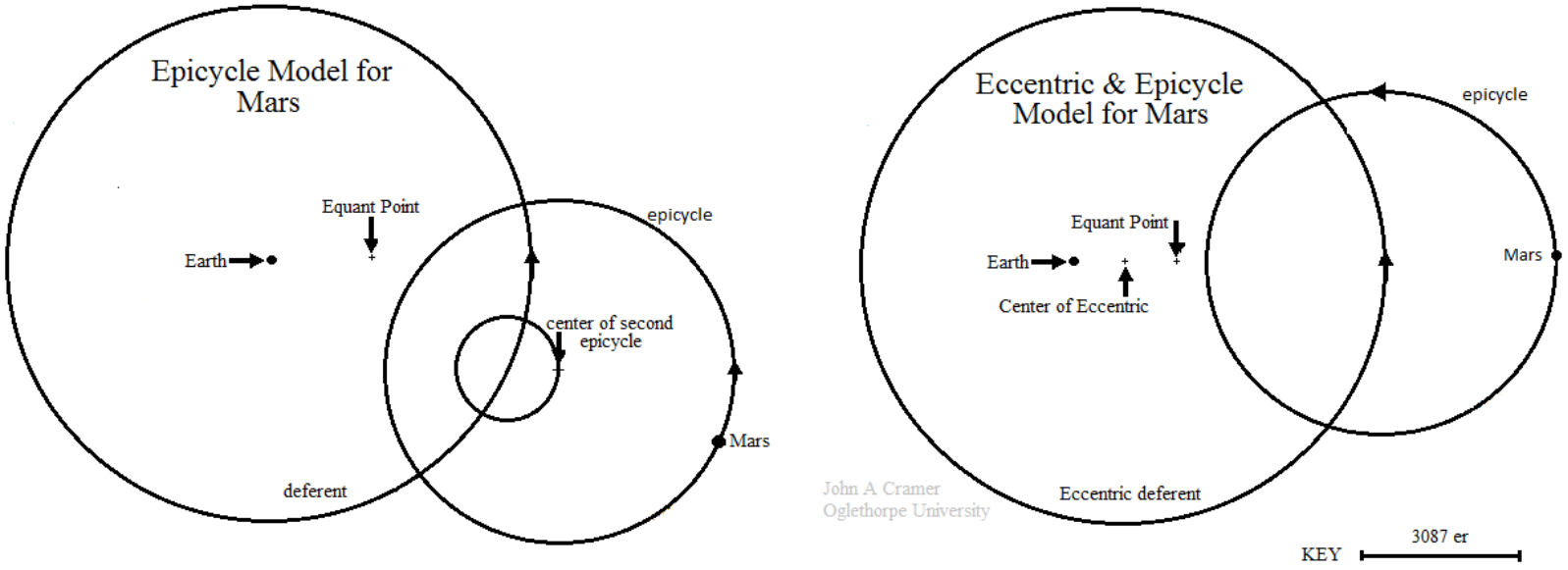


The model for Jupiter is much the same although the relative sizes are different. For example, the epicycle of Jupiter is actually only a little larger than that of Saturn but, in these figures, it



seems much larger because the deferent of Jupiter is about 2/3 that of Saturn.

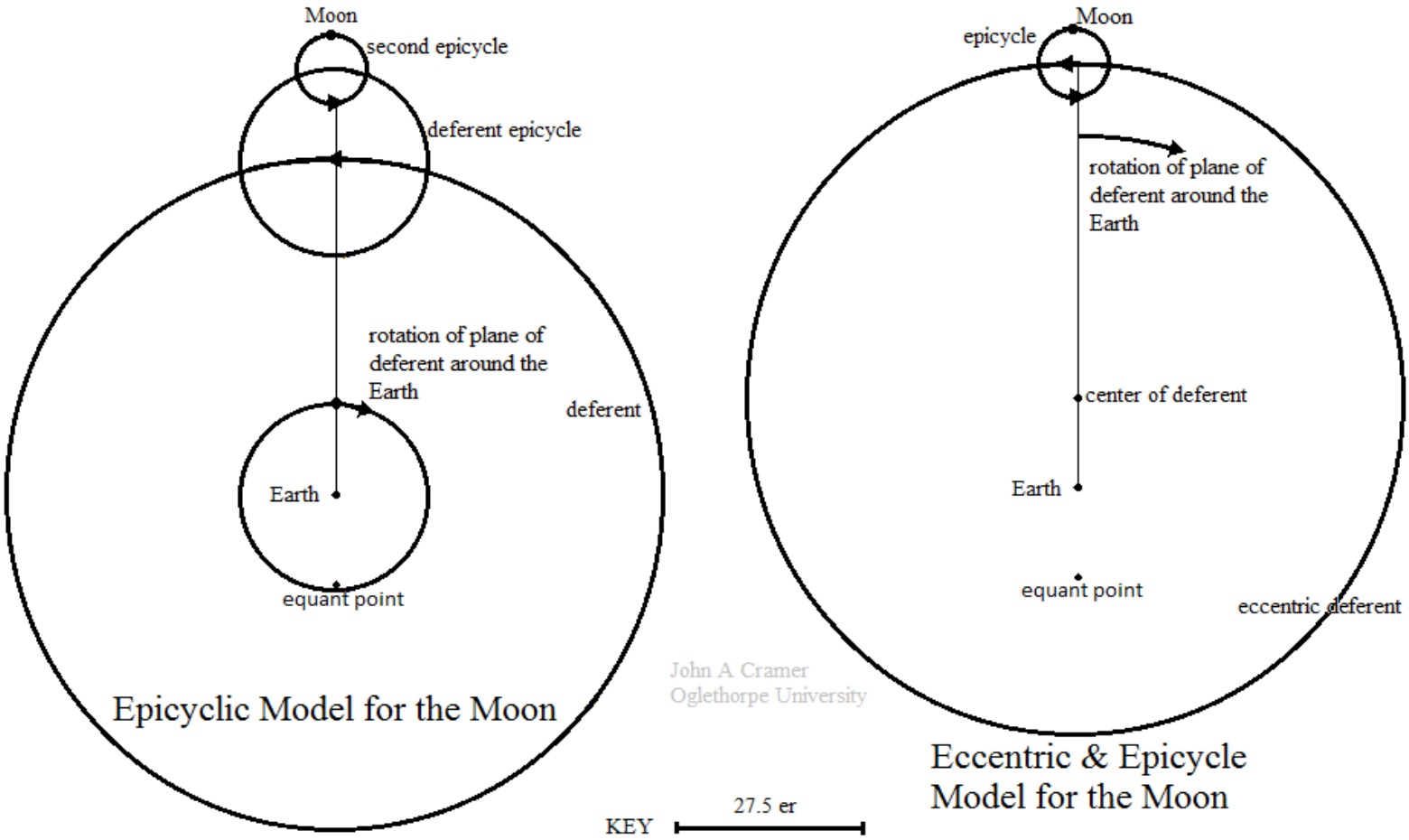
How we count circles here is problematic. The eccentric plus epicycle of the standard model makes three for each of the three planets. But how do we count the additional equant? It is not really a circle but a control on the rate of rotation on a circle. On the other hand, the distance between the equant point and the center of the eccentric is a distance that must be determined separately although, admittedly Ptolemy always makes the distance equal to that between the earth and the center of the eccentric. If we count equants as circles, the Ptolemaic system now has 19 circles but 16 circles is also a possible count.



Mars has a similar problem to that of Venus; the relative size of its epicycle is absurdly great. Like Venus, Mars is far larger (7 X, in fact, where 5 X is more correct) when close to Earth compared to its size at furthest distance from Earth. Here, again, Ptolemy may have thought Mars too small for it to be seen.

Mercury and the Moon yet remain. These last two "planets" were easily the most troubling for the Ptolemaic system, as their figures will show. The standard model works for neither of them and Ptolemy had to exert all his genius to devise models that were at all workable. He did not truly succeed in either case.

The Moon also required a new model. Here the simple epicycle model gave distance between the Sun and Moon that varied in a way that suggested to Ptolemy that he needed the entire eccentric deferent to rotate in space at the same rate but in the opposite sense from the rotation of the epicycle. Thus, the Moon would arrive at the "bottom" of the figure "inside" the eccentric deferent and close to Earth. Ptolemy also used an equant point but, unlike the others, this one is an equal distance on the other side of the earth from the center of the eccentric deferent!

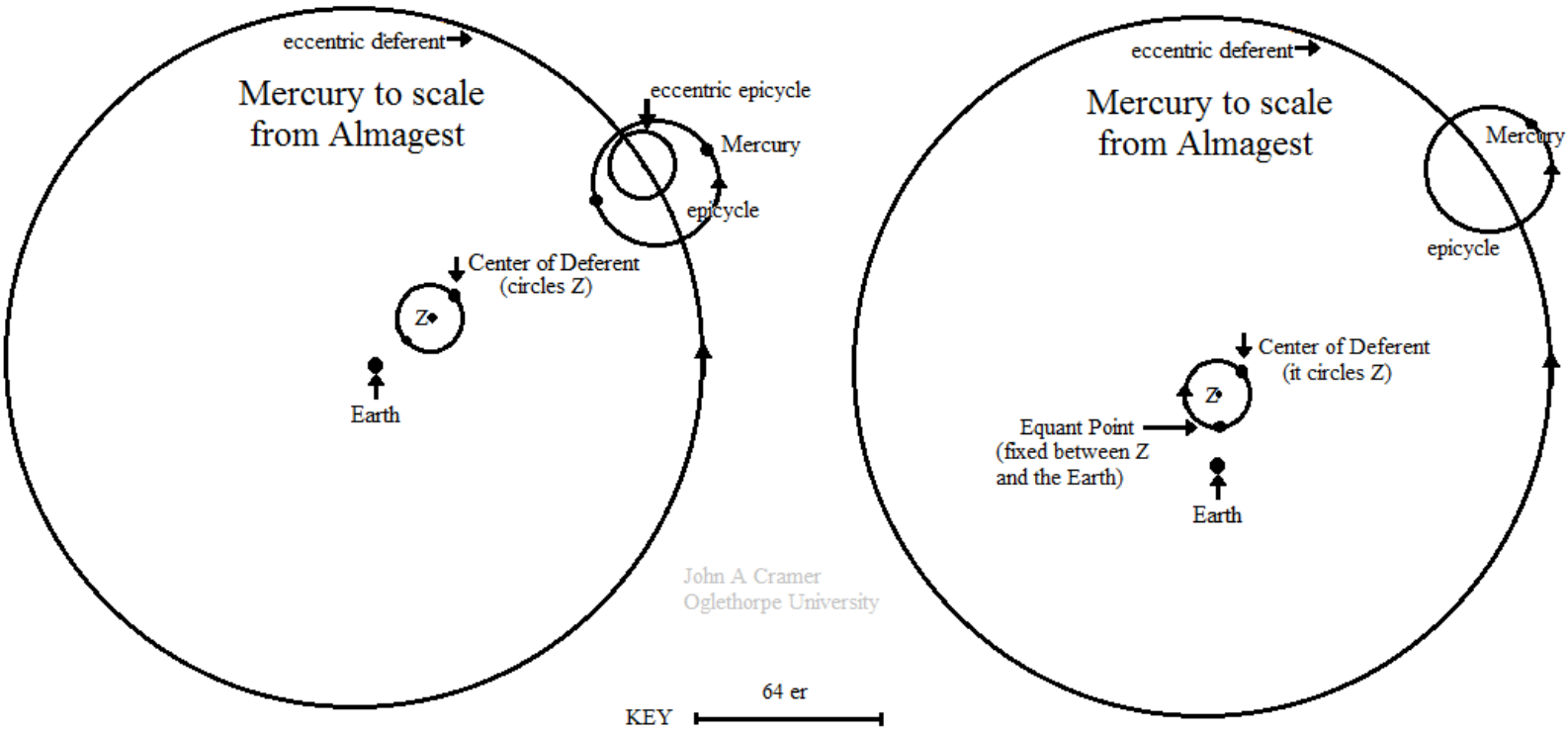


Once again the Ptolemaic system generates an absurd change in size, the moon changes size by a factor of two which should be common knowledge were it true. Mars and Venus might be

thought too small but the argument does not apply to the Moon. How Ptolemy missed this or why he refrained from mentioning it is one of the great failings of the Ptolemaic system.

It is difficult to make a count of circles here. An eccentric and an epicycle total three. The rotation of the eccentric deferent at a uniform rate seems to demand a fourth circle with a fifth from the precession of the equinoxes. The running total of the system circles is then perhaps 25, or as few as 20, circles.

Mercury's is far and away the most peculiar of all the models. Because of its proximity to the Sun, it is difficult and dangerous to observe. Thus, what little data Ptolemy had was rather inaccurate. Additionally, Mercury has the most eccentric orbit about the Sun of all these "planets." Ptolemy found a way to combine circles that, more or less, predicted the motion of Mercury but needed a rotating eccentric to do this, as he did also with the Moon. Mercury required a further complication; the center of the eccentric deferent had to be made to circle another point, Z in the figure, at the same rate as the planet on the epicycle but in the opposite direction. And the equant point was equally space between the Earth and Z.



With an eccentric deferent, an epicycle and the weird motion of the center of the eccentric deferent, we must attach at least 4 to 7 circles to this model. The final total for the Ptolemaic system is then between 32 to 24 circles. Not counting equants, probably the right choice although no less an expert than Copernicus would count them, gives the most frequently cited result of 27 circles. Ptolemy himself gave different counts in different places!

Ptolemy was well aware that the planets do not all move in the same plane. The plane of the Earth - Sun motions is called "the plane of the ecliptic." The other planetary orbits are at small angles, the angles of inclination, away from this. Ptolemy discussed and dismissed any need to correct for the angle of inclination at first but he concluded the Almagest with corrections to "latitude" using the known inclinations of orbital planes.