Comparing Inquiry-Based and Explicit Instruction in High School Geometry

Rodney Sizemore
Kennesaw State University

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Comparing Inquiry-Based and Explicit Instruction in High School Geometry

By

Charles Rodney Sizemore

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Comparing Inquiry-Based and Explicit Instruction in High School Geometry

By

Charles Rodney Sizemore

Dissertation Chair: Dr. David Glassmeyer
Committee Member: Dr. Mei-Lin Chang
Committee Member: Dr. Rachel Gaines
ABSTRACT

The use of inquiry-based and explicit instructional methods in mathematics has been researched and have found that the most appropriate instructional method depends on many factors including the subject matter being taught, the students’ prior knowledge, and the students’ special education status. However, teachers’ use of these methods has not been investigated in two important topics in high school geometry: arc length/sector area and graphing circles – topics which reveal how several important mathematical concepts connect from elementary and middle school, to geometry, then to pre-calculus. This quasi-experimental study compared the effects of inquiry-based and explicit teaching methods on arc length/sector area and graphing circles in terms of student achievement growth and students’ assessment of their learning gains. One group of students was taught arc length and sector area using inquiry-based methods while the other group was taught using explicit methods. An independent samples t-test compared growth in procedural fluency and conceptual understanding for all students as well as for certain subgroups based on prior achievement and special education status. The procedure was repeated with graphing circles with the instructional methods swapped for each group. Overall, results showed significantly higher student achievement growth in arc length and sector area procedural fluency under explicit instruction, but other overall differences were not significant. Therefore, teachers can use either instructional method to promote student achievement growth in conceptual understanding of arc length/sector area, conceptual understanding of graphing circles and procedural fluency in graphing circles.
Teachers should consider using explicit instruction to teach arc length and sector area procedural fluency.
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# TABLE OF CONTENTS

ABSTRACT ........................................................................................................................................ ii

ACKNOWLEDGEMENTS .................................................................................................................... iv

LIST OF TABLES ................................................................................................................................. xi

LIST OF FIGURES ............................................................................................................................... xii

CHAPTER

I. INTRODUCTION ............................................................................................................................... 13
   Purpose of the Study .......................................................................................................................... 15
   Research Questions .......................................................................................................................... 16
   Definitions ....................................................................................................................................... 18

II. LITERATURE REVIEW .................................................................................................................... 20
   Theoretical Foundation .................................................................................................................... 20
      Foundation for Inquiry-Based Instruction: Constructivism ......................................................... 20
      Foundations for Explicit Instruction: Behaviorism and Cognitive Load Theory ....................... 23
   Comparing Inquiry-Based and Explicit Instruction ......................................................................... 25
      The Most Appropriate Method Depends on Many Factors ......................................................... 25
      The Use of Inquiry-Based and Explicit Instruction in High School Geometry ......................... 29
   Summary and Conclusions .............................................................................................................. 31

III. METHODOLOGY ........................................................................................................................... 33
   Research Questions ......................................................................................................................... 33
   Research Design and Rationale ....................................................................................................... 34
   Participants and Setting .................................................................................................................. 35
      Role of the Researcher .................................................................................................................. 35
      Population ...................................................................................................................................... 36
      Procedure for Recruitment .......................................................................................................... 38
      Sampling ........................................................................................................................................ 38
   Instrumentation ............................................................................................................................... 40
      Achievement Growth Measures ................................................................................................. 40
      Test Development Procedures .................................................................................................... 42
Student Assessment of Their Learning Gains Surveys .......................................................... 45
Procedures .................................................................................................................................. 46
Instruction ....................................................................................................................................... 46
Procedures for Measuring Student Achievement Growth ......................................................... 49
Procedures for Measuring Student-Reported Learning Gains .................................................. 50
Procedures for Analyzing Open-Ended Survey Responses ...................................................... 50
Data Analysis ............................................................................................................................... 51
Steps Taken to Overcome Design Limitations .......................................................................... 54

IV. FINDINGS ............................................................................................................................... 56

Achievement Growth .................................................................................................................. 57
  Research Question 1: Arc Length and Sector Area Procedural Fluency ........................... 60
  Research Question 2: Arc Length and Sector Area Conceptual Understanding .............. 61
  Research Question 3: Graphing Circles Procedural Fluency ............................................. 62
  Research Question 4: Graphing Circles Conceptual Understanding .............................. 64
Student Perceptions of Their Learning Gains ........................................................................ 65
  Research Question 5: Student-Reported Learning Gains ................................................. 65
  Helpfulness of Each Learning Activity ................................................................................ 66
  Open-Ended Survey Question Responses ........................................................................ 67
    Responses Concerning Inquiry-Based Instruction ............................................................ 67
    Responses Concerning Explicit Instruction ...................................................................... 68
Summary ....................................................................................................................................... 69

V. DISCUSSION, CONCLUSION, AND IMPLICATIONS ................................................................ 72

Discussion .................................................................................................................................... 73
  Research Question 1: Arc Length and Sector Area Procedural Fluency ......................... 73
  Research Question 2: Arc Length and Sector Area Conceptual Understanding ............. 74
  Research Question 3: Graphing Circles Procedural Fluency ........................................... 76
  Research Question 4: Graphing Circles Conceptual Understanding ............................. 77
  Research Question 5: Student-Reported Learning Gains ................................................. 78
Limitations .................................................................................................................................... 78
Implications ................................................................................................................................. 80
Future Research ......................................................................................................................... 81

REFERENCES .............................................................................................................................. 83

APPENDIX

A. IRB APPROVAL ....................................................................................................................... 96

B. STUDENT ASSENT FORM ................................................................................................... 98

C. PARENT CONSENT FORM .................................................................................................. 102
D.  ARC LENGTH/SECTOR AREA TEST ...............................................................105
    Standards .................................................................................................106
    Learning Outcomes ..................................................................................106
    Test Blueprint ..........................................................................................107
    Arc Length and Sector Area Test ..............................................................108

E.  GRAPHING CIRCLES TEST .........................................................................115
    Standards .................................................................................................116
    Learning Outcomes ..................................................................................116
    Test Blueprint ..........................................................................................117
    Graphing Circles Test ..............................................................................118

F.  STUDENT SURVEY ON ARC LENGTH/SECTOR AREA ..............................124

G.  STUDENT SURVEY ON GRAPHING CIRCLES .............................................127

H.  COMPARISON OF INQUIRY-BASED AND EXPLICIT INSTRUCTION
    FOR ARC LENGTH AND SECTOR AREA ..................................................130

I.  COMPARISON OF INQUIRY-BASED AND EXPLICIT INSTRUCTION
    FOR GRAPHING CIRCLES .......................................................................134

J.  INQUIRY-BASED LESSON PLANS FOR ARC LENGTH/SECTOR AREA ......138
    Day 1 Warm-Up .......................................................................................139
    Day 1 Classwork: Arc Length and Sector Area .......................................140
    Day 1 Homework .....................................................................................142
    Day 2 Warm-Up .......................................................................................143
    Day 2 Classwork: Using Arc Length and Sector Area ............................144
    Day 2 Homework .....................................................................................146
    Day 3 Warm-Up .......................................................................................147
    Day 3 Classwork: Other Ways to Set Up Arc Length and Sector Area Problems 148
    Day 3 Homework .....................................................................................149
    Day 4 Warm-Up .......................................................................................150
    Day 4 Classwork: Desmos Arc Length and Sector Area Activity ..............151
    Day 4 Homework .....................................................................................152
    Day 5 Warm-Up .......................................................................................153
    Day 5 Classwork: How Many Radians in a Circle? .................................154
    Day 5 Homework .....................................................................................155
    Day 6 Warm-Up .......................................................................................156
    Day 6-7 Classwork: How Many Radians in a Circle? (continued) ............157
    Day 6 Homework .....................................................................................158
    Day 7 Warm-Up .......................................................................................159
    Day 7 Classwork: Converting Between Radians and Degrees .................160
    Day 7 Homework: DeltaMath Practice ......................................................161
Day 8 Warm-Up ...........................................................................................................162
Day 8 Classwork: Desmos Arc Length and Sector Area Review .........................163
Day 8 Homework ........................................................................................................164

K. EXPPLICIT LESSON PLANS FOR ARC LENGTH/SECTOR AREA ..........................165

Day 1 Warm-Up ...........................................................................................................166
Day 1 Classwork and Homework: Arc Length and Sector Area ............................167
Day 2 Warm-Up ...........................................................................................................169
Day 2 Classwork and Homework: Using Arc Length and Sector Area .................170
Day 3 Warm-Up ...........................................................................................................172
Day 3 Classwork: Foldable .........................................................................................173
Day 3 Classwork and Homework: Other Ways to Set Up Arc Length and Sector
Area Problems ...........................................................................................................175
Day 4 Warm-Up ...........................................................................................................177
Day 4 Classwork and Homework: Solving for the Radius or Angle in ALSA
Problems ....................................................................................................................178
Day 5 Warm-Up ...........................................................................................................180
Day 5 Classwork and Homework: What Are Radians? ..........................................181
Day 6 Warm-Up ...........................................................................................................183
Day 6 Classwork: Foldable .........................................................................................184
Day 6 Classwork and Homework: DeltaMath Practice ...........................................186
Day 7 Warm-Up ...........................................................................................................188
Day 7 Classwork and Homework: Converting Between Radians and Degrees
Puzzle ..........................................................................................................................189
Day 8 Warm-Up ...........................................................................................................192
Day 8 Classwork and Homework: Test Review .......................................................193

L. INQUIRY-BASED LESSON PLANS FOR GRAPHING CIRCLES ..........................195

Day 1 Warm-Up ...........................................................................................................196
Day 1 Classwork: Equations of Circles .....................................................................197
Day 1 Homework .........................................................................................................200
Day 2 Warm-Up ...........................................................................................................201
Day 2 Classwork: Equations of Circles (continued) ..................................................202
Day 2 Homework .........................................................................................................205
Day 3 Warm-Up ...........................................................................................................206
Day 3 Classwork: Equations of Circles (continued) ..................................................207
Day 3 Homework .........................................................................................................209
Day 4 Warm-Up ...........................................................................................................210
Day 4 Classwork: If You’re Given Points on a Circle ..................................................211
Day 4 Homework .........................................................................................................212
Day 5 Warm-Up ...........................................................................................................213
Day 5 Classwork: Standard to General Form .............................................................214
Day 5 Homework .........................................................................................................215
Day 6 Warm-Up ...........................................................................................................216
Day 6-7 Classwork: Completing the Square ................................................................. 217
Day 6 Homework .......................................................................................................... 221
Day 7 Warm-Up ........................................................................................................... 222
Day 7 Homework ......................................................................................................... 223
Day 8 Warm-Up ........................................................................................................... 225
Day 8 Classwork: Test Review .................................................................................... 226
Day 8 Homework ......................................................................................................... 228

M.  EXPLICIT LESSON PLANS FOR GRAPHING CIRCLES ........................................... 229

Day 1 Warm-Up .......................................................................................................... 230
Day 1 Classwork and Homework: Graphing Circles .................................................... 231
Day 2 Warm-Up .......................................................................................................... 234
Day 2 Classwork and Homework: If You’re Given Points on a Circle ....................... 235
Day 3 Warm-Up .......................................................................................................... 237
Day 3 Classwork: Graphing Circles Foldable ............................................................. 238
Day 4 Warm-Up .......................................................................................................... 241
Day 4 Classwork: DeltaMath Practice ........................................................................ 242
Day 5 Warm-Up .......................................................................................................... 243
Day 5 Classwork and Homework: Converting from Standard to General Form ....... 244
Day 6 Warm-Up .......................................................................................................... 246
Day 6 Classwork and Homework: Completing the Square ........................................ 247
Day 7 Warm-Up .......................................................................................................... 250
Day 7 Classwork and Homework: DeltaMath Practice ............................................. 251
Day 8 Warm-Up .......................................................................................................... 252
Day 8 Classwork: Circles Review on Mini-Dry Erase Boards .................................... 253
Day 8 Homework: Test Review .................................................................................. 255

N.  STUDENT SURVEY OPEN-ITEM RESPONSES .......................................................... 257

Responses from the Inquiry-Based Group on Arc Length and Sector Area ............... 258
Responses from the Explicit Group on Arc Length and Sector Area ....................... 260
Responses from the Inquiry-Based Group on Graphing Circles ................................ 262
Responses from the Explicit Group on Graphing Circles .......................................... 264
LIST OF TABLES

Table 1  Examples of Studies Comparing Inquiry-Based and Explicit Instruction in Mathematics ..........................................................26

Table 2  Equivalency of Groups ..................................................................................................................39

Table 3  Interrater Reliability for Arc Length and Sector Area Test .........................................................44

Table 4  Interrater Reliability for Graphing Circles Test ..............................................................................44

Table 5  Reliability for Surveys ..................................................................................................................46

Table 6  Research Alignment Table: Research Questions, Measures and Key Variables, and Corresponding Statistical Analysis ..........................................................52

Table 7  Student Attendance During Study .................................................................................................55

Table 8  Organization of Research Questions .............................................................................................56

Table 9  Summary of Achievement Growth Results ..................................................................................58

Table 10 Specific Inquiry-Based Activities Students Found Helpful .........................................................67

Table 11 Ways Students Found Inquiry-Based Instruction Helpful ..........................................................68

Table 12 Specific Explicit Activities Students Found Helpful ....................................................................68

Table 13 Summary of Achievement Growth Results ..................................................................................69

Table 14 Procedural Fluency and Conceptual Understanding Growth for All Students .................73
LIST OF FIGURES

Figure 1  Arc Length and Sector Area Connect Elementary School and Pre-Calculus Concepts.................................................................14

Figure 2  Graphing Circles Connect Middle Elementary School and Pre-Calculus Concepts..............................................................................15

Figure 3  Calendar for Study ................................................................................................................................................35

Figure 4  Composition of Groups................................................................................................................................................37

Figure 5  Test Development ..................................................................................................................................................42

Figure 6  Beginning of the First Inquiry-Based Lesson on Arc Length and Sector Area ........................................47

Figure 7  Second Question Posed to Students in the First Inquiry-Based Lesson on Arc Length and Sector Area ........................................................................48

Figure 8  Beginning of the First Explicit Lesson on Arc Length and Sector Area .................................................................49

Figure 9  Arc Length and Sector Area Procedural Fluency Achievement Growth.........................................................60

Figure 10 Arc Length and Sector Area Conceptual Understanding Achievement Growth ........................................62

Figure 11 Graphing Circles Procedural Fluency Achievement Growth ..............................................................................63

Figure 12 Graphing Circles Conceptual Understanding Achievement Growth .................................................................65
CHAPTER I
INTRODUCTION

Developing students’ mathematical thinking is a primary goal of all mathematics teachers. However, the best instructional method to develop mathematical thinking has been a subject of heated debates in mathematics education for decades. The debate gained more attention with the 1989 publication of the National Council of Teachers of Mathematics’ *Curriculum and Evaluation Standards*, which encouraged the use of constructivist methods and a focus on mathematical reasoning. There was much opposition to the *Standards* by those who favored more traditional instruction; this debate became known as the Math Wars (Schoenfeld, 2004). At the heart of this debate is a fundamental disagreement about how students should be taught mathematics. Should students be taught by constructing knowledge through exploration, or should they be taught by being given new information and having its meaning and implications explained to them?

Mathematics researchers have studied the effects of different instructional methods on many topics (Alsup, 2005; Alsup & Sprigler, 2003; Boaler, 1998; Brune, 2010; Geier et al., 2008; Kogan & Laursen, 2014; Lewis, 2009; Marshall & Horton, 2011; Mensah-Wonkyi & Adu, 2016; Thompson, 1992; Zafra-Gómez et al., 2015). Some have begun to embrace a view that students learn best using a combination of methods, depending on the standards being taught, the instructional goals, and the students’ prior knowledge (Clark, 2009; Gresalfi & Lester, 2009; Hmelo-Silver et al., 2007; Kirschner, 2009; Mayer, 2009).
Instructional methods have not been investigated in two important areas of high school geometry: arc length/sector area and graphing circles. These topics require research because they provide links between elementary, middle, and high school mathematics (Burger et al., 2020). Arc length and sector are the high school bridge that connect the elementary school ideas of proportions, circumference, and area with the pre-calculus concepts of radians. The progression from arc length and sector area to radians is shown in Figure 1. In elementary and middle school, students study ratio, proportions, circumference, and area of circles as separate topics. In geometry, arc length and sector area link those concepts together because arc length and sector area are proportions of the circumference and area of a circle. Then later in pre-calculus, students encounter radians, which are a specific application of arc length and are used to measure angles. One radian is defined as the central angle formed when the length of an arc is equal to one radius of the circle (Burger et al., 2020).

**Figure 1**

*Arc Length and Sector Area Connect Elementary School and Pre-Calculus Concepts*

Similarly, Figure 2 shows how equations of circles link the middle school concept of Pythagorean Theorem to the pre-calculus topic of conic sections. In middle school, students use
the Pythagorean Theorem to find the length of the hypotenuse of a right triangle, which is also the distance between two points. In geometry, students are taught that the equation for a circle is a specific application of the Pythagorean Theorem because a circle is the set of all points a certain distance from the center (Burger et al., 2020). Then later in pre-calculus, students are taught that other conic sections such as the equation for an ellipse are also applications of the Pythagorean Theorem; for example, an ellipse is defined as the set of all points the sum of whose distances from two points are equal (Young, 2018).

Figure 2

Graphing Circles Connect Middle Elementary School and Pre-Calculus Concepts

Purpose of the Study

If the most effective instructional method depends on the standards being taught, the instructional goals, and the students’ prior knowledge as some have suggested (Clark, 2009; Gresalfi & Lester, 2009; Hmelo-Silver et al., 2007; Kirschner, 2009; Mayer, 2009), then research must be conducted for a variety of standards and in a variety of situations. This study attempted to fill in a gap in the literature in that previous studies have not examined the effect of
instructional methods in arc length/sector area and graphing circles with on-grade-level suburban high school students. This study contributed data to the existing research base on the use of instructional methods by comparing student achievement growth and student-perceived learning gains when taught about arc length/sector area and graphing circles using inquiry-based and explicit instruction. It compared student achievement growth in procedural fluency and conceptual understanding for all students as well as for subgroups based on prior achievement and special education status. Achievement growth, as measured by student growth in procedural fluency and conceptual understanding on pre-/post-tests, was compared based on the instructional method, and students were surveyed about their learning gains.

**Research Questions**

**Research Question #1:** Is there a significant difference in student achievement growth in procedural fluency in the unit of arc length and sector area between inquiry-based instruction and explicit instruction?

\[ H_0: \] There is no significant difference in student achievement growth in procedural fluency in the unit of arc length and sector area between inquiry-based instruction and explicit instruction.

\[ H_A: \] There is a significant difference in student achievement growth in procedural fluency in the unit of arc length and sector area between inquiry-based instruction and explicit instruction.

**Research Question #2:** Is there a significant difference in student achievement growth in conceptual understanding in the unit of arc length and sector area between inquiry-based instruction and explicit instruction?
H₀: There is no significant difference in student achievement growth in conceptual understanding in the unit of arc length and sector area between inquiry-based instruction and explicit instruction.

Hₐ: There is a significant difference in student achievement growth in conceptual understanding in the unit of arc length and sector area between inquiry-based instruction and explicit instruction.

Research Question #3: Is there a significant difference in student achievement growth in procedural fluency in the unit of graphing circles between inquiry-based instruction and explicit instruction?

H₀: There is no significant difference in student achievement growth in procedural fluency in the unit of graphing circles between inquiry-based instruction and explicit instruction.

Hₐ: There is a significant difference in student achievement growth in procedural fluency in the unit of graphing circles between inquiry-based instruction and explicit instruction.

Research Question #4: Is there a significant difference in student achievement growth in conceptual understanding in the unit of graphing circles between inquiry-based instruction and explicit instruction?

H₀: There is no significant difference in student achievement growth in conceptual understanding in the unit of graphing circles between inquiry-based instruction and explicit instruction.

Hₐ: There is a significant difference in student achievement growth in conceptual understanding in the unit of graphing circles between inquiry-based instruction and explicit instruction.
Research Question #5: Is there a significant difference in student-reported learning gains between inquiry-based instruction and explicit instruction?

\( H_0: \) There is no significant difference in student-reported learning gains between inquiry-based instruction and explicit instruction.

\( H_A: \) There is a significant difference in student-reported learning gains between inquiry-based instruction and explicit instruction.

Definitions

This section defines achievement growth, which is what was measured for four of the research questions in this study. It defines inquiry-based and explicit instruction, which are the two types of instruction used in this study. Finally, it defines procedural fluency and conceptual understanding, which are the two types of knowledge being measured.

**Achievement growth:** “the amount of improvement, or growth, between the beginning (pretest) and end (post-test) of instruction” (Kubiszyn & Borich, 2016, p. 219).

**Inquiry-based instructional methods:** students construct their own knowledge by exploring concepts, discovering patterns, and integrating new with existing knowledge with little explicit instruction (Bruner, 1975; Dewey, 1916; Piaget, 1972; Vygotsky, 1978). Inquiry-based methods encourage collaboration and dialogue as ways to build understanding. Typical inquiry-based classes spend the majority of class time on student-centered activities such as working in groups and giving and listening to presentations by other students, with teachers providing scaffolding through carefully selected problems, giving feedback to students, and providing mini-lessons when needed (Laursen et al., 2011). This type of instruction is sometimes referred to as guided inquiry (Clark et al., 2012; Kuhlthau et al., 2007; Mayer, 2004).
**Explicit instructional methods:** learning is a stimulus-response reaction and students must be trained how to respond (Skinner, 1965; Watson, 1913). Explicit methods are supported by cognitive load theory which suggest that the most efficient way for a person to gain knowledge is to be told new information (Sweller et al., 2011). Explicit instruction often involves teachers telling and showing students everything they need to know to solve a problem.

**Procedural knowledge:** “rules, algorithms, or procedures used to solve mathematical tasks” (Hiebert & Lefevre, 1986, p. 6). Procedural fluency is “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (National Research Council, 2001, p. 116).

**Conceptual understanding:** “connecting concepts to specific procedures – for example, knowing why certain procedures work for certain problems or knowing the purpose of each step in a procedure” (Crooks & Alibali, 2014, p. 371).
CHAPTER II
LITERATURE REVIEW

The first section of this chapter examines the theoretical foundations for inquiry-based instruction and the theoretical foundations for explicit instruction. The second section describes several studies comparing inquiry-based and explicit mathematics instruction in a variety of settings and shows that the most appropriate type of instruction may depend on a variety of factors including the subject matter being taught, the students’ prior knowledge, and the students’ special education status. The third section examines in more detail the use of inquiry-based and explicit instruction in high school geometry.

Theoretical Foundation

This section gives a brief overview of the theoretical frameworks for inquiry-based and explicit instruction. The primary theoretical framework for inquiry-based instruction is constructivism (Doolittle & Camp, 1999; Hulett et al., 2004; Khalaf, 2018; Lazonder & Harmsen, 2016; Mayer, 1992, 1996; Spronken-Smith, 2012), and the primary theoretical frameworks for explicit instruction are behaviorism and cognitive load theory (Doolittle & Camp, 1999; Lenjani, 2015).

Foundation for Inquiry-Based Instruction: Constructivism

Inquiry-based instruction is an instructional method in which students spend the majority of class time on student-centered activities such as working in groups and giving and listening to presentations by other students, with teachers providing scaffolding through carefully selected
problems, giving feedback to students, and providing mini-lessons when needed (Laursen et al., 2011). Many of these activities of inquiry-based instruction can be traced to constructivist principles espoused by Dewey, Piaget, Vygotsky, and Bruner.

Constructivism is a theory of learning which says that students build their own knowledge by engaging in active processes to integrate new information with previous understandings and experiences (Clements & Battista, 1990; Confrey, 1990; Confrey & Kazak, 2006; Dengate & Lerman, 1995; Goldin, 1990; Leino, 1994; Narayan et al., 2013; Noddings, 1990; Shapiro, 2013; Windschitl, 2002). Naylor and Keogh (1999) explained that from a constructivist perspective, “Learning involves an active process in which learners construct meaning by linking new ideas with their existing knowledge” (p. 93).

Inquiry-based instruction dates back at least to the time of Socrates, who asked his students probing questions to help students discover ideas (Kennedy et al., 2015). Modern inquiry-based instruction traces many of its roots to Dewey (Barrow, 2006; Duffy & Raymer, 2010; Herman et al., 2015; Khalaf, 2018; Page & Painter, 2019; Pedaste et al., 2015; Schön, 1992; Stoller, 2018). Dewey argued that the purpose of education was to teach students to think rather than to be able to recite information previously discovered by others. Therefore, Dewey (1910) said schools should promote “the essentials of thinking” and suggested that the appropriate way to do that was through “systematic and protracted inquiry” (p. 13). He said that the inquiry experiences should “[generate] an experience that has educative quality with particular individuals at a particular time” (Dewey, 1938, p. 46). He defined two components of inquiry-based learning. First, he said that instructional experiences should be designed so that students discover connections. He said that education should involve “[giving] the pupils something to do, not something to learn; and the doing is of such a nature as to demand thinking,
or the intentional noting of connections; learning naturally results” (Dewey, 1916, p. 191).

Second, he said that during the educational experiences that students should make sense of new knowledge: “… the two limits of every unit of thinking are a perplexed, troubled, or confused situation at the beginning and a cleared-up, unified, resolved situation at the close” (Dewey, 1933, p. 106).

Piaget concurred with Dewey and built on Dewey’s theories. Piaget (1972) said that the role of teachers was to “guide [students]” as they “do their own experimenting and their own research” (p. 27). Piaget built on Dewey’s theories by proposing a model for how the brain stores information. In his book *The Language and Thought of the Child*, Piaget (1926) proposed that the brain organizes groups of related information into schemas. Piaget (1972) said that intellectual growth occurs as the brain experiences new information and integrates it into existing schemas, a process he referred to as assimilation. Piaget argued that the process of assimilation occurs through an individual’s experiences: “… for a child to understand something, he must construct it himself” (p. 27).

Vygotsky added to Piaget’s theory of constructivism by suggesting that learning has a social component. Vygotsky’s (1978) theory of social constructivism says that “children grow into the intellectual life of those around them” (p. 88). Vygotsky said that individuals internalize information only after they have learned it by interacting with others. In his book *Mind and Society*, Vygotsky (1978) wrote,

Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological), and then inside the child (intrapsychological)... All the higher functions originate as actual relations between human individuals (p. 57).
Vygotsky (1962) said that after children have learned through interactions with others, then they will be able to apply the knowledge on their own: “What the child can do in cooperation today he can do alone tomorrow” (p. 188).

Brunner contributed to the development of inquiry-based instruction by introducing scaffolding to the constructivist methods of Dewey, Piaget, and Vygotsky. Scaffolding is the process of providing temporary supports to help students accomplish tasks they would not be able to solve by themselves, with the objective of eventually allowing the student to complete such tasks unassisted (Wood et al., 1976). Brunner (1975) described the process as “supporting the child in achieving an intended outcome, entering only to assist or reciprocate or 'scaffold' the action” (p. 12). The process of scaffolding by the teacher is a common component of guided inquiry (Kuhlthau et al., 2007).

**Foundations for Explicit Instruction: Behaviorism and Cognitive Load Theory**

Explicit instruction is “a systematic method for presenting material in small steps, pausing to check for student understanding, and eliciting active and successful participation from all students” (Rosenshine, 1986, p. 60) and involves teachers telling and showing students everything they need to know to solve a problem or understand a concept (Hudson et al., 2006; Lenjani, 2015; Strickland & Maccini, 2010). Many of these characteristics of explicit instruction are founded in behaviorism and supported by cognitive load theory.

Behaviorism is a theory which considers learning to be a conditioned response to a stimulus (Skinner, 1965; Watson, 1913, 1930). From a behaviorist perspective, a teacher asking a question is a stimulus, and a student providing an answer is the response (Durwin & Reese-Weber, 2016). Skinner (1954) described how to use stimulus-response conditioning to improve classroom instruction. His methods for teaching humans were based on earlier experiments with
animals: “… it is possible to shape three or four well-defined responses in a single demonstration period… Extremely complex performances may be reached through successive stages in the shaping process” (Skinner, 1954, p. 86). Edwards et al. (2016) said, “Research in the behaviorist tradition… conceptualized mathematical learning as the formation and strengthening of those stimulus-response associations” (p. 58).

Explicit instructional methods are also supported by cognitive load theory, which says that the most efficient way for a person to gain knowledge is to be told new information. Cognitive load theory is based on implications of human cognitive architecture such as limited working memory. Paas et al. (2010) said, “Cognitive load theory is concerned with the learning of complex cognitive tasks, in which learners are often overwhelmed by the number of interactive information elements that need to be processed simultaneously before meaningful learning can commence” (p. 116). Sweller et al. (2011) said, “Instruction needs to consider the limitations of working memory so that information can be stored effectively in long-term memory” (p. 7). The instructional implications of a limited working memory are that new information should be broken down into small chunks and that extraneous information should be eliminated (Paas et al., 2010).

Clark et al. (2012) said that the problem-solving process, especially for novice learners, “overburdens limited working memory and diverts working-memory resources away from storing information in long-term memory” (p. 10). He said that as a result of overburdened working memory, “partial or minimally guided instruction typically is ineffective for novices” (p. 9); therefore, novice learners should be explicitly told the information they need rather than having them discover it.
Comparing Inquiry-Based and Explicit Instruction

Clark (2009) summarized the areas of agreement and disagreement between advocates of inquiry-based and explicit instruction. Both groups agree on modeling, focusing attention on important details, using authentic problems, assessing students’ application of knowledge, using varied practice, providing feedback, and gradually fading support. The main area of disagreement is whether students should be required to construct their own solutions when good solutions are already known to exist.

The Most Appropriate Method Depends on Many Factors

There have been many studies comparing the effects of explicit and inquiry instruction in mathematics, with mixed results depending on the methodology and the topic being covered; some of those studies are described in Table 1. As a result of the mixed results, mathematics researchers have begun to embrace a view that students learn best using a combination of methods (Alfieri et al., 2011), depending on the standards being taught and on the students’ prior knowledge. The Report of the Task Group on Instructional Practices of the National Mathematics Advisory Council recommended that teachers “employ instructional approaches and tools that are best suited to the mathematical goals, recognizing that a deliberate and conscious mix of strategies will be needed” (Benbow et al., 2008, p. xxiv). Clark stated, “We must collaborate to produce a clear taxonomy of instructional support that specifies the appropriateness of each type [of instruction] for different learning goals, tasks, and learners” (2009, p. 175). Klahr (2009) said, “Even the most zealous constructivist would acknowledge that there exist conditions of time, place, topic, learner, and context, when it is optimal to simply tell students something” (p. 291).
Table 1

Examples of Studies Comparing Inquiry-Based and Explicit Instruction in Mathematics

<table>
<thead>
<tr>
<th>Author(s) and Year</th>
<th>Students</th>
<th>Methods</th>
<th>Overall Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alsup (2005)</td>
<td>Pre-service elementary school teachers in Math Concepts I and II</td>
<td>One group was taught using traditional lecture-based instruction and the other group was taught using constructivist methods, followed by survey to compare “math anxiety, mathematics teaching efficacy beliefs, and perceptions of autonomy” (p. 4). Compared rating scale responses using analysis of covariance and paired two-sample t-tests.</td>
<td>Both groups improved on autonomy, math anxiety, and self-efficacy, but constructivist methods only produced statistically higher outcomes in feelings of autonomy.</td>
</tr>
<tr>
<td>Alsup and Sprigler (2003)</td>
<td>8th grade public school students in western U.S.</td>
<td>In the first year of this study, eighth grade students were taught using the traditional Houghton-Mifflin Mathematics series, in the second year the next group of eighth graders was taught using the CORD Applied Mathematics series, and the third year the following group of eighth graders was taught using a combination of the two series. Students were administered the Stanford Achievement Test Ninth Edition (Stanford 9) near the end of the year. Compared scores using ANOVA.</td>
<td>No difference in total SAT 9 or SAT 9 problem-solving scores, but students using the traditional curriculum had higher procedure scores.</td>
</tr>
<tr>
<td>Bando et al. (2019)</td>
<td>17,006 students in preschool and grades 3 and 4 in four Latin American countries</td>
<td>Baseline standardized test scores were compared with standardized test scores after receiving seven months of inquiry-based instruction in mathematics.</td>
<td>Mathematics test scores increased over baseline scores by 0.18 standard deviations after one year of inquiry-based instruction and by 0.39 standard deviations after four years of inquiry-based instruction.</td>
</tr>
<tr>
<td>Author(s) and Year</td>
<td>Students</td>
<td>Methods</td>
<td>Overall Conclusion</td>
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<tr>
<td>Boaler (1998)</td>
<td>A total of about 300 mathematics students in two schools in England as students progressed through grades 9, 10, and 11</td>
<td>Students at one school were taught using self-paced workbooks while students at the other school were taught using constructivist approaches.</td>
<td>In tests of applied knowledge in both 9th and 10th grade, students who learned using constructivist approaches outscored those who learned using explicit instruction. In a 10th grade traditional test, there was no difference between groups. In an 11th grade standardized test used for post-secondary pursuits, students at the school using constructivist methods outscored students who used workbooks.</td>
</tr>
<tr>
<td>Kogan and Laursen (2014)</td>
<td>Thousands of students in several undergraduate mathematics classes at four universities</td>
<td>Compared grades in subsequent mathematics classes after taking inquiry-based and non-inquiry-based classes. Used matched samples based on SAT scores, academic major, year in college, gender, and ethnicity.</td>
<td>Students who took inquiry-based classes performed at least as well in subsequent mathematics classes as students who took non-inquiry-based classes. Also found that inquiry-based classes helped close gaps for low-achieving students and female students.</td>
</tr>
<tr>
<td>Hill and Parker (2006)</td>
<td>Approximately 3200 students at Michigan State University</td>
<td>Compared college mathematics performance of students whose high schools used the inquiry-based Core-Plus Mathematics Program with those whose high schools did not use Core-Plus.</td>
<td>Students who came from high schools using the Core-Plus program “were less well prepared than either graduates in the Control Group (who came from a broad mix of curricula) or graduates of their own high schools before the implementation of Core-Plus mathematics” (p. 920).</td>
</tr>
<tr>
<td>Author(s) and Year</td>
<td>Students</td>
<td>Methods</td>
<td>Overall Conclusion</td>
</tr>
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</tr>
<tr>
<td>Mayer (1998)</td>
<td>2369 students in first-year algebra in a large district</td>
<td>Compared scores on district-wide traditional algebra tests for students who were taught using inquiry-based methods with students who were taught using traditional methods</td>
<td>Inquiry-based instruction did not lower overall scores. Also found that higher achieving students benefitted more from inquiry-based instruction than other students.</td>
</tr>
<tr>
<td>McCaffrey et al. (2001)</td>
<td>5426 tenth grade students in a district implementing both integrated and traditional mathematics curricula, and both used inquiry-based methods</td>
<td>Compared growth in Stanford 9 mathematics test scores from 9th to 10th grade</td>
<td>Using inquiry-based practices on a traditional curriculum did not result in gains in student achievement while using inquiry-based practices on a curriculum geared for use with inquiry-based practices did result in gains in student achievement. Students in the integrated math scored slightly lower than students in the traditional classes, but the differences were generally not statistically significant.</td>
</tr>
<tr>
<td>Pegg (2019)</td>
<td>147 university-level students in an beginning calculus course in Greece</td>
<td>75 students were taught conic sections using explicit methods and 72 were taught conic sections using constructivist methods, then post-test ratings of “excellent”, “very good”, “good”, “satisfactory” and “failure” were compared</td>
<td>Using a grade point average-type calculation, the constructivist group scored 15.5% higher than the explicit group</td>
</tr>
<tr>
<td>Wilson and Sindelar (1991)</td>
<td>62 learning disabled elementary school students</td>
<td>Students were divided into groups and given either explicit instruction or inquiry-based instruction in solving addition and subtraction word problems. Authors compared the number of problems solved correctly on a post-test and used the pre-test as the covariate in an ANCOVA.</td>
<td>The learning-disabled students who were explicitly taught strategies for solving addition and subtraction word problems scored significantly higher than those taught using inquiry-based methods.</td>
</tr>
</tbody>
</table>
The Use of Inquiry-Based and Explicit Instruction in High School Geometry

This section focuses on previous studies that have examined the use of inquiry-based and explicit instruction in high school geometry. To search for studies comparing the use of inquiry-based and explicit instruction in high school geometry, I searched EBSCO, ProQuest, JSTOR, ERIC, Google Scholar, and Academia.edu for articles containing as many of the terms inquiry-based instruction, explicit instruction, high school, and geometry as possible. These searches returned papers from a variety of disciplines and types of instruction. Disciplines primarily included mathematics (including high school geometry and algebra as well as college-level, middle school, and elementary school mathematics) and science. Types of instruction included inquiry-based instruction (fully guided, guided, minimally guided, and unguided), explicit instruction, problem-based learning, discovery-based learning, active learning, and cooperative group work.

I started at the top of each list of returned articles from each database and read the abstract. If the abstract seemed relevant to my research, I downloaded the article into Endnote X9, then read and highlighted the article, paying particular attention to the references. If any references were relevant to my research, I also downloaded, read, and highlighted them. I continued this process until the search results were no longer relevant. This process found three previous studies comparing the use of inquiry-based and explicit instruction in high school geometry.

One of the studies, a dissertation by Thompson (1992), examined different instructional methods and their effects on student achievement growth and attitude when studying congruent triangles. The three methods studied were guided inquiry small group work using pencil and paper activities, guided inquiry small group work using a computer, and whole class explicit
instruction. The study examined students taking geometry at five high schools in Montana during the four-week unit. Each of the existing classes was assigned a treatment type. Since the classes were created prior to the study, the author used a nonequivalent control group design. To measure achievement growth, the author created a criterion-referenced test based on the unit objectives. All students were tested three times: once as a pre-test, once as a post-test, and once four weeks after the unit ended to measure retention. Results were mixed. There was no significant difference in achievement growth overall, but there was a significant difference in favor of both forms of guided inquiry small group work if only low cognitive level items were considered. There was a significant difference in retention when all items were considered and when only high cognitive level items were considered in favor of guided inquiry small group work using pencil and paper activities over whole class explicit instruction. There was no significant difference in attitudes toward geometry.

The second study, published in a peer-reviewed journal, by Mensah-Wonkyi and Adu (2016) examined the effects of inquiry-based instruction on student achievement growth and student-perceived motivation in studying theorems about circles. The quasi-experimental study involved students taking high school geometry in Ghana. The authors did not state the length of the intervention. Thirty-eight students were in the group which received explicit instruction, and 41 were in the group which received inquiry-based instruction. Students were given a pre-test, and the authors used an independent samples t-test to determine that there was no significant difference in understanding of circle theorems before the study began. Students were given a post-test. The authors used an independent samples t-test to determine that there was a significant difference in post-test scores in favor of the inquiry-based group. Scores for the explicit-instruction group increased from 12.61 to 18.65, but scores for the inquiry-based group
increased from 10.71 to 33.00. Students were given a Likert-type survey asking about “how motivating they find their mathematics classroom learning environment” (p. 63). On a scale of 1-3 with 1 being low and 3 being high, students in the explicit instruction group rated the overall motivation in their classes as 1.6, while students in the inquiry-based group rated the overall motivation in their classes as 2.7.

A third study, a dissertation by Lewis (2009), examined the impact of inquiry-based instruction on End of Course Test (EOCT) scores of high school students in a southern Title I school. Lewis’ study used a posttest-only design to compare 2006-2007 of EOCT scores of her students taught using inquiry-based methods with 2005-2006 EOCT scores of other teachers, who did not use inquiry-based methods. The group taught using inquiry-based instruction had a higher mean score ($n = 127, M = 64.2, SD = 6.7$), but it was not significantly higher than scores of the group taught using traditional instruction ($n = 155, M = 63.8, SD = 4.8$).

**Summary and Conclusions**

This chapter placed the research into inquiry-based and explicit instructional methods into theoretical contexts. It examined the constructivist foundations of inquiry-based instruction as well as the behaviorist and cognitive load theory foundations of explicit instruction. This chapter presented numerous studies comparing inquiry-based and explicit instruction in mathematics which showed that the most appropriate instructional method may depend on a variety of factors. Finally, this chapter described in detail three studies comparing inquiry-based and explicit instruction in high school geometry. Two studies compared student achievement growth in congruent triangles and theorems about circles, while a third study only examined EOCT scores.
Researchers have stated that the most effective instructional method depends on a variety of factors including the standards being taught, the instructional goals, and the students’ prior knowledge (Clark, 2009; Gresalfi & Lester, 2009; Hmelo-Silver et al., 2007; Kirschner, 2009; Mayer, 2009). Previous research has not identified the most effective instructional methods for several important topics in high school geometry, including arc length/sector area and graphing circles. This study attempted to fill in the gaps to identify the most effective methods to teach arc length/sector area and graphing circles to high school geometry students.
CHAPTER III
METHODOLOGY

This study examined the use of inquiry-based and explicit instruction in two areas of high school geometry which had not been previously studied: arc length/sector area and graphing circles. The study compared student achievement growth and student-perceived learning gains when groups were taught these topics using different methods. This chapter reviews the research questions then describes the research design, participants and setting, instrumentation, procedures, and data analysis plan for this study.

Research Questions

1. Is there a significant difference in student achievement growth in procedural fluency in the unit of arc length and sector area between inquiry-based instruction and explicit instruction?

2. Is there a significant difference in student achievement growth in conceptual understanding in the unit of arc length and sector area between inquiry-based instruction and explicit instruction?

3. Is there a significant difference in student achievement growth in procedural fluency in the unit of graphing circles between inquiry-based instruction and explicit instruction?

4. Is there a significant difference in student achievement growth in conceptual understanding in the unit of graphing circles between inquiry-based instruction and explicit instruction?
5. Is there a significant difference in student-reported learning gains between inquiry-based instruction and explicit instruction?

**Research Design and Rationale**

This study used a quasi-experimental design and convenience sample of 10th grade geometry students (Creswell & Creswell, 2017). A calendar for the study is shown in Figure 3. In February 2020, one group of students was taught arc length and sector area using inquiry-based instruction, and another group was taught using explicit instruction. The unit on arc length and sector area lasted approximately two weeks. Students in both groups were given a pre- and post-test to measure achievement growth (the tests are discussed in the “Instruments” section), and students were given a survey to measure student-reported learning gains at the end of the unit (the survey is also discussed in the “Instruments” section). For the second unit, the instructional methods were swapped, and the process was repeated with graphing circles. The unit on graphing circles began in February 2020 and lasted approximately two weeks. The amount of time spent on each unit was guided by the state’s and district’s pacing guides. The length of this study should be sufficient to determine the effects of inquiry-based instruction according to Lazonder and Harmsen (2016), whose meta-analysis said that “findings do not point to restrictions regarding the duration of future studies on guided inquiry learning” (p. 705).
### Participants and Setting

The participants in the study are described in the next four sections. The first section describes the role of the researcher, the second section describes the population, the third section describes the recruitment procedures, and the fourth section describes the sample that was obtained after recruitment.

#### Role of the Researcher

I am a high school geometry teacher, and I conducted this research at the school in which I teach. The year the study was conducted, I co-taught two classes with another geometry teacher.
teacher. I have taught at this school for eight years, and my co-teacher and I have taught together for seven years. My co-teacher actively assisted with this research; our roles are described in the Procedures section.

**Population**

The population was students in four classes taking on-level geometry at a suburban high school in north Georgia. A total of 100 students in four classes were invited to participate in the study. Of those 100 students, 64% were White, 21% were Black, 10% were Hispanic, 3% were multiracial, and 2% were Asian. Fifty-five percent were male and 45% were female. Seventy-six percent were general education students, and 24% received special education services in the co-taught classroom. Overall, 23% of the school’s population was eligible for free or reduced-price lunch (Georgia Department of Early Care and Learning, 2019).

Students enrolled in the on-level geometry course based on the recommendation of their teacher from the previous year. The co-taught classes included both general education and special education students who were pre-assigned to the classes. The general education students were assigned randomly from the pool of all students taking non-honors geometry. Special education students were assigned to the classes by each student’s IEP team based on a variety of factors including attention span, short- or long-term memory, math calculation, and math problem-solving skills.

Two classes were my co-taught geometry classes, and two classes were general-education-only classes “borrowed” from my co-teacher for this study. The use of borrowed classes allowed the study to include more participants, but the use of borrowed classes can have drawbacks such as classroom norms already being established (Singh et al., 2000). To lessen
potential effects of borrowed classes, each group in the experiment had one of my co-taught classes and one borrowed class as shown in Figure 4.

**Figure 4**

*Composition of Groups*

- All four classes were regularly taught using both inquiry-based and explicit methods prior to the study
- All four classes were taught by the researcher during the study to ensure consistency of delivery

I taught all classes to ensure consistency of delivery. The week before the experiment began, I began teaching the two borrowed classes so the students and I could become familiar with each other and to lessen any effects that might result from the novelty of having a new teacher. All four classes were regularly taught using a combination of inquiry-based and explicit...
instruction, so students were accustomed to both types of instruction. I have used many of the activities (both inquiry-based and explicit) in previous years. The only difference the students observed was that instead of a blend of inquiry-based and explicit instruction, they experienced only one type of instruction for each two-week period.

**Procedure for Recruitment**

After distributing consent and assent forms, which are included in Appendix B and Appendix C, my co-teacher or I described the study to the students, indicating participation in the study was voluntary and would not impact their performance in the course. After having a chance to read the assent and consent forms, students were given the opportunity to ask questions. Consent forms were sent home via students for parent signatures. My co-teacher or I collected signed consent and assent forms and gave each parent and student a copy for their records. For my own students, my co-teacher obtained consent/assent to reduce the perception of coercion.

This research was conducted as a normal part of the students’ geometry instruction. Letters were sent home asking for permission to use the data collected in the research. Pre-tests, post-tests, and surveys were given to all students, but data were only used from students whose parents return the permission forms.

**Sampling**

Students who returned parent consent and student assent forms and who had previous semester assessment grades available composed the sample. Sixty of the 100 students returned both forms and had previous semester assessment grades available, representing 60% of the population. The sample closely mirrored the population: 70% White (64% for the population), 18% Black (21% for the population), 8% Hispanic (10% for the population), 2% multiracial (3%
for the population), and 2% Asian (2% for the population). Fifty-two percent were male (55% for the population), and 48% were female (45% for the population). Seventy-seven percent were general education students (75% for the population), and 23% received special education services in the co-taught classroom (24% for the population). The percent eligible for free or reduced-price lunch was not available for the sample.

Groups were checked for equivalence on six characteristics: first semester assessment grades, days present during the study, and pre-test scores on researcher-developed tests in procedural fluency and conceptual understanding of arc length/sector area and graphing circles. First semester assessment grades were checked because previous grades and prior achievement are important predictors of future achievement (Hattie, 2008). Attendance was compared for the 20 days of the study to determine if groups were present for the same amount of instruction. Pre-test scores were checked to determine if either group had more prior knowledge about arc length/sector area or graphing circles.

One-way ANOVA was conducted in SPSS version 26 to test the mean difference among these groups. The results, given in Table 2, showed that the groups were not significantly different in terms of first semester assessment grades, prior knowledge, or attendance.

Table 2

<table>
<thead>
<tr>
<th>Equivalency of Groups</th>
<th>Means</th>
<th>t-value or F-value</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Semester Assessment Grades</td>
<td>Group A = 80.20 (SD = 10.26)</td>
<td>F(1,58) = .02</td>
<td>p = .95</td>
</tr>
<tr>
<td></td>
<td>Group B = 80.40 (SD = 12.81)</td>
<td></td>
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<tr>
<td>Days Present During Study</td>
<td>Group A = 18.80 (SD = 2.17)</td>
<td>F(1,58) = .02</td>
<td>p = .89</td>
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<tr>
<td></td>
<td>Group B = 18.73 (SD = 1.66)</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Means</td>
<td>( t )-value or  ( F )-value</td>
<td>Significance</td>
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</tr>
<tr>
<td>Arc Length and Sector Area</td>
<td>Group A = 14.72 (SD = 23.54)</td>
<td>( F(1,58) = .75 )</td>
<td>( p = .39 )</td>
</tr>
<tr>
<td>Pre-Test</td>
<td>Group B = 10.00 (SD = 18.49)</td>
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<tr>
<td>Procedural Fluency</td>
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<tr>
<td></td>
<td>Group A = 2.22 (SD = 5.76)</td>
<td>( F(1, 58) = .00 )</td>
<td>( p = 1.00 )</td>
</tr>
<tr>
<td></td>
<td>Group B = 2.22 (SD = 5.76)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arc Length and Sector Area</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-Test</td>
<td>Group A = 1.67 (SD = 4.32)</td>
<td>( t_{40} = -1.05 )</td>
<td>( p = .30 )</td>
</tr>
<tr>
<td>Conceptual Understanding</td>
<td>Group B = 3.75 (SD = 9.93)</td>
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<tr>
<td>Graphing Circles</td>
<td>Group A = 1.11 (SD = 4.47)</td>
<td>( F(1, 58) = .08 )</td>
<td>( p = .78 )</td>
</tr>
<tr>
<td>Pre-Test</td>
<td>Group B = 1.48 (SD = 5.64)</td>
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<tr>
<td>Procedural Fluency</td>
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</table>

**Note.** \( n = 60 \)

**Instrumentation**

Two types of instruments were used in this study: (1) researcher-developed criterion-reference tests to measure student achievement growth in procedural fluency and conceptual understanding in arc length/sector area and in graphing circles, and (2) researcher-modified online surveys to measure students’ perceptions of the amounts of learning that took place about arc length/sector area and graphing circles. The next two sections describe those instruments.

**Achievement Growth Measures**

This section defines achievement growth, describes methods for measuring procedural fluency and conceptual understanding, and describes the test development procedures.
Achievement growth in this study is defined as “the amount of improvement, or growth, between the beginning (pretest) and end (post-test) of instruction” (Kubiszyn & Borich, 2016, p. 219). Researcher-developed criterion-reference tests gave separate scores for procedural fluency and conceptual understanding. The difference between pre- and post-tests gave a score for procedural fluency growth and a score for conceptual understanding growth for each student for each unit.

Procedural fluency was measured as suggested by Crooks and Alibali (2014) who said, “the way in which procedural knowledge is measured has become relatively standardized: participants solve a set of problems, and a score is calculated based on how many correct answers they obtain or on the specific procedures they used to arrive at those answers” (p. 345). For example, in the case of arc length and sector area, students were given diagrams or word problems and asked to calculate the appropriate measures. For graphing circles, students were given equations that needed graphing, graphs that needed equations, and equations to convert between standard and general conic form.

Conceptual understanding was measured in two ways. First, conceptual understanding was measured by having students evaluate the appropriateness of procedures for solving problems (Crooks & Alibali, 2014). For example, in the case of arc length and sector area, students were given several problems with equations and asked if the equations were acceptable ways to solve them. Conceptual understanding was also measured explicitly by having students explain procedures and reasoning (Rittle-Johnson & Schneider, 2015). In the case of graphing circles, students were asked to explain connections between concepts such as equations of circles and the Pythagorean Theorem. The next section of this chapter describes the test development process in more detail.
Test Development Procedures

I developed the tests on arc length/sector area and graphing circles following Kubiszyn and Borich’s (2016) model as summarized in Figure 5.

Figure 5

Test Development

The standards for arc length/sector area from the state curriculum were used to produce learning outcomes. The learning outcomes were used to produce test blueprints. Then the first drafts of the tests were produced from the blueprints. The standards, learning outcomes, and test blueprints are given in Appendix D and Appendix E.

The tests were then given to an expert panel to review for content validity. The panelists were selected for their knowledge of and experience with inquiry-based mathematics instruction and current geometry standards. The panel consisted of (1) an instructor at a state university
with a Ph. D. in Teaching and Learning who teaches geometry and methods of teaching secondary mathematics, (2) a former math specialist for a Regional Educational Service Agency who regularly conducted in-service trainings for teachers in inquiry-based instructional methods, and (3) a current high school geometry teacher with 13 years of experience who uses a mixture of inquiry-based and explicit instruction. Each panelist was given the learning outcomes, test blueprints and rubrics. Each panelist was then asked to determine if each item measured procedural knowledge or conceptual understanding, if it addressed the stated learning outcome for that question, and if they had any comments about each question. Based on that feedback, several questions were revised. The revised tests were then piloted by a group of students who had previously passed geometry. Several questions were revised again considering some of the students’ responses to them.

The piloted tests were scored separately by me and my co-teacher using the rubrics and checked for interrater reliability using SPSS version 26. Two questions on the arc length and sector area test and two questions on the graphing circles test were revised as a result of the interrater reliability check. On the arc length and sector area test, one question was broken into three parts, and the layout and format for the answer choices was revised for another item. On the graphing circles test, one question was broken into three separate questions to make scoring clearer, and the rubric for another item was revised for clarity.

Final versions of the tests and rubrics are provided in Appendix D and Appendix E. The pre- and post-tests were identical except that the numbers were changed to ensure they were the same level of difficulty (International Training and Education Center for Health, 2008). Table 3 and Table 4 show the Cohen’s kappa values for each item on the final versions of the tests.
Table 3

*Interrater Reliability for Arc Length and Sector Area Test*

<table>
<thead>
<tr>
<th>Item Number</th>
<th>Kappa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Overall Agreement 1.000</td>
</tr>
<tr>
<td>2</td>
<td>Overall Agreement 1.000</td>
</tr>
<tr>
<td>3</td>
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</tr>
<tr>
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</tr>
<tr>
<td>5</td>
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</tr>
<tr>
<td>6</td>
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</tr>
<tr>
<td>7</td>
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</tr>
<tr>
<td>8</td>
<td>Overall Agreement .840</td>
</tr>
<tr>
<td>9</td>
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</tr>
<tr>
<td>10A</td>
<td>Overall Agreement 1.000</td>
</tr>
<tr>
<td>10B</td>
<td>Overall Agreement 1.000</td>
</tr>
<tr>
<td>11</td>
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</tr>
</tbody>
</table>

Table 4

*Interrater Reliability for Graphing Circles Test*

<table>
<thead>
<tr>
<th>Item Number</th>
<th>Kappa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Overall Agreement 1.000</td>
</tr>
<tr>
<td>2</td>
<td>Overall Agreement 1.000</td>
</tr>
<tr>
<td>3</td>
<td>Overall Agreement 1.000</td>
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<tr>
<td>4</td>
<td>Overall Agreement 1.000</td>
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<tr>
<td>5</td>
<td>Overall Agreement .950</td>
</tr>
<tr>
<td>6</td>
<td>Overall Agreement .832</td>
</tr>
<tr>
<td>7A</td>
<td>Overall Agreement .805</td>
</tr>
<tr>
<td>7B</td>
<td>Overall Agreement .903</td>
</tr>
<tr>
<td>8A</td>
<td>Overall Agreement .869</td>
</tr>
<tr>
<td>8B</td>
<td>Overall Agreement .961</td>
</tr>
<tr>
<td>9A</td>
<td>Overall Agreement .934</td>
</tr>
<tr>
<td>9B</td>
<td>Overall Agreement .933</td>
</tr>
</tbody>
</table>
Student Assessment of Their Learning Gains Surveys

Student perceptions of learning were measured using a survey based on the modified Student Assessment of their Learning Gains – Modified (SALG-M) (Laursen et al., 2011). The original version of the Student Assessment of their Learning Gains (SALG) survey (Seymour et al., 2000) “asks students to assess and report on their own learning, and on the degree to which specific aspects of the course have contributed to that learning” (Mathieu et al., 2020). The SALG has been tested with students in mathematics, chemistry, engineering, biology, physics, psychology, statistics, business, and communications. According to SALG’s authors, “it may be adapted for any pedagogical approach or discipline” (Seymour et al., 2000, p. 1).

Laursen et al. (2011) adapted the SALG to create the SALG-M as part of an evaluation of inquiry-based learning in college mathematics classes. The SALG-M also allows users to customize the survey, and it has been customized for use in other studies (Ethnography & Evaluation Research, 2010; Mullins et al., 2019). The surveys are shown in Appendix C and Appendix D.

The survey used for this study contained five sections. Sections 1, 3, and 5 were used to answer research question 5 (students’ assessment of their learning gains). Section 1 asked three questions about how effectively the overall instructional approach helped their learning. Section 3 asked students to what extent they understand or could explain concepts within the unit. Section 5 asked students about their gains in confidence that they understood the material. Since the survey used in this study differed from the original SALG and SALG-M, the reliability was calculated, and the Cronbach’s alpha values for sections 1, 3, and 5 are shown in Table 5. Section 2 asked students to rate how helpful particular instructional activities were, and section 4 asked students to comment on the activities in the unit and how helpful they were.
Table 5

Reliability for Surveys

<table>
<thead>
<tr>
<th>Survey Section</th>
<th>Cronbach’s alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.875</td>
</tr>
<tr>
<td>3</td>
<td>.935</td>
</tr>
<tr>
<td>5</td>
<td>.933</td>
</tr>
</tbody>
</table>

Surveys were given to both groups at the end of each unit using the Qualtrics web-based survey tool on school-issued laptops during class time. Web-based surveys administered during class time were used because they can increase response rate, as indicated by Young et al. (2019), who found that the combination of the use of online surveys and providing class time to complete them has increased college course evaluation responses to over 73%.

Procedures

Instruction

This section describes the instruction used for each group. My co-teacher and I developed the lessons, and we have used similar lessons with high school geometry students for several years. The inquiry-based lessons spent the majority of class time on student-centered activities such as working in groups and giving and listening to presentations by other students, with teachers providing scaffolding through carefully selected problems, giving feedback to students, and providing mini-lessons when needed (Laursen et al., 2011). The explicit lessons, in contrast, had the teachers telling and showing students everything they need to know to understand and solve a problem (Clark et al., 2012).

An example of the difference between instruction given to the two groups can be seen in the first day of the study. Both groups began with a warm-up asking them to calculate area and
circumference. After the warm-up, students in the inquiry-based group were placed in groups of 2-3 and given the question shown in Figure 6.

**Figure 6**

*Beginning of the First Inquiry-Based Lesson on Arc Length and Sector Area*

Would you get more pizza if you ate two slices of a Personal Pan Pizza or one slice of a large pizza?

While students were working, my co-teacher and I walked around the room to observe student work and asked guiding questions if necessary. My co-teacher and I made mental notes about which groups used which methods while circulating around the room. After most groups had finished or nearly finished, several groups were asked to show their solutions on the document camera and explain their reasoning to the rest of the class. Groups were chosen to reflect the variety of procedures that students used.

Then, students remained in their same groups to answer one of the questions shown in Figure 7. Groups were encouraged to pick different questions. After all groups had finished at least one of the questions, several groups were asked to show their solutions on the document camera and explain their reasoning to the rest of the class, but this time groups were chosen to highlight the differences between the three questions the groups had to choose from. This
concluded the first lesson on arc length and sector area. Students were then given six homework problems to practice using the procedures they had developed in class.

**Figure 7**

*Second Question Posed to Students in the First Inquiry-Based Lesson on Arc Length and Sector Area*

---

**Answer ONE of the following questions about this clock:**

1. How far does the end of the minute hand travel in 20 minutes?
2. How far does the end of the hour hand travel between 1:00 and 5:00?
3. How far does the end of the hour hand travel between 1:00 and 1:30?

Explicit instruction was very different. After the warm-up, students were given a guided notes sheet, part of which is shown in Figure 8. I projected the guided notes sheet on the board and gave students the definitions and units for arc length and sector area. The guided notes included several formulas that could be used to solve for arc length and sector area. I demonstrated solving one arc length and one sector area problem at the board. Students then
worked on nine similar problems in class while my co-teacher and I circled the room answering questions and checking for understanding.

**Figure 8**

*Begnning of the First Explicit Lesson on Arc Length and Sector Area*

**Definitions**

- **Arc Length**: the distance around part of the edge of a circle
  - Units for Arc Length: It's distance, so units will be in, cm, yd, etc.

**Formulas**:

\[
\text{Arc Length} = \frac{2\pi r \theta}{360^\circ}
\]

\[
\text{Arc Length} = 2\pi r \left(\frac{\text{part}}{\text{whole}}\right)
\]

\[
\frac{\text{Arc Length}}{\text{Circumference}} = \frac{\text{part}}{\text{whole}}
\]

Appendix H describes the classwork and homework each day for both groups during the arc length and sector area unit, and Appendix I describes the same information for the graphing circles unit. All warm-ups, classwork, and homework for the inquiry-based lessons are given in Appendix J, and the warm-ups, classwork, and homework for the explicit lessons are given in Appendix K.

**Procedures for Measuring Student Achievement Growth**

Each student was given a pre- and a post-test on arc length and sector area one week prior to the beginning of each unit. All tests were scored independently by my co-teacher and me, then the scores for each item were compared. If a score were different due to an obvious error in scoring or data entry, the error was corrected. If the difference were due to a difference of opinion of the raters, the difference was noted, and a consensus score agreed upon. The
difference between consensus pre- and post-test scores were used as measures of achievement growth in procedural fluency and conceptual understanding. The process was repeated for graphing circles.

**Procedures for Measuring Student-Reported Learning Gains**

Responses to survey items were assigned a 1-6 value with 1 representing lower scores and 6 representing higher scores. Student-reported learning gains were calculated by adding the assigned values for questions 1, 3, and 5 on the survey. The minimum possible amount of student-reported learning was 13 points, and the maximum was 78 points.

A t-test was used to determine if there were a difference in student-reported learning gains between inquiry-based and explicit instruction. The survey responses for Group A in the unit on arc length/sector area and the responses for Group B in the unit on graphing circles provided the data for learning gains under inquiry-based instruction. The survey responses for Group A in the unit on graphing circles and the responses for Group B in the unit on arc length/sector area provided the data for learning gains under explicit instruction. The total scores for questions 1, 3, and 5 on the survey were used as the amount of student-reported learning gains.

**Procedures for Analyzing Open-Ended Survey Responses**

First, I categorized responses based on whether students found each type of instruction (inquiry-based or explicit) to be “Helpful”, “Not Helpful”, or “Neutral” to their learning. Responses that did not fit one of those categories were not analyzed. Then, if the response included a mention of a specific activity (e.g., group work or teacher explanation) or if it included a description of how the instruction was helpful or not (e.g., improving recall or increasing understanding), those pieces of information were coded using open coding. For
example, the statement “Most group activities made my thinking process less limited, as it allowed me to hear direct input from another persons [sic] cognitive process of each question and assignment” was categorized as “Helpful / Group Work / Multiple Solutions”. The statement “Without being shown enough on how to do it like there wasn’t enough examples shown for me to really understand well enough” was coded as “Not Helpful / Little Direct Instruction / Understanding”. Finally, responses were grouped by codes to show themes in the responses.

**Data Analysis**

Data was input into SPSS version 26. For research questions 1-4, a one-way ANOVA was conducted to determine if the mean achievement growth was significantly different between groups. The samples were independent, and student achievement growth was the dependent variable. For research question 5, a one-way ANOVA was conducted to determine if student-reported learning gains were significantly different between groups. The samples were independent, and the student-reported amount of learning was the dependent variable. I used an alpha value of .05 for all tests of significance (Coolidge, 2012). A table of Research Questions and Type of Measurement for Variables is given in Table 6.
<table>
<thead>
<tr>
<th>Research Question</th>
<th>Research Design</th>
<th>Key Variables</th>
<th>Measures and Key Variables</th>
<th>Scale</th>
<th>Measurement Type</th>
<th>Proposed Data Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Is there a significant difference in student achievement growth in procedural fluency in the unit of arc length and sector area between inquiry-based instruction and explicit instruction?</td>
<td>Quasi-Experimental Design</td>
<td>Performance on test on arc length and sector area</td>
<td>Common arc length and sector area assessment score</td>
<td>0 to 100</td>
<td>Continuous variable</td>
<td>Independent sample t-test comparing student achievement growth</td>
</tr>
<tr>
<td>(2) Is there a significant difference in student achievement growth in conceptual understanding in the unit of arc length and sector area between inquiry-based instruction and explicit instruction?</td>
<td>Quasi-Experimental Design</td>
<td>Performance on test on arc length and sector area</td>
<td>Common arc length and sector area assessment score</td>
<td>0 to 100</td>
<td>Continuous variable</td>
<td>Independent sample t-test comparing student achievement growth</td>
</tr>
<tr>
<td>(3) Is there a significant difference in student achievement growth in procedural fluency in the unit of graphing circles between inquiry-based instruction and explicit instruction?</td>
<td>Quasi-Experimental Design</td>
<td>Performance on test on graphing circles</td>
<td>Common graphing circles assessment score</td>
<td>0 to 100</td>
<td>Continuous variable</td>
<td>Independent sample t-test comparing student achievement growth</td>
</tr>
<tr>
<td>(4) Is there a significant difference in student achievement growth in conceptual understanding in the unit of graphing circles between inquiry-based instruction and explicit instruction?</td>
<td>Quasi-Experimental Design</td>
<td>Performance on test on graphing circles</td>
<td>Common graphing circles assessment score</td>
<td>0 to 100</td>
<td>Continuous variable</td>
<td>Independent sample t-test comparing student achievement growth</td>
</tr>
</tbody>
</table>
(5) Is there a significant difference in student-reported learning gains between inquiry-based instruction and explicit instruction?

<table>
<thead>
<tr>
<th>Study</th>
<th>Amount of perceived learning by students</th>
<th>Student learning survey</th>
<th>Interval Variable with the Construct Mean Score</th>
<th>Independent sample t-test comparing student-reported learning gains</th>
</tr>
</thead>
</table>
Steps Taken to Overcome Design Limitations

A potential limitation involved me being both the teacher and the researcher. This created the opportunity for me to unintentionally introduce bias into the study. To reduce experimenter expectancy bias, blinding methods were implemented: pre- and post-tests were coded with identifiers instead of student names, the two groups of tests were shuffled together before being graded, and a second scorer was used to check for reliability (Suter, 2012). Bias was also reduced by having a clearly defined data collection and analysis procedure, by checking the instruments for validity, and by using a very similar pre- and post-tests to ensure reliability (Smith & Noble, 2014).

Student attendance was another limitation because all students were not present for all of the instruction. To ensure that differences in student attendance did not affect the results of the study, attendance was checked for each group and found to be not significantly different between groups (see Table 2). Therefore, whatever effect student attendance had on performance, it should have affected both groups similarly. There were also two students who were absent much more than other students as shown in Table 7: one student (who was in Group A) was only present 10 days, and one student (who was in Group B) was only present 12 days. I decided to include their data because having a few students with poor attendance is typical, and I wanted the results of this study to reflect typical classroom situations.
Table 7

*Student Attendance During Study*

<table>
<thead>
<tr>
<th>Days Present</th>
<th>No. Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>28</td>
</tr>
<tr>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>18</td>
<td>7</td>
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<td>17</td>
<td>1</td>
</tr>
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<td>16</td>
<td>4</td>
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<td>15</td>
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<td>12</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
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CHAPTER IV
FINDINGS

This study examined the use of inquiry-based and explicit instruction in two areas of high school geometry, arc length/sector area and graphing circles, by comparing student achievement growth and student-perceived learning gains when groups were taught these topics using different methods. For the first part of the study, one group was taught arc length and sector area using inquiry-based methods while the other group was taught the same topic using explicit methods. For the second phase of the study, both groups were taught graphing circles, but the instructional methods were swapped.

The first section of this chapter gives the results of the achievement growth research questions. The second section of this chapter examines student-reported learning gains as well students’ responses to the free-response question on the survey. The research questions are organized as shown in Table 8.

Table 8

Organization of Research Questions

<table>
<thead>
<tr>
<th>Research Topic</th>
<th>Research Question No.</th>
</tr>
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<tbody>
<tr>
<td>Arc Length and Sector Area</td>
<td></td>
</tr>
<tr>
<td>• Procedural Fluency</td>
<td>RQ 1</td>
</tr>
<tr>
<td>• Conceptual Understanding</td>
<td>RQ 2</td>
</tr>
<tr>
<td>Graphing Circles</td>
<td></td>
</tr>
<tr>
<td>• Procedural Fluency</td>
<td>RQ 3</td>
</tr>
<tr>
<td>• Conceptual Understanding</td>
<td>RQ 4</td>
</tr>
<tr>
<td>Student-Reported Learning Gains</td>
<td>RQ 5</td>
</tr>
</tbody>
</table>
Achievement Growth

Research questions 1-4 dealt with achievement growth, which was defined as the difference between pre-test and post-test scores. To answer each achievement growth question, a one-way ANOVA was conducted to test the mean difference between groups. A MANOVA indicated significant interaction between instructional method and prior achievement for arc length and sector area, $F(4, 106) = 2.77, p = .03$, Wilks’ $\Lambda = .82$. Therefore, to examine the relationship between instructional method, prior achievement, and achievement growth, additional univariate analyses were conducted. Prior achievement was divided into three subgroups: students whose previous semester assessment grades were in the highest quartile, students whose previous semester assessment grades were in the middle 50%, and students whose previous semester assessment grades were in the lowest quartile. Although there was no significant interaction between instructional method and prior achievement for graphing circles, $F(4, 106) = 1.93, p = .11$, Wilks’ $\Lambda = .87$, one-way univariate analyses were conducted on the subgroups for completeness. There were 15 students in the highest quartile (seven in Group A and eight in Group B), 30 students in the middle 50% (16 in Group A and 14 in Group B), and 15 students in the lowest quartile (seven in Group A and eight in Group B). An additional univariate analysis was also conducted for the subgroup of students receiving special education services. There were 14 students who received special education services, seven in each group. The achievement growth results are summarized in Table 9. In Table 9, F-values are reported if the variances were equal, and t-values were reported if the variances were not equal.
### Table 9

**Summary of Achievement Growth Results**

<table>
<thead>
<tr>
<th></th>
<th>Inquiry-Based</th>
<th>Explicit</th>
<th>$t$-value or $F$-value</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Arc Length and Sector Area</strong></td>
<td>(Group A)</td>
<td>(Group B)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Procedural Fluency</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>All Students</strong></td>
<td>58.89 (SD = 33.33)</td>
<td>80.00 (SD = 22.27)</td>
<td>$t_{51} = -2.89$</td>
<td>.01*</td>
</tr>
<tr>
<td>Special Education Students</td>
<td>54.76 (SD = 36.91)</td>
<td>86.90 (SD = 17.91)</td>
<td>$F (1, 12) = 4.30$</td>
<td>.06</td>
</tr>
<tr>
<td>Top Quartile Students</td>
<td>57.14 (SD = 23.29)</td>
<td>83.33 (SD = 24.80)</td>
<td>$F (1, 13) = 4.40$</td>
<td>.06</td>
</tr>
<tr>
<td><strong>Middle 50% Students</strong></td>
<td>61.98 (SD = 36.51)</td>
<td>83.93 (SD = 17.44)</td>
<td>$t_{22} = -2.14$</td>
<td>.04*</td>
</tr>
<tr>
<td>Bottom Quartile Students</td>
<td>53.57 (SD = 37.84)</td>
<td>69.79 (SD = 26.70)</td>
<td>$F (1, 13) = .94$</td>
<td>.35</td>
</tr>
<tr>
<td><strong>Conceptual Understanding</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Students</td>
<td>16.11 (SD = 17.77)</td>
<td>25.00 (SD = 22.64)</td>
<td>$F (1, 58) = 2.86$</td>
<td>.10</td>
</tr>
<tr>
<td>Special Education Students</td>
<td>2.38 (SD = 11.50)</td>
<td>26.19 (SD = 21.21)</td>
<td>$F (1, 12) = 6.82$</td>
<td>.02*</td>
</tr>
<tr>
<td>Top Quartile Students</td>
<td>7.14 (SD = 13.11)</td>
<td>29.17 (SD = 19.42)</td>
<td>$F (1, 13) = 6.41$</td>
<td>.03*</td>
</tr>
<tr>
<td>Middle 50% Students</td>
<td>23.96 (SD = 17.18)</td>
<td>16.67 (SD = 11.32)</td>
<td>$t_{26} = 1.39$</td>
<td>.18</td>
</tr>
<tr>
<td>Bottom Quartile Students</td>
<td>7.14 (SD = 16.27)</td>
<td>35.41 (SD = 35.00)</td>
<td>$t_{10} = -2.05$</td>
<td>.07</td>
</tr>
</tbody>
</table>
### Inquiry-Based vs Explicit

<table>
<thead>
<tr>
<th></th>
<th>Inquiry-Based (Group B)</th>
<th>Explicit (Group A)</th>
<th>t-value or F-value</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Graphing Circles</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Procedural Fluency</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Students</td>
<td>51.25 (SD = 32.39)</td>
<td>62.50 (SD = 31.83)</td>
<td>F (1, 58) = 1.84</td>
<td>.18</td>
</tr>
<tr>
<td>Special Education Students</td>
<td>50.00 (SD = 19.09)</td>
<td>51.79 (SD = 24.40)</td>
<td>F (1, 12) = .02</td>
<td>.88</td>
</tr>
<tr>
<td>Top Quartile Students</td>
<td>68.75 (SD = 18.90)</td>
<td>48.21 (SD = 26.45)</td>
<td>F (1, 13) = 3.06</td>
<td>.10</td>
</tr>
<tr>
<td>Middle 50% Students</td>
<td>53.57 (SD = 32.68)</td>
<td>74.22 (SD = 29.04)</td>
<td>F (1, 28) = 3.36</td>
<td>.08</td>
</tr>
<tr>
<td>Bottom Quartile Students</td>
<td>29.69 (SD = 33.37)</td>
<td>50.00 (SD = 36.08)</td>
<td>F (1, 13) = 1.28</td>
<td>.28</td>
</tr>
<tr>
<td><strong>Conceptual Understanding</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Students</td>
<td>43.70 (SD = 25.76)</td>
<td>37.78 (SD = 27.31)</td>
<td>F (1, 58) = .75</td>
<td>.39</td>
</tr>
<tr>
<td>Special Education Students</td>
<td>42.85 (SD = 18.63)</td>
<td>34.92 (SD = 26.00)</td>
<td>F (1, 12) = .43</td>
<td>.52</td>
</tr>
<tr>
<td><strong>Top Quartile Students</strong></td>
<td><strong>55.56 (SD = 23.00)</strong></td>
<td><strong>25.40 (SD = 22.87)</strong></td>
<td><strong>F (1, 13) = 6.45</strong></td>
<td><strong>.03</strong> *</td>
</tr>
<tr>
<td>Middle 50% Students</td>
<td>42.06 (SD = 25.85)</td>
<td>48.61 (SD = 27.18)</td>
<td>F (1, 28) = .45</td>
<td>.51</td>
</tr>
<tr>
<td>Bottom Quartile Students</td>
<td>34.72 (SD = 26.85)</td>
<td>25.40 (SD = 23.76)</td>
<td>F (1, 13) = .50</td>
<td>.49</td>
</tr>
</tbody>
</table>

*p < .05
Research Question 1: Arc Length and Sector Area Procedural Fluency

Research Question #1: Is there a significant difference in student achievement growth in procedural fluency in the unit of arc length and sector area between inquiry-based instruction and explicit instruction?

Achievement growth for arc length and sector area procedural fluency was consistently higher in favor of explicit instruction. For the All Students group in arc length and sector area procedural fluency, the group receiving explicit instruction had mean achievement growth 21.11 percentage points higher than the inquiry-based group as shown in Figure 9, and the difference was significant ($t_{51} = -2.89, p = .01$). Therefore, the null hypothesis that there was no difference in achievement growth in procedural fluency in the unit of arc length and sector area between inquiry-based instruction and explicit instruction was rejected.

Figure 9

*Arc Length and Sector Area Procedural Fluency Achievement Growth*
In examining subgroups, the middle 50% subgroup showed significantly higher achievement growth under explicit instruction ($t_{22} = -2.14, p = .04$). The subgroup of special education students approached significance ($F (1, 12) = 4.30, p < .10$), as did the top quartile subgroup ($F (1, 13) = 4.40, p < .10$). Students in the bottom quartile scored higher under explicit instruction, but the difference was not significant.

**Research Question 2: Arc Length and Sector Area Conceptual Understanding**

Research Question #2: Is there a significant difference in student achievement growth in conceptual understanding in the unit of arc length and sector area between inquiry-based instruction and explicit instruction?

Achievement growth for arc length and sector area conceptual understanding was generally higher in favor of explicit instruction. For the All Students group in arc length and sector area conceptual understanding, the group receiving explicit instruction had mean achievement growth 8.89 percentage points higher than the inquiry-based group as shown in Figure 10, but the difference was not significant ($F (1, 58) = 2.86, p = .10$). Therefore, the null hypothesis that there was no difference in achievement growth in conceptual understanding in the unit of arc length and sector area between inquiry-based instruction and explicit instruction was accepted.
In examining subgroups, special education students’ achievement grew significantly more under explicit instruction ($F(1, 12) = 6.82, p=.02$), as did students in the top quartile ($F(1, 13) = 6.41, p=.03$). Achievement growth for students in the bottom quartile approached significance in favor of explicit instruction ($t_{10} = -2.05, p<.10$). Students in the middle 50% showed higher growth when taught using inquiry-based instruction, but the difference was not significant.

**Research Question 3: Graphing Circles Procedural Fluency**

Research Question #3: Is there a significant difference in student achievement growth in procedural fluency in the unit of graphing circles between inquiry-based instruction and explicit instruction?
Achievement growth for graphing circles procedural fluency was generally higher in favor of explicit instruction. For the All Students group in graphing circles procedural fluency, the group receiving explicit instruction had mean achievement growth 11.25 percentage points higher than the inquiry-based group as shown in Figure 11, but the difference was not significant \(F (1, 58) = 1.84, p=.18\). Therefore, the null hypothesis that there was no difference in achievement growth in procedural fluency in the unit of graphing circles between inquiry-based instruction and explicit instruction was accepted.

**Figure 11**

*Graphing Circles Procedural Fluency Achievement Growth*

Achievement growth for students in the middle 50% subgroup approached significance \(F (1, 28) = 3.36, p<.10\). The special education subgroup as well as students in the top and
bottom quartiles showed higher achievement growth under explicit instruction, but the differences were not significant.

**Research Question 4: Graphing Circles Conceptual Understanding**

Research Question #4: Is there a significant difference in student achievement growth in conceptual understanding in the unit of graphing circles between inquiry-based instruction and explicit instruction?

Achievement growth for graphing circles conceptual understanding was generally higher in favor of inquiry-based instruction. For the All Students group in graphing circles conceptual understanding, the group receiving inquiry-based instruction had mean achievement growth 5.92 percentage points higher than the explicit group as shown in Figure 12, but the difference was not significant \((F(1, 58) = .75, p=.39)\). Therefore, the null hypothesis that there was no difference in achievement growth in conceptual understanding in the unit of graphing circles between inquiry-based instruction and explicit instruction was accepted.
Students in the top quartile showed significantly higher achievement growth under inquiry-based instruction \( (F(1, 13) = 6.45, p<.03) \). Students receiving special education services and students in the bottom quartile also scored higher when taught using inquiry-based methods, but the differences were not significant. Students in the middle 50% scored higher when taught using explicit methods, but the difference was not significant.

**Student Perceptions of Their Learning Gains**

**Research Question 5: Student-Reported Learning Gains**

Research Question #5: Is there a significant difference in student-reported learning gains between inquiry-based instruction and explicit instruction?
Students reported higher learning gains when taught using explicit instruction, but the difference was not significant. Mean student-reported learning gains for inquiry-based instruction was 57.30 (SD = 14.09), and mean student-reported learning gains for explicit instruction was 61.20 (SD = 13.11). One-way ANOVA was conducted in SPSS to test the mean difference among these instructional methods. The results showed that there was no significant difference between instructional methods, $F(1, 172) = 3.56, p = .06$. Therefore, the null hypothesis that there was no difference in student-reported learning gains between inquiry-based instruction and explicit instruction was accepted. Note: Survey results could not be broken down by the subgroups because the surveys were anonymous.

**Helpfulness of Each Learning Activity**

Section 2 of the survey asked students to rate how helpful particular instructional activities were. The survey included activities such as watching the teacher solve problems and solving problems on their own (which were aimed at the explicit group) and working in small groups and hearing other students explain their work (which were aimed at the inquiry-based group). The intention was that students would select the “Did Not Happen” option if their instructional method did not include an activity. For example, all students in the inquiry-based group should have selected “Did Not Happen” when asked how helpful it was to watch the teacher solve problems because I did not solve problems during inquiry-based instruction; instead, groups presented and explained their work to the rest of the class using a document camera. However, only 4% of students in the inquiry-based arc length and sector area group selected “Did Not Happen” for that question. Since students clearly did not answer Section 2 as intended, those results are not being reported.
Open-Ended Survey Question Responses

Students were asked the following question on the survey at the end of each unit: “Please comment on the activities we did in this unit and whether or not they helped you remember key ideas.” Their complete responses are given in Appendix N. The intent of the question was to find out what aspects of each type of instruction students found helpful.

Responses Concerning Inquiry-Based Instruction

Forty-seven students said inquiry-based instruction was helpful to them. Thirty-six of them cited specific inquiry-based activities or aspects of inquiry-based instruction they found helpful. Activities which more than one student cited are shown in Table 10.

Table 10

Specific Inquiry-Based Activities Students Found Helpful

<table>
<thead>
<tr>
<th>Activity</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group work</td>
<td>18</td>
</tr>
<tr>
<td>Activities in general</td>
<td>5</td>
</tr>
<tr>
<td>Circle plate</td>
<td>5</td>
</tr>
<tr>
<td>Finding other ways to solve problems</td>
<td>2</td>
</tr>
<tr>
<td>Circle plate and practice work</td>
<td>1</td>
</tr>
</tbody>
</table>

Group work was the most frequently cited aspect of inquiry-based instruction students cited as helpful. Two representative examples of students’ comments about helpful aspects of inquiry-based instruction were “Teaching other students how to do the things helped me memorize them” and “We worked in groups and it really helped me understand other peoples [sic] ideas on the problems in math.”
Twenty-two students described how inquiry-based instruction was helpful to them (as the two students quoted in the previous paragraph did). Ways which were cited by more than one student are shown in Table 11.

**Table 11**

*Ways Students Found Inquiry-Based Instruction Helpful*

<table>
<thead>
<tr>
<th>Activity</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improved recall</td>
<td>8</td>
</tr>
<tr>
<td>Seeing multiple solutions</td>
<td>5</td>
</tr>
<tr>
<td>Improved understanding</td>
<td>2</td>
</tr>
<tr>
<td>Improved understanding and recall</td>
<td>2</td>
</tr>
<tr>
<td>Improved recall and seeing multiple solutions</td>
<td>1</td>
</tr>
</tbody>
</table>

Twelve students said inquiry-based instruction was not helpful. Three said having little direct instruction was not helpful, and two said group work was not helpful. Four students said inquiry-based instruction was not helpful because they did not understand the work.

**Responses Concerning Explicit Instruction**

Thirty-two students said explicit instruction was helpful to them. Seventeen of them cited specific explicit activities or aspects of explicit instruction they found helpful. Activities which more than one student cited are shown in Table 12.

**Table 12**

*Specific Explicit Activities Students Found Helpful*

<table>
<thead>
<tr>
<th>Activity</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher explanation</td>
<td>6</td>
</tr>
<tr>
<td>Repetition</td>
<td>5</td>
</tr>
<tr>
<td>Foldables</td>
<td>2</td>
</tr>
<tr>
<td>Independent work</td>
<td>2</td>
</tr>
</tbody>
</table>
Teacher explanation was the most frequently cited aspect of explicit instruction that students cited as helpful, and the repetition was the second most frequently cited. Representative quotes from students were, “I believe having the teacher go over a few practice problems in class helped me understand the work better” and “Yep, namely the practice work drilled the equations into my brain.”

Nine students described how explicit instruction was helpful to them, as the two students quoted in the previous paragraph did. Only two ways were cited by more than one student: four students said it increased their recall, and three said it increased their understanding.

Only two students said explicit instruction was not helpful. There was no common theme in their responses.

Summary

Which instructional method produced greater gains varied depending on the topic and subgroup. Table 13 summarizes the academic achievement results by highlighting which group had higher achievement growths and the percentage points difference between groups.

Table 13

Summary of Achievement Growth Results

<table>
<thead>
<tr>
<th></th>
<th>Group with Higher Outcome</th>
<th>Points Difference Between Groups</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arc Length and Sector Area</td>
<td>Explicit</td>
<td>21.11</td>
<td>.01&quot;</td>
</tr>
<tr>
<td>Procedural Fluency</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| All Students             | Explicit                  | 21.11                            | .01"
<p>| Special Education Students | Explicit                | 32.14                            | .06 |
| Top Quartile Students    | Explicit                  | 26.19                            | .06 |
| Middle 50% Students      | Explicit                  | 21.95                            | .04&quot;|
| Bottom Quartile Students | Explicit                  | 16.22                            | .35 |</p>
<table>
<thead>
<tr>
<th></th>
<th>Group with Higher Outcome</th>
<th>Points Difference Between Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conceptual Understanding</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| All Students           | Explicit                  | 8.89                             | .10  
| *Special Education Students* | Explicit                  | **23.81**                         | **.02***  
| *Top Quartile Students* | Explicit                  | **22.03**                         | **.03***  
| Middle 50% Students    | Inquiry                   | 7.29                             | .18  
| Bottom Quartile Students | Explicit              | 28.27                             | .07  

**Graphing Circles**

|                        |                           |                                  |  
| **Procedural Fluency** |                           |                                  |  
| All Students           | Explicit                  | 11.25                            | .18  
| Special Education Students | Explicit              | 1.79                             | .88  
| Top Quartile Students  | Inquiry                   | 20.54                            | .10  
| Middle 50% Students    | Explicit                  | 20.65                            | .08  
| Bottom Quartile Students | Explicit              | 20.31                            | .28  

|                        |                           |                                  |  
| **Conceptual Understanding** |                           |                                  |  
| All Students           | Inquiry                   | 5.92                             | .39  
| Special Education Students | Inquiry               | 7.93                             | .52  
| *Top Quartile Students* | Inquiry                  | **30.16**                        | **.03***  
| Middle 50% Students    | Explicit                  | 6.55                             | .51  
| Bottom Quartile Students | Inquiry              | 9.32                             | .49  

*p<.05

In arc length and sector area procedural fluency, overall students taught using explicit instruction had higher outcomes for the All Students group as well as for each of the four subgroups. The differences were significant for the All Students group and for the middle 50% group.
In arc length and sector area conceptual understanding, students taught using explicit instruction had higher outcomes for the All Students group as well as for three of the subgroups. The differences were significant for students receiving special education services and for the top quartile. The middle 50% scored higher when taught with inquiry-based instruction, but the difference was not significant.

In graphing circles procedural fluency, students taught using explicit instruction had higher outcomes for the All Students group and for three of the subgroups. The top quartile scored higher when taught using inquiry-based instruction. None of the differences were significant.

In graphing circles conceptual understanding, students taught using inquiry-based instruction had higher outcomes for the All Students group and for three of the subgroups. The difference for students in the top quartile was significant. The middle 50% scored higher when taught using explicit instruction, but the difference was not significant.
CHAPTER V
DISCUSSION, CONCLUSION, AND IMPLICATIONS

This study of 60 suburban high school students over two 2-week periods compared inquiry-based and explicit instruction of arc length/sector area and graphing circles. It compared growth in both procedural fluency and conceptual understanding as measured by pre- and post-tests, as well as student-reported learning gains. The results of this study, summarized in Table 14, show a significant difference in favor of explicit instruction for arc length and sector area procedural fluency but no significant difference for the other areas studied. An analysis of certain subgroups (Special Education, Top Quartile, Middle 50%, and Bottom Quartile) found some significant differences in favor of inquiry-based and some significant differences in favor of explicit instruction, which supports the idea that the most appropriate instructional method depends a variety of factors including the subject matter being taught, the students’ prior knowledge, and the students’ special education status. This chapter includes discussion of the results as well as limitations and implications of the study and recommendations for future research.
Table 14

*Procedural Fluency and Conceptual Understanding Growth for All Students*

<table>
<thead>
<tr>
<th></th>
<th>Inquiry-Based</th>
<th>Explicit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Arc Length / Sector Area</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedural Fluency</td>
<td>58.89 (SD = 33.33)</td>
<td>80.00 (SD = 22.27)*</td>
</tr>
<tr>
<td>Conceptual Understanding</td>
<td>16.11 (SD = 17.77)</td>
<td>25.00 (SD = 22.64)</td>
</tr>
<tr>
<td><strong>Graphing Circles</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedural Fluency</td>
<td>51.25 (SD = 32.39)</td>
<td>62.50 (SD = 31.83)</td>
</tr>
<tr>
<td>Conceptual Understanding</td>
<td>43.70 (SD = 25.76)</td>
<td>37.78 (SD = 27.31)</td>
</tr>
</tbody>
</table>

*p < .05

**Discussion**

**Research Question 1: Arc Length and Sector Area Procedural Fluency**

Achievement growth in procedural fluency of arc length and sector area favored explicit instruction. The All Students group as well as students in each of the four subgroups showed higher achievement growth under explicit instruction. Results were significant for the All Students group and for students in the Middle 50% group.

Other researchers have found higher achievement growth in procedural fluency under explicit instruction. For example, Alsup and Sprigler (2003) found that students using a traditional curriculum showed significantly higher procedural fluency scores than students using the CORD Applied Mathematics series, which the teacher whose students participated in the study attributed to a focus on “facts and procedures” in their district (p. 694). A focus on “facts and procedures” could also be a contributing factor in this study because the participants were
mainly exposed to explicit instruction in eighth and ninth grade, thus making the students more comfortable with that type of instruction due to the mere exposure effect (Zajonc, 1968).

In another study, McCaffrey et al. (2001) also found that inquiry-based instruction produced higher student achievement on the Stanford 9 mathematics test than traditional instruction in courses designed for inquiry-based methods (specifically the Interactive Mathematics Program and College Preparatory Mathematics program), but they found no difference in student achievement between instructional methods for traditional mathematics courses. They said that the “use of reform practices was unrelated to achievement [on the Stanford 9] in the more traditional algebra and geometry courses” (p. 493). My study had a similar situation in that my course was designed as a traditional course, but inquiry-based methods were compared in the study.

Another possible explanation for explicit instruction producing greater achievement gains on arc length and sector area procedural fluency in this study was the similarity between the test and the practice problems. While both the inquiry-based and explicit groups were assigned practice problems that were very similar to those on the test, the explicit group practiced more of them (43 for the inquiry-based group versus 72 for the explicit group), possibly increasing the fluency though repetition (Kratochwill & Bijou, 1987).

**Research Question 2: Arc Length and Sector Area Conceptual Understanding**

Achievement growth in conceptual understanding of arc length and sector area produced mixed results. Students in the All Students group and all subgroups except the Middle 50% subgroup showed higher achievement growth under explicit instruction, and the results were significant for the Special Education and Top Quartile subgroups. Students in the Middle 50%
subgroup showed higher achievement growth under inquiry-based instruction, but the result was not significant.

Special education students’ scoring higher under explicit instruction is consistent with a meta-analysis by Kroesbergen and Van Luit (2003) which examined mathematics interventions for special education students in elementary school. Their meta-analysis found explicit instruction had higher mean effect sizes for special education students than inquiry-based instruction. They suggested that working with peers may be less effective than explicit instruction because “peers are less capable of perceiving the needs of other students than teachers” (p. 111). Since inquiry-based instruction in this study involved special education students working with peers, Kroesbergen and Van Luit’s explanation of greater achievement growth under explicit instruction may apply to special education students in arc length and sector area conceptual understanding in my study.

One surprising aspect of this study was that the inquiry-based All Students group did not outperform students in the explicit group even though much more class time was spent on arc length and sector area conceptual understanding with inquiry-based than with explicit instruction. I designed most of the lessons in this study so that approximately equal amounts of time were spent on each aspect of arc length and sector area under each instructional method. While the explicit group spent part of one class period being told what a radian is, the inquiry-based group spent two entire class periods developing the definition of a radian. This difference in time was because the inquiry-based students did an exploration involving paper plates, pipe cleaners, and hot glue. Despite the extra time allocated for the exploration, the time spent making the radian circles somewhat limited our time for discussing their connection to the definition of a radian.
Overall, arc length and sector area conceptual understanding growth was particularly small compared to other results as shown in Table 14. Low growth in this area may be attributable to a disconnect between practice work and the conceptual understanding items on the test. Half of the arc length and sector area conceptual understanding questions required students to state whether an equation was set up in a way that would correctly calculate arc length or sector area, then justify their answer. For a question to be counted as correct, the student had to correctly state both whether the equation would work and provide a correct explanation based on similarity or proportions. The arc length and sector area conceptual understanding questions on the test were less like the problems students had solved in class, where students had been given problems and told to set them up as many ways as they could. Therefore, students in both groups could have done the practice work correctly but not set up equations all the possible ways, causing them to miss those items on the test. This disconnect between the practice work and the arc length and sector area conceptual understanding items on the test could make this measure less valid than the other areas measured.

**Research Question 3: Graphing Circles Procedural Fluency**

Achievement growth in procedural fluency of graphing circles produced mixed results. Students in the All Students group and all subgroups except the Top Quartile subgroup showed higher achievement growth under explicit instruction, but students in the Top Quartile subgroup showed higher growth under inquiry-based instruction. Results were not significant for the All Students group or any of the subgroups.

The result that graphing circles procedural fluency growth was not significantly different was surprising. The graphing circles procedural questions are very calculation-oriented, so I
expected that this area would favor explicit instruction due to more fluency through repetition as occurred when examining procedural fluency for arc length and sector area.

The significantly higher achievement for students in the top quartile is consistent with research by Mayer (1998), who found that inquiry-based instruction can help higher achieving students more than other students. Clark et al. (2012), explaining why more advanced students may benefit more from inquiry-based instruction, suggested that “often, only the brightest and most well-prepared students make the discovery” (p. 8).

**Research Question 4: Graphing Circles Conceptual Understanding**

Achievement growth in conceptual understanding of graphing circles produced mixed results. Students in the All Students group and in all subgroups except the Middle 50% subgroup showed higher achievement growth under inquiry-based instruction. Results favored inquiry-based instruction for student in the Top Quartile subgroup, and the difference was significant.

The All Students group’s higher growth under inquiry-based instruction is consistent with previous research such as that by (Bando et al., 2019), (Boaler, 1998), and (Pegg, 2019). Interestingly, students in the current study showed more conceptual understanding growth (though not statistically significantly) under inquiry-based instruction for graphing circles but not for arc length and sector area. One explanation for the different results for arc length/sector area conceptual understanding and graphing circles conceptual understanding is the potential disconnect between the practice work and test items for arc length and sector area conceptual understanding (as noted in the discussion of Research Question 2 above). The conceptual understanding items on the graphing circles test did not exhibit the same disconnect. A second explanation for the different results for arc length/sector area conceptual understanding and graphing circles conceptual understanding involves the activities used in each unit aimed at
improving conceptual understanding. The radian circle activity (discussed in Research Question 2 above) took up two entire class periods, and students were still somewhat limited in time for discussing their connection to the definition of a radian. In contrast, the graphing circles conceptual activities were shorter, leaving more time for small group and large group discussion.

**Research Question 5: Student-Reported Learning Gains**

Students reported higher learning gains when taught using explicit instruction, but the difference was not significant. This result is different than Laursen et al. (2014) found among their sample of college students, where inquiry-based learning students reported larger learning gains than non-inquiry-based learning students.

There are at least two possible reasons students in the current study may have reported higher learning gains under explicit instruction. First, many of their mathematics teachers from previous years relied on explicit instruction, so students may have more positive impressions of explicit instruction simply because they are more familiar with it due to the mere exposure effect (Zajonc, 1968). Second, there was more of a written product at the end of each day under explicit instruction, so students may have seen the larger number of solved problems under explicit instruction and associated that with “learning more”.

**Limitations**

There are five limitations to the design and implementation of this study. This section describes five limitations and the measures that were used to address them, if possible.

The first limitation of this study is that it only examined arc length/sector area and graphing circles in on-level co-taught and general education classes in a suburban high school as described in Chapter 2. Since this research examined only 60 students at one school with one
teacher, the findings are only generalizable to populations similar to the population in this study, which is described in Chapter 3.

A second limitation of this study is that the participants were not randomly assigned, which may result in reduced validity. Classes were used that were created by the school prior to the beginning of the school year. Therefore, the study is quasi-experimental rather than experimental. To minimize this limitation, groups were compared and found to be equivalent on a variety of characteristics including first semester assessment grades, prior knowledge, and attendance as described in more detail in Chapter 3.

A third limitation of the study deals with the instruments and may result in reduced validity. The researcher-created tests may not have measured procedural fluency and conceptual understanding as accurately as possible because some of the conceptual understanding questions could be answered through memorization. To minimize this limitation, the questions were reviewed by an expert panel and piloted with a different group of students as described in more detail in Chapter 3.

A fourth limitation involved the use of borrowed classes and may result in reduced reliability. When using borrowed classes, there is a risk that a borrowed classroom has pre-established “mindset and norms” that may be less conducive to the study (Singh et al., 2000, p. 27). The year the study was conducted, I only had two co-taught classes, so I borrowed two general-education classes from my co-teacher to increase the sample size. To minimize the effects of borrowed classrooms, the classes were grouped so that each group contained one borrowed class and one class that was not borrowed, as described in Chapter 3.

The fifth limitation of this study involved me being both the teacher and the researcher. This created the opportunity for me to unintentionally introduce experimenter expectancy bias.
into the study. Experimenter expectancy bias is “[bias] that influences researchers in ways that lead them to create conditions favoring expected findings” (Suter, 2012, p. 187). Several things were done to address this issue. First, the instruments were reviewed by an expert panel to ensure content validity. Second, student names were removed from all tests and replaced with a random number, then the tests were shuffled before being graded. Third, a detailed scoring rubric was used. Fourth, all tests were scored separately both by me and my co-teacher; then, we compared our scores on each test to ensure reliability. Finally, the mixed results obtained from the study suggest that experimenter bias was likely not a factor.

**Implications**

This research has implications for mathematics teachers, students, textbook publishers, teacher educators, and school administrators.

Overall results of this study found no differences in student achievement growth in most of the areas examined. Therefore, teachers can use either instructional method to promote student achievement growth in conceptual understanding of arc length/sector area, conceptual understanding of graphing circles and procedural fluency in graphing circles. Teachers should consider using explicit instruction to teach arc length and sector area procedural fluency.

An analysis of results by subgroups revealed information with implications for some teachers. Teachers of special education students should consider explicit instruction when teaching arc length and sector area conceptual understanding. Teachers of students in the top quartile should consider using explicit instruction for arc length/sector area conceptual understanding and inquiry-based instruction for graphing circles conceptual understanding.

Textbook publishers should consider producing materials that use a variety of instructional methods. Too often, publishers produce textbooks that are clearly geared toward
just one instructional method. Publishers should also consider presenting multiple ways to teach each topic to enable teachers to select the most appropriate method based on their students. As an example for arc length and sector area, a publisher could include instructions for the radian circle activity for teachers whose students are primarily in the 25th to 75th percentile, and they would also include explicit definitions, formulas, and examples for teachers of honors and special education students.

Teacher educators should consider teaching prospective teachers to become proficient in both inquiry-based and explicit instruction and to help them recognize when one method may be more appropriate than the other. School administrators should communicate the expectation that teachers utilize inquiry-based and explicit instructional methods and support that expectation through staff development.

**Future Research**

First, replicating this research with a larger sample would increase the size of the subgroups. Sixty students participated in this study, but when examining the results in finer detail, the subgroups were very small. Each quartile had a total of only 15 students, and the Special Education subgroup had a total of only 14 students. Somewhat surprisingly, even with subgroups that small, significant results appeared. Perhaps with larger subgroups, more results will be found to be significant.

Second, this research should be extended to include other topics in high school geometry. So far, the list of researched high school geometry topics includes congruent triangles (Thompson, 1992), theorems about circles (Mensah-Wonkyi & Adu, 2016), arc length/sector area (this study) and graphing circles (this study). Similarity, proofs, trigonometry, surface area, and volume have not yet been researched; investigating them could provide information to
determine if the results of the current study are consistent with other high school geometry topics.

Third, future studies should refine this study to capture additional information about student engagement and performance. High-achieving students and unmotivated students seemed to me to be more actively involved in learning under inquiry-based instruction. The high-achieving students were anxious to explore concepts and push their abilities. The students who were usually unmotivated and reluctant to engage were much more engaged under inquiry-based instruction. Although I expected this higher level of engagement to translate into higher achievement growth, this study did not support that in general. Therefore, additional research is needed to determine how to quantify the benefits of more engaged students.
REFERENCES


https://doi.org/10.1037/a0021017


https://doi.org/10.2307/749717

[http://scholarworks.boisestate.edu/td/161/](http://scholarworks.boisestate.edu/td/161/)


[https://doi.org/10.2307/749909](https://doi.org/10.2307/749909)


Georgia Department of Early Care and Learning. (2019). *FY2019 free and reduced lunch school data: All schools.*


Georgia Department of Education. (2016). *Georgia Standards of Excellence: Geometry.* Atlanta, GA: Georgia Department of Education Retrieved from

https://www.georgiastandards.org/Georgia-Standards/Frameworks/Geometry-Standards.pdf


Klahr, D. (2009). “To every thing there is a season, and a time to every purpose under the heavens”: What about direct instruction? In S. Tobias & T. M. Duffy (Eds.), *Constructivist instruction: Success or failure?* (pp. 291-310). Routledge.


https://doi.org/10.1007/s10755-013-9269-9


https://doi.org/10.1007/978-1-4899-3620-2_8


https://www.colorado.edu/eer/research-areas/student-centered-stem-education/inquiry-based-learning-college-mathematics


https://doi.org/10.5951/jresematheduc.45.4.0406


http://dx.doi.org/10.3102/0034654315627366


https://doi.org/10.46827/ejse.v0i0.13


https://scholarworks.waldenu.edu/dissertations/721/


https://doi.org/10.1016/j.edurev.2015.02.003


https://doi.org/10.1093/oxfordhb/9780199642342.013.014


https://doi.org/10.1177/0895904803260042


[https://doi.org/10.1136/eb-2014-101946](https://doi.org/10.1136/eb-2014-101946)

Spronken-Smith, R. (2012). *Experiencing the process of knowledge creation: The nature and use of inquiry-based learning in higher education* [Conference paper]. International Colloquium on Practices for Academic Inquiry, University of Otago, Dunedin, NZ.

[https://doi.org/10.2307/90026737](https://doi.org/10.2307/90026737)

[https://doi.org/10.1177/1053451210369519](https://doi.org/10.1177/1053451210369519)
Suter, W. N. (2012). *Introduction to educational research: A critical thinking approach* (2nd ed.). SAGE Publications, Inc. [https://doi.org/10.4135/9781483384443](https://doi.org/10.4135/9781483384443)


Thompson, E. O. (1992). *Three methods of instruction in high school geometry and the effects they have on achievement, retention, and attitude* [Doctoral dissertation, Montana State University-Bozeman]. Bozeman, MT. [https://scholarworks.montana.edu/xmlui/bitstream/handle/1/7097/31762101715587.pdf?sequence=1](https://scholarworks.montana.edu/xmlui/bitstream/handle/1/7097/31762101715587.pdf?sequence=1)


Appendix A

IRB APPROVAL
From: irb@kennesaw.edu <irb@kennesaw.edu>
Sent: Sunday, January 26, 2020 8:28 PM
To: David Glassmeyer <dglassmeyer@kennesaw.edu>
Cc: irb@kennesaw.edu; David Glassmeyer <dglassmeyer@kennesaw.edu>
Subject: Study 20-329: A Comparison of Inquiry-Based and Explicit Instruction in High School Geometry

1/26/2020

Charles Sizemore, Student

RE: Your application dated 1/23/2020, Study #20-329: A Comparison of Inquiry-Based and Explicit Instruction in High School Geometry

Hello Mr. Sizemore,

Your application for the new study listed above has been administratively reviewed. This study qualifies as exempt from continuing review under DHHS (OHRP) Title 45 CFR Part 46.101(b)(1) - Research in established or commonly accepted educational settings. The consent procedures described in your application are in effect. You are free to conduct your study.

NOTE: All surveys, recruitment flyers/emails, and consent forms must include the IRB study number noted above, prominently displayed on the first page of all materials.

Please note that all proposed revisions to an exempt study require submission of a Progress Report and IRB review prior to implementation to ensure that the study continues to fall within an exempted category of research. A copy of revised documents with a description of planned changes should be submitted to irb@kennesaw.edu for review and approval by the IRB.

Please submit a Progress Report to close the study once it is complete.

Thank you for keeping the board informed of your activities. Contact the IRB at irb@kennesaw.edu or at (470) 578-6407 if you have any questions or require further information.

Sincerely,

Christine Ziegler, Ph.D.
KSU Institutional Review Board, Director of Human Subjects Research

c: dglassmeyer@kennesaw.edu
Appendix B

STUDENT ASSENT FORM
Research Study Assent Form

Study Title: Comparing Inquiry-based and Explicit Instruction in High School Geometry (Kennesaw State University Study # 20-329)

Researchers: Rodney Sizemore, csizemo2@students.kennesaw.edu

My name is Mr. Rodney Sizemore, and I am working with Dr. David Glassmeyer on a research study involving your geometry class. I am a teacher at XXXXXXXXXXXX and a student at Kennesaw State University. I would like to invite you to take part in a research study. Your parent(s) know we are talking with you about the study, but it is up to you to decide if you want to be in the study. This form will tell you about the study to help you decide whether or not you want to take part in it.

Why is this study being done?
The purpose of the study is to help me learn which ways students learn about geometry best. I want to make sure you understand how to solve problems, but I want to make sure you understand how it fits in with what you already know.

You are being asked to take part because you are in XXXXXXXXXXXX geometry class.

What am I being asked to do?
This spring, all students in geometry will learn about arc length, sector area, and equations of circles. I am asking you to allow me to use data from those assessments so I can see how students learn those topics best. There will also be two surveys where you can tell me your opinion about how you learned.

If you don’t want to participate in the study, I will not use your data in the study. However, you will still learn about arc length, sector area, and equations of circles, and you will still have the same activities, homeworks, and tests as the other students in your class.

If you change your mind, you can tell me at any time. If you wish to see a copy of the results, I will share them with you.

What are the benefits to me for taking part in the study?
These topics might make more sense to you. For example, you may learn how these topics fit together and how they fit in with things you already know.

Additionally, you get to let us know what you thought about the way we taught the units. Your input will help us decide how we (and other people) teach this in the future.

Are there any risks to me if I am in this study?
There are no foreseeable risks outside the normal risks that occur in educational settings. Your name will not be published, and any link to your specific contribution to
the data collected will be minimized. For example, I’ll use things like “Student 1” instead of actual student names.

**Will my information be kept private?**
The data for this study will be kept private and confidential to the extent allowed by federal and state law. The data (copies of student work and surveys) for this study will be kept in a secure location and destroyed after three years.

When we tell other people or write articles about what we learned in the study, we won’t include your name or that of anyone else who took part in the study.

**Are there any costs or payments for being in this study?**
There will be no costs to you for taking part in this study.

You will not receive money or any other form of compensation for taking part in this study.

**What are my rights as a research study volunteer?**
Your participation in this research study is completely voluntary. You do not have to be a part of this study if you don’t want to. There will be no penalty to you if you choose not to take part and no one will be upset or angry at you. You may choose not to answer any questions you don’t want to answer, and you can change your mind and not be in the study at any time.

**Who can I talk to if I have questions?**
If you have questions at any time, you can ask the researchers and you can talk to your parent about the study. We will give you a copy of this form to keep. If you want to ask us questions about the study, contact

- Mr. Rodney Sizemore, csizemo2@students.kennesaw.edu, 770-229-8897, or
- Dr. David Glassmeyer, dglassme@kennesaw.edu, 470-578-7867.

The Kennesaw State University Institutional Review Board has reviewed this study to make sure that the rights and safety of people who take part in the study are protected. If you have questions about your rights in the study, or you are unhappy about something that happens to you in the study, you can contact them at (678) 797-2268 or irb@kennesaw.edu.

**What does my signature on this consent form mean?**
Your signature on this form means that:

- You understand the information given to you in this form
- You have been able to ask the researcher questions and state any concerns
- The researcher has answered your questions and concerns
- You believe you understand the research study and the potential benefits and risks that are involved.
Statement of Consent
I give my voluntary consent to take part in this study. I will be given a copy of this consent document for my records.

____________________________  ________________
Signature of Participant       Date

____________________________
Printed Name of Participant

Statement of Person Obtaining Informed Consent
I have carefully explained to the person taking part in the study what he or she can expect.

I certify that when this person signs this form, to the best of my knowledge, he or she understands the purpose, procedures, potential benefits, and potential risks of participation.

I also certify that he or she:
- Speaks the language used to explain this research
- Reads well enough to understand this form or, if not, this person is able to hear and understand when the form is read to him or her
- Does not have any problems that could make it hard to understand what it means to take part in this research.

____________________________________________________________________
Name of parent who gave consent for child to participate

____________________________  ________________
Signature of Person Obtaining Consent       Date
Appendix C

PARENT CONSENT FORM
PARENTAL CONSENT FORM

Title of Research Study: Comparing Inquiry-based and Explicit Instruction in High School Geometry (Kennesaw State University Study # 20-329)

Researcher’s Contact Information:
Rodney Sizemore, 770-229-8897, csizemo2@students.kennesaw.edu
Dr. David Glassmeyer, 470-578-7867, dglassme@kennesaw.edu

Your child is being invited to take part in a research study conducted by Mr. Rodney Sizemore, a teacher at XXXXXXXXXX and student at Kennesaw State University, under the supervision of Dr. David Glassmeyer, also of Kennesaw State University. Before you decide to allow your child to participate in this study, you should read this form and ask questions if you do not understand. Participation in this research study is completely voluntary.

Description of Project
The purpose of the study is to compare two commonly used instructional methods to determine which method helps students better understand the meanings behind geometric concepts and which method helps students better perform the calculations associated with those concepts. The two methods being studied are inquiry-based instruction (where students will work in small groups to explore relationships between new and previous material) and explicit instruction (where students will be told what relationships exist).

The material that will be covered is a required part of the geometry curriculum. Participants in this study will complete the same lessons, quizzes, and tests over a four-week period as non-participants in their class. If you allow your child’s data to be used in the study, it will be collected from students’ tests and from student surveys.

Explanation of Procedures
All students in XXXXXXXXXX classes will work on lessons about arc length, sector area, and equations of circles as a normal part of the geometry curriculum. Two commonly accepted instructional methods will be used. One instructional method will be used for arc length and sector area, and the other instructional method will be used for equations of circles. At the end of the study, I will compare the results to see which method helped students learn better.

Time Required
Participants will not be required to spend any time for this study outside of regular school expectations. Non-participants and participants will be required to complete the same activities, homeworks, and tests.
Risks or Discomforts
There are no foreseeable risks outside the normal risks that occur in educational settings. Your student’s name will not be published and any link to your student’s specific contribution to the data collection will be minimized. For example, “Student 1” would be used instead of a student’s real name.

Benefits
This study will benefit students and teachers by providing data on how to best teach several important topics in geometry. Your student will also have the opportunity to let us know what they thought about the way we taught the units.

Confidentiality
The data for this study will be confidential. The data (copies of student work and survey results) will be kept in a secure location for three years, after which time it will be destroyed. There will be no identifiable data included in the research results.

Use of Online Surveys
Students will take two online surveys asking about their learning. Data collected online will be handled in a confidential manner, and the IP addresses will not be collected by the survey program.

Inclusion Criteria for Participation
Participants will be high school students in XXXXXXXXXX geometry classes.

Consent to Participate
I give my consent for my child, __________________________________________, to participate in the research project described above. I understand that this participation is voluntary and that I may withdraw my consent at any time without penalty. I also understand that my child may withdraw his/her assent at any time without penalty.

____________________________________________
Signature of Parent or Authorized Representative, Date

____________________________________________
Signature of Investigator, Date

__________________________
PLEASE SIGN BOTH COPIES OF THIS FORM, KEEP ONE AND RETURN THE OTHER TO THE INVESTIGATOR

Research at Kennesaw State University that involves human participants is carried out under the oversight of an Institutional Review Board. Address questions or problems regarding these activities to the Institutional Review Board, Kennesaw State University, 585 Cobb Avenue, KH3417, Kennesaw, GA 30144-5591, (470) 578-6407.
Appendix D

ARC LENGTH/SECTOR AREA TEST
Standards

MGSE9-12.G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector (Georgia Department of Education, 2016).

Learning Outcomes

1. Students will solve for arc length and sector area.
   1a. Students will solve for arc length.
   1b. Students will solve for sector area.

2. Students will identify arc length and sector area as an application of similarity and proportions.

3. Students will convert between degrees and radians.
   3a. Students will convert from degrees to radians.
   3b. Students will convert from radians to degrees.

4. Students will explain the connection between radians and arc length.
<table>
<thead>
<tr>
<th>Content Outline</th>
<th>Categories</th>
<th>(Number of Items)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The student will solve for arc length and sector area.</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2. The student will identify arc length and sector area as an application of similarity and proportions.</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>3. The student will convert between degrees and radians.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4. The student will explain the connection between radians and arc length.</td>
<td>2 (1 with 2 parts plus 1 additional question)</td>
<td></td>
</tr>
</tbody>
</table>
Arc Length and Sector Area Test

| Formulas |
|------------------|------------------|------------------|
| Circumference of a Circle | Arc Length of a Circle | Area of a Circle |
| \( C = \pi d \text{ or } C = 2\pi r \) | Arc Length = \( \frac{2\pi r \theta}{360} \) | Area = \( \pi r^2 \) |
| Area of a Sector of a Circle | \( \text{Area of Sector} = \frac{\pi r^2 \theta}{360} \) |

Procedural Fluency (6 questions, 12 points total)

You must show all work to receive credit.

1. Find the length of the highlighted arc. [Learning Outcome 1a]

![Diagram of a circle with a highlighted arc and a central angle of 270° and a radius of 9 inches.]

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 pts</td>
<td>Includes values correctly substituted into a correct formula and correct answer. Note: Correct answer is (27\pi/2) inches or 42.41 inches. Award 2 points even if units are omitted.</td>
</tr>
<tr>
<td>1 pt</td>
<td>Values are correctly substituted into a correct formula but incorrect final answer.</td>
</tr>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>

2. Find the area of the highlighted sector. [Learning Outcome 1b]

![Diagram of a circle with a highlighted sector and a central angle of 270° and a radius of 9 meters.]

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 pts</td>
<td>Includes values correctly substituted into a correct formula, and correct answer. Note: Correct answer is (243\pi/4) square meters or 190.85 square meters. Award 2 points even if units are omitted or if the student rounded to a different number of decimals or rounded incorrectly.</td>
</tr>
<tr>
<td>1 pt</td>
<td>Values are correctly substituted into a correct formula but incorrect final answer.</td>
</tr>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>
3. The windshield wiper blade on a car is 19 inches long. Each swipe of the wiper makes an angle of 135°. About how far does the tip of the windshield wiper travel with each swipe of the blade? [Learning Outcome 1a]

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 pts</td>
<td>Includes values correctly substituted into a correct formula and correct answer. Note: Correct answer is $57\pi/4$ inches or 44.77 inches. Award 2 points even if units are omitted or if the student rounded to a different number of decimals or rounded incorrectly.</td>
</tr>
<tr>
<td>1 pt</td>
<td>Values are correctly substituted into a correct formula but incorrect final answer.</td>
</tr>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>

4. What is the measure of the shaded part of the circle below? [Learning Outcome 1b]

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 pts</td>
<td>Includes values correctly substituted into a correct formula and correct answer. Note: Correct answer is $75\pi/2$ square centimeters or 117.81 square centimeters. Award 2 points even if units are omitted or if the student rounded to a different number of decimals or rounded incorrectly.</td>
</tr>
<tr>
<td>1 pt</td>
<td>Values are correctly substituted into a correct formula but incorrect final answer.</td>
</tr>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>
5. Convert to radians. Leave $\pi$ in your answer and reduce fractions to lowest terms. Remember to show your work. [Learning Outcome 3a]

$330^\circ$

<table>
<thead>
<tr>
<th>2 pts</th>
<th>Includes values correctly substituted into a correct formula and correct answer. Note: Correct answer is $11\pi/6$. Award 2 points even if the student only shows multiplying $330^\circ$ by $\pi/180^\circ$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pt</td>
<td>Values are correctly substituted into a correct formula but incorrect final answer. Also award 1 point if the student converted $11\pi/6$ to a decimal.</td>
</tr>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>

6. Convert $\frac{5\pi}{3}$ radians to degrees. Remember to show your work. [Learning Outcome 3b]

<table>
<thead>
<tr>
<th>2 pts</th>
<th>Includes values correctly substituted into a correct formula and correct answer. Note: Correct answer is $300^\circ$. Award 2 points even if the student only shows multiplying $5\pi/3$ by $180^\circ/\pi$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pt</td>
<td>Values are correctly substituted into a correct formula but incorrect final answer.</td>
</tr>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>
7. A student has a pizza with a 6” radius and would like to know how many square inches of pizza are in two slices. She has set up the equation below. [Learning Outcome 2]

\[
\text{Sector Area} = \pi (6)^2 \times \frac{2}{8}
\]

Will the equation work? Circle one: Yes/No  [Learning Outcome 2]

Explain why this equation does or doesn’t work. Note: Saying that it does or does not give the correct answer is not sufficient.

<table>
<thead>
<tr>
<th>1 pt</th>
<th>Correct answer is Yes. Must include explanation that the area of two slices is 2/8 (or ¼) of the total area of the pizza.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>
8. A student would like to find the area of the indicated sector below and has set up the equation given.

\[
\frac{\text{Sector Area}}{\pi (18)^2} = \frac{120}{360}
\]

Will the equation work? Circle one: Yes/No  [Learning Outcome 2]

Explain why this equation does or doesn’t work. Note: Saying that it does or does not give the correct answer is not sufficient.

<table>
<thead>
<tr>
<th>1 pt</th>
<th>Correct answer is Yes. Must include explanation that the area of the sector is the fractional part (or 120/360 or 1/3) of the area of the circle.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>


9. A student would like to find the distance the tip of a 10 cm long minute hand on a clock travels in 15 minutes and has set up the equation below. [Learning Outcome 2]

\[
\frac{\text{Arc Length}}{2\pi(10)} = \frac{15}{360}
\]

Will the equation work? Circle one: Yes/No  [Learning Outcome 2]

Explain why this equation does or doesn’t work. Note: Saying that it does or does not give the correct answer is not sufficient.

<table>
<thead>
<tr>
<th>1 pt</th>
<th>Correct answer is No. Must include explanation that the second fraction is incorrect and state either (1) the fraction should be 15/60 or (2) the fraction is not the portion of the circle that the hand travels.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>
10. Use the figure below to answer parts (a) and (b). [Learning Outcome 4]

![Diagram of a circle with an angle θ and radius r]

(a) What is the measure of angle θ in radians? _______________________

| 1 pt | Note: Answer is 1. |
| 0 pts | Anything else. |

(b) Why? _____________________________________________________________________

| 1 pt | Includes at least one of the following: |
| | • The diagram shows one radian because the arc length is one radius. |
| | • The formula for converting is $\theta = \frac{\text{arc length}}{\text{radius}}$. |
| 0 pts | Anything else. |

11. Why are there $2\pi$ radians in a circle? [Learning Outcome 4]

______________________________________________________________________________

| 1 pt | Includes at least one of the following: |
| | • The number of radians in the circle is equal to the circumference divided by the radius. |
| | • The circumference of the circle is $2\pi r$. Dividing the circumference by $r$ leaves $2\pi$. |
| | • Because the radius will wrap around the circle $2\pi$ times. |
| | • The formula for converting is $\theta = \frac{\text{arc length}}{\text{radius}}$. |
| 0 pts | Anything else. |
Appendix E

GRAPHING CIRCLES TEST
Standards

MGSE9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

MGSE9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0,2)$ (Georgia Department of Education, 2016).

Learning Outcomes

1. Students will graph circles in standard form.

2. Students will convert between standard and general conic form of a circle.
   2a. Students will convert from standard to general conic form of a circle.
   2b. Students will convert general conic to standard form of a circle.

3. Students will explain when the distance and midpoint formulas are needed to find the equation of a circle.

4. Students will explain the how the equation of a circle is derived from the Pythagorean Theorem.
## Test Blueprint

<table>
<thead>
<tr>
<th>Content Outline</th>
<th>Categories</th>
<th>Procedural Fluency</th>
<th>Conceptual Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The student will graph circles in standard form.</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2. The student will convert between standard and general conic form of a circle.</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3. The student will explain when the distance and midpoint formulas are needed to find the equation of a circle.</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>4. The student will explain the how the equation of a circle is derived from the Pythagorean Theorem.</td>
<td></td>
<td>1 (3 parts)</td>
<td></td>
</tr>
</tbody>
</table>
Graphing Circles Test

Formula

Equation of a Circle

\[(x - h)^2 + (y - k)^2 = r^2\]

Procedural Fluency (5 questions, 8 points total)

You must show all work to receive credit.

1. Graph the equation below. [Learning Outcome 1]

\[(x - 2)^2 + (y + 4)^2 = 9\]

1 pt  Shows a circle with center (2, –4) and radius 3. No other work is required to earn 1 point for this item.

0 pts  Anything else.
2. The equation of a circle is \((x - 2)^2 + (y - 2)^2 = 4\). Tell whether each point is on the circle, inside of the circle, or outside of the circle. [Learning Outcome 1]

- a) (1, 2) 
- b) (1, 4) 
- c) (2, 0) 
- d) (4, 2) 

2 pts  Shows the graph of a circle centered at (2, 2) with a radius of 2. Shows all four correct answers.  
Note: Correct answers are a) in; b) out; c) on; d) on.

1 pt  Shows the graph of a circle centered at (2, 2) with a radius of 2. Shows three correct answers.

0 pts  Anything else.

3. Write the equation of the following circle in standard form: [Learning Outcome 1]

Center: \((-3, 10)\)  
Radius: 8

\[
\begin{align*}
A) & \ (x - 10)^2 + (y + 3)^2 = 64 & B) & \ (x + 3)^2 + (y - 10)^2 = 64 \\
C) & \ (x - 3)^2 + (y + 10)^2 = 64 & D) & \ (x + 10)^2 + (y - 3)^2 = 64
\end{align*}
\]

1 pt  Correct answer is B. No other work is required to earn 1 point for this item.

0 pts  Anything else.
4. Convert the equation below from standard form to general conic form. [Learning Outcome 2a]
\[(x + 8)^2 + (y + 2)^2 = 121\]

<table>
<thead>
<tr>
<th>2 pts</th>
<th>Shows expansion of ((x + 8)^2) and ((y + 2)^2). It is not necessary to show a separate step of moving 121 to the other side of the equation as long as the final answer is correct. Note: Correct answer is (x^2 + y^2 + 16x + 4y - 53 = 0).</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pt</td>
<td>Work is shown as described above. One term is incorrect, but the other terms are correct. Also award 1 point if the student did not move all terms to one side of the equation but all other terms are correct.</td>
</tr>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>

5. Convert the equation below from general conic form to standard form. [Learning Outcome 2b]
\[x^2 + y^2 - 10x - 20y + 109 = 0\]

<table>
<thead>
<tr>
<th>2 pts</th>
<th>Shows completing the square terms for (x) and (y). It is not necessary to show a separate step of moving 109 to the other side of the equation as long as the final answer is correct. Note: Correct answer is ((x - 5)^2 + (y - 10)^2 = 16).</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pt</td>
<td>Work is shown as described above. One term is incorrect, but the other terms are correct.</td>
</tr>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>
Conceptual Understandings (4 questions, 9 points total)

6. Using the graph below, explain as many relationships as you can between the Pythagorean Theorem and the equation for a circle. [Learning Outcome 4]

Pythagorean Theorem: \(a^2 + b^2 = c^2\)

Equation for a circle in standard form: \((x - h)^2 + (y - k)^2 = r^2\)

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
</table>
| 3 pts  | Includes ALL THREE of the following:  
|        |   • Explanation that \((x - h)\) or \((x - (-3))\) or \(|x - h|\) is the length of the horizontal leg of the triangle or is equal to \(a\).  
|        |   • Explanation that \((y - k)\) or \((y - 2)\) or \(|y - k|\) is the length of the vertical leg of the triangle or is equal to \(b\).  
|        |   • Explanation that the radius of the circle is the hypotenuse of the triangle or is equal to \(c\). |
| 2 pts  | Includes exactly TWO of the above. |
| 1 pt   | Includes exactly ONE of the above. |
| 0 pts  | Includes NONE of the above. |
7. Which formula(s) will be needed to write the equation for this circle? [Learning Outcome 3]

   Center: \((13, 8)\)
   Point on Circle: \((12, 5)\)

a) Will you need to use the midpoint formula? Circle one: Yes/No

   Why? _______________________________________________________________

   1 pt | Correct answer is No. Must include explanation that you are given the center or that
   0 pts | the midpoint formula is not needed to find the radius.

b) Will you need to use the distance formula? Circle one: Yes/No

   Why? _______________________________________________________________

   1 pt | Correct answer is Yes. Must include explanation that you need to use the distance
   0 pts | formula to find the radius.

8. Which formula(s) will be needed to write the equation for this circle? [Learning Outcome 3]

   Ends of a diameter: \((15, -5)\) and \((-3, 9)\)

a) Will you need to use the midpoint formula? Circle one: Yes/No

   Why? _______________________________________________________________

   1 pt | Correct answer is Yes. Must include explanation that you need to use the midpoint
   0 pts | formula to find the center.

b) Will you need to use the distance formula? Circle one: Yes/No

   Why? _______________________________________________________________

   1 pt | Correct answer is Yes. Must include explanation that you need to use the distance
   0 pts | formula to find the radius.
9. Which formula(s) will be needed to write the equation for this circle? [Learning Outcome 3]

Center: \((1, 15)\)
Radius: 3

a) Will you need to use the midpoint formula? Circle one: Yes/No

Why? _______________________________________________________________

<table>
<thead>
<tr>
<th>1 pt</th>
<th>Correct answer is No. Must include explanation that you are given the center or that all information needed for the formula is already given.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>

b) Will you need to use the distance formula? Circle one: Yes/No

Why? _______________________________________________________________

<table>
<thead>
<tr>
<th>1 pt</th>
<th>Correct answer is No. Must include explanation that you are given the radius or that all information needed for the formula is already given.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 pts</td>
<td>Anything else.</td>
</tr>
</tbody>
</table>
Appendix F

STUDENT SURVEY ON ARC LENGTH/SECTOR AREA
Student Survey on Arc Length and Sector Area Unit

This survey asks you to think about your overall experiences in the two-week unit we just completed on arc length and sector area. It is only asking about your experiences during the last two weeks in class. Please answer each question honestly to provide us with feedback about what helped you learn (Laursen et al., 2011).

1. How much did the following aspects of the unit help your learning?

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Great help</th>
<th>Much help</th>
<th>Moderate help</th>
<th>Some help</th>
<th>A little help</th>
<th>No help</th>
</tr>
</thead>
<tbody>
<tr>
<td>The overall approach to teaching and learning about arc length and sector area</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How class topics, activities, &amp; assignments fit together</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The mental stretch required of you</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. How much did the following activities help your learning about arc length/sector area?

<table>
<thead>
<tr>
<th>Activity</th>
<th>Great help</th>
<th>Much help</th>
<th>Moderate help</th>
<th>Some help</th>
<th>A little help</th>
<th>No help</th>
<th>DID NOT HAPPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watching the teacher solve arc length and sector area problems</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Listening to the teacher explain the connections between proportions, arc length, and radians</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doing 1-2 pages of each type of practice problem on your own</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working in small groups to figure out formulas for arc length and sector area</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Working in small groups to see the connections between proportions, arc length, and radians</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hearing other students explain their work</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explaining your work to other students</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doing a few practice problems on your own</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. As a result of your work in the unit on arc length and sector area, to what extent do you understand or could you explain the following concepts?

<table>
<thead>
<tr>
<th></th>
<th>A Great Deal (90% and above)</th>
<th>Majority of the Content (70%-89%)</th>
<th>Half the Content (50%-69%)</th>
<th>Some of the Content (30%-49%)</th>
<th>A Little of the Content (10%-29%)</th>
<th>Almost None of the Content (0%-9%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>How well can you solve problems for arc length and sector area?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>How well do you understand how arc length and sector area relates to proportions?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>How well do you know the steps to convert between degrees and radians?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>How well do you understand how arc length and sector area connects to radians?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>How well can you explain to other people how arc length and sector area connects to proportions?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>How well can you explain to other people how arc length and sector area relates to radians?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>

4. Please comment on the activities we did in this unit and whether or not they helped you remember key ideas.

5. As a result of your work in this unit, what gains did you make in the following?

<table>
<thead>
<tr>
<th></th>
<th>Strongly Agree</th>
<th>Somewhat Agree</th>
<th>Slightly Agree</th>
<th>Slightly Disagree</th>
<th>Somewhat Disagree</th>
<th>Strongly Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence that you can solve problems about arc length and sector area</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Confidence that you understand connections between circumference, arc length, and radians</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Confidence that you will remember what you have learned about arc length and sector area</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Comfort in communicating about arc length and sector area</td>
<td>○</td>
<td>○</td>
<td>○</td>
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<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>

6. What period is this?

Period: _______
Appendix G

STUDENT SURVEY ON GRAPHING CIRCLES
Student Survey on Graphing Circles

This survey asks you to think about your overall experiences in the two-week unit we just completed on graphing circles. It is only asking about your experiences during the last two weeks in class. Please answer each question honestly to provide us with feedback about what helped you learn (Laursen et al., 2011).

1. How much did the following aspects help your learning about graphing circles?

<table>
<thead>
<tr>
<th></th>
<th>Great help</th>
<th>Much help</th>
<th>Moderate help</th>
<th>Some help</th>
<th>A little help</th>
<th>No help</th>
</tr>
</thead>
<tbody>
<tr>
<td>The overall approach to teaching and learning about graphing circles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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</tr>
</tbody>
</table>

2. How much did the following activities help your learning about graphing circles?

<table>
<thead>
<tr>
<th></th>
<th>Great help</th>
<th>Much help</th>
<th>Moderate help</th>
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<th>A little help</th>
<th>No help</th>
<th>DID NOT HAPPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watching the teacher solve graphing circles problems</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Listening to the teacher explain the connections between the Pythagorean Theorem and graphing circles</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doing 1-2 pages of each type of practice problem on your own</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working in small groups to figure out formulas for circles in standard and general form</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working in small groups to see the connections between the Pythagorean Theorem and graphing circles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hearing other students explain their work</td>
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<tr>
<td>Explaining your work to other students</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Doing a few practice problems on your own</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. As a result of your work in the unit on graphing circles, what gains did you make in your understanding of each of the following?

<table>
<thead>
<tr>
<th></th>
<th>A Great Deal (90% and above)</th>
<th>Majority of the Content (70%-89%)</th>
<th>Half the Content (50%-69%)</th>
<th>Some of the Content (30%-49%)</th>
<th>A Little of the Content (10%-29%)</th>
<th>Almost None of the Content (0%-9%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>How well do you know how to graph circles in standard form?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>How well do you know how to convert between standard and general conic form of a circle?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>How well do you understand how the Pythagorean Theorem relates to graphing circles?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>How well do you understand when the distance and midpoint formulas are needed to find the equation of a circle?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>How well can you explain to other people how the Pythagorean Theorem relates to graphing circles?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>How well can you explain to other people when the distance and midpoint formulas are needed to find the equation of a circle?</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>

4. Please comment on the activities we did in this unit and whether or not they helped you remember key ideas.


5. As a result of your work in this unit, what gains did you make in the following?

<table>
<thead>
<tr>
<th></th>
<th>Strongly Agree</th>
<th>Somewhat Agree</th>
<th>Slightly Agree</th>
<th>Slightly Disagree</th>
<th>Somewhat Disagree</th>
<th>Strongly Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence that you can solve problems about graphing circles</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Confidence that you understand connections between standard and general forms of a circle</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Confidence that you will remember what you have learned about graphing circles</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Comfort in communicating about graphing circles</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>

6. What period is this?

Period: _____
Appendix H

COMPARISON OF INQUIRY-BASED AND EXPLICIT INSTRUCTION FOR ARC LENGTH
AND SECTOR AREA
<table>
<thead>
<tr>
<th>Day</th>
<th>Inquiry-Based</th>
<th>Explicit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Classwork:</strong> Students worked in groups of 2-3 to develop formulas for arc length and sector area. &lt;br&gt;<strong>Homework:</strong> Six problems on calculating arc length and sector area</td>
<td><strong>Classwork/Homework:</strong> Students were given a guided notes sheet and told definitions and formulas for arc length and sector area. After seeing the me work two examples, students worked nine more on their own.</td>
</tr>
<tr>
<td>2</td>
<td><strong>Classwork:</strong> Students worked in groups of 2-3 to apply arc length and sector area formulas to situations that were new to them. &lt;br&gt;<strong>Homework:</strong> Six problems on calculating arc length and sector area like those done Day 1-2</td>
<td><strong>Classwork/Homework:</strong> Students were given a graphic organizer that contained formulas, definitions, and key words to look for. After seeing the me work two examples, students worked 12 additional problems on their own.</td>
</tr>
<tr>
<td>3</td>
<td><strong>Classwork:</strong> Students worked in groups of 2-3 to set up an arc length and a sector area problem as many ways as they could. &lt;br&gt;<strong>Homework:</strong> Seven arc length and sector area problems like those done Days 1-3</td>
<td><strong>Classwork/Homework:</strong> Using their notes from the previous two days, students filled in a foldable to help them differentiate between arc length and sector area problems. Next, students watched me solve three arc length and sector area problems three different ways. Then, students solved nine problems on their own using methods other than the standard formula.</td>
</tr>
<tr>
<td>4</td>
<td><strong>Classwork:</strong> Students worked on an online Desmos activity involving visually estimating what angle would produce a certain sector area given the area of the entire circle. Then they discovered how to calculate the exact angle. &lt;br&gt;<strong>Homework:</strong> Eight problems involving solving for the angle or radius in arc length and sector area problems.</td>
<td><strong>Classwork/Homework:</strong> After seeing the me solve two arc length and sector area problems for the angle and the radius, students solved 14 problems on their own.</td>
</tr>
<tr>
<td>Day</td>
<td>Inquiry-Based</td>
<td>Explicit</td>
</tr>
<tr>
<td>-----</td>
<td>---------------</td>
<td>----------</td>
</tr>
</tbody>
</table>
| 6   | **Classwork:** Students started a discovery activity to see how many radii will fit around the edge of a circle.  
Homework: None | **Classwork/Homework:** Students were given a guided notes sheet with the definition of a radian, a pictorial representation of a radian, and the formulas to convert between radians and degrees. After seeing the me convert radians to degrees and degrees to radians, students solved 10 additional problems on their own. |
| 6   | **Classwork:** Students completed the discovery activity to see how many radii will fit around the edge of a circle, then completed a Desmos activity to compare with their results.  
Homework: None | **Classwork/Homework:** Using their notes from the previous day, students filled in a foldable to help them remember the definition of a radian and the formulas for converting between radians and degrees. Next, students completed 18 conversion practice problems on Deltamath.com. |
| 7   | **Classwork:** Students worked in groups of 2-3 to discover how to convert between radians and degrees.  
Homework: Ten problems on Deltamath.com converting between radians and degrees. | **Classwork/Homework:** Students put together radian/degree puzzles. Puzzle pieces had radians and/or degrees written on the sides. The puzzle was complete when all pieces that touched were equal. |
| 8   | **Classwork:** Students worked individually on open-ended review questions. After each question, the class talked about the answers as a large group.  
Homework: Nine problems reviewing the unit. | **Classwork/Homework:** Students worked on a 20-question worksheet reviewing the unit. |
| 9   | Unit Test  
Survey | Unit Test  
Survey |
<table>
<thead>
<tr>
<th>Day</th>
<th>Inquiry-Based</th>
<th>Explicit</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Make-up day for students who missed the test or did not complete the survey. Pre-test for graphing circles</td>
<td>Make-up day for students who missed the test or did not complete the survey. Pre-test for graphing circles</td>
</tr>
</tbody>
</table>
Appendix I

COMPARISON OF INQUIRY-BASED AND EXPLICIT INSTRUCTION FOR GRAPHING CIRCLES
<table>
<thead>
<tr>
<th>Day</th>
<th>Inquiry-Based</th>
<th>Explicit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Classwork:</strong> Students worked in groups of 2-3 to derive the equation for a circle from the Pythagorean Theorem.</td>
<td><strong>Classwork/Homework:</strong> Students were given a guided notes sheet which included a diagram and an explanation about how the equation of a circle is derived from the Pythagorean Theorem. Students were shown how to graph two circles and given 10 problems to do on their own.</td>
</tr>
<tr>
<td></td>
<td><strong>Homework:</strong> None</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td><strong>Classwork/Homework:</strong> Students were given a guided notes sheet where the steps were listed to find the equation of a circle given the center and a point on the circle and given the endpoints of the diameter. Two examples were demonstrated, and students were given 12 problems to complete on their own.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td><strong>Classwork:</strong> Using their notes for the previous two days, students completed a graphic organizer about how to write equations of circles. They were then given three problems to complete using their graphic organizer.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td><strong>Classwork:</strong> Students worked in groups of 2-3 to create formulas for circles given the center and point on a circle or the ends of the diameter.</td>
<td><strong>Classwork/Homework:</strong> Students were given 16 problems on Deltamath.com involving writing equations of circles given a graph or the center and a point on the circle.</td>
</tr>
<tr>
<td></td>
<td><strong>Homework:</strong> Six problems finding the equations of circles given the center and a point on the circle or the ends of the diameter.</td>
<td></td>
</tr>
<tr>
<td>Day</td>
<td>Inquiry-Based</td>
<td>Explicit</td>
</tr>
<tr>
<td>-----</td>
<td>-------------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
| 5   | Classwork: Students worked in groups of 2-3 to rearrange the equation of a circle in standard form to arrive at the equation of a circle in general conic form.  
Homework: Five problems converting equations of circles from standard form to general conic form. | Classwork/Homework: Students were shown two examples of how to convert from standard form to general conic form of a circle, then given 8 problems to complete on their own. |
| 6   | Classwork: Using yesterday’s example of converting from standard form to general conic form, students worked in groups of 2-3 to discover methods to convert from general conic form to standard form of a circle.  
Homework: Seven problems converting from general conic form to standard form of a circle. | Classwork/Homework: Students were shown the steps for completing the square and shown how to use that to convert from general conic form to standard form of a circle. They were then given 9 problems to complete on their own. |
| 7   | Classwork/Homework: Students were given 10 problems on Deltamath.com involving converting from general conic form to standard form of a circle. |                                                                 |
| 8   | Classwork: Students worked in groups of 2-3 to determine which circle was bigger given different types of information about it.  
Homework: Six problems reviewing the unit. | Classwork: Students worked individually on mini dry-erase boards on 9 problems reviewing the unit.  
Homework: Nine problems reviewing the unit. |
| 9   | Unit Test  
Survey | Unit Test  
Survey |
| 10  | Make-up day for students who missed the test or did not complete the survey. | Make-up day for students who missed the test or did not complete the survey. |
Appendix J

INQUIRY-BASED LESSON PLANS FOR ARC LENGTH/SECTOR AREA
Day 1 Warm-Up

Find the distance around the circle.

Find the space inside the circle.
Would you get more pizza if you ate two slices of a Personal Pan Pizza or one slice of a large pizza?
Answer ONE of the following questions about this clock:

1. How far does the end of the **minute** hand travel in 20 minutes?
2. How far does the end of the **hour** hand travel between 1:00 and 5:00?
3. How far does the end of the **hour** hand travel between 1:00 and 1:30?

Fill in these blanks:

<table>
<thead>
<tr>
<th>If you are talking about…</th>
<th>… there are this many parts in the whole.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pie cut into 6 slices</td>
<td></td>
</tr>
<tr>
<td>Minutes on a clock</td>
<td></td>
</tr>
<tr>
<td>Hours on a clock</td>
<td></td>
</tr>
<tr>
<td>Degrees in a circle</td>
<td></td>
</tr>
</tbody>
</table>
Day 1 Homework

1. Find the length of the indicated arc.
   ![Image of a circle with a 315° arc and 14 km radius]

2. Find the area of the indicated sector.
   ![Image of a circle with a 225° sector and 11 in radius]

3. Find the length of the indicated arc.
   ![Image of a circle with a 5 cm arc]

4. What is the area of the shaded region?
   ![Image of a circle with a shaded region and 110° sector]

5. Find the area of the shaded region.
   ![Image of a circle with a shaded region]

6. Find the length of the indicated arc.
   ![Image of a circle with a 5 cm arc]

Key:
1. \( \frac{49\pi}{2} \) km
2. \( \frac{605\pi}{8} \) in²
3. \( \frac{10\pi}{3} \) cm
4. \( \frac{625\pi}{36} \) cm²
5. \( \frac{125\pi}{8} \) cm²
6. \( 5\pi \) cm
Day 2 Warm-Up

1) Find the length of the indicated arc.

![Diagram of a circle with a dashed arc and a radius of 4 cm.]

2) Find the area of the indicated sector.

![Diagram of a circle with a sector shaded and an angle of 150°.]

Key: 1) \( \frac{8\pi}{3} \) 2) 135\( \pi \) km\(^2\)
Day 2 Classwork: Using Arc Length and Sector Area

Review from yesterday:

Answer ONE of the following questions about this clock:

1. How far does the end of the minute hand travel in 20 minutes?
2. How far does the end of the hour hand travel between 1:00 and 5:00?
3. How far does the end of the hour hand travel between 1:00 and 1:30?
Answer ONE of the following questions about the figure below:

1. Find the \textbf{perimeter} of the shaded region.
2. Find the \textbf{area} of the shaded region.
**Day 2 Homework**

1. Find the perimeter of the shaded region.

2. A circular hot tub will have a diameter of 10 ft. Decorative brick will be placed around the edge of the hot tub except in a 60° section, which is where the steps will be. How many feet of decorative brick are needed?

3. A landscape company is planting flowers in a circular flowerbed with a diameter of 10’. A 120° sector will be planted with purple flowers, and the rest of the circle will be planted with orange flowers. What is the area of the flowerbed that will be planted in orange flowers?

4. A gardener is putting border around a circular flower bed. The radius of the flower bed is 5 feet. He runs out of border, leaving a 45° section without border. How much of the flower bed has no border?

5. Shaq and his friends order a pizza. While everyone else is watching the game, Shaq eats 210° of the extra-large pizza, which had a radius of 11 in. How much pizza is left for his friends?

6. A lawn sprinkler sprays water in a circular pattern with a radius of 20 feet. If the sprinkler is set to rotate in a 250° arc, how many square feet will it water?

**KEY:**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 21.42</td>
<td>2) 26.18 ft</td>
<td>3) $\frac{50}{3}\pi$</td>
<td>4) 3.93 ft</td>
<td>5) 158.39 in²</td>
<td>6) 872.67 ft²</td>
</tr>
</tbody>
</table>
## Day 3 Warm-Up

1) Find the perimeter of the shaded region in terms of $\pi$.

2) A circular hot tub will have a diameter of 12 ft. Decorative brick will be placed around the edge of the hot tub except in an $80^\circ$ section, which is where the steps will be. How many feet of decorative brick are needed?

<table>
<thead>
<tr>
<th>Key: 1) $4\pi + 10$ 2) 8.38 ft.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Set up as many equations as you can to solve for the area of the shaded sector below:

2. Set up as many equations as you can to solve for the distance the tip of an 8 cm long minute hand travels in 15 minutes.
Day 3 Homework

<table>
<thead>
<tr>
<th>Arc Length and Sector Area Practice (C.5)</th>
<th>2. A circle with a radius of 7 is cut into 8 parts. Find the area of 3 parts in terms of ( \pi ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. SET UP THE FOLLOWING PROBLEM AS MANY WAYS AS YOU CAN. You do not need to solve it. “A pizza delivery store in New York (because that is where the best pizza is made) receives an order for a 16&quot; diameter pizza with ( \frac{1}{10} ) of the pizza topped with pepperoni (New Yorkers are very demanding about their pizzas). How much of the pizza will have pepperoni?”</td>
<td>![Pizza Diagram]</td>
</tr>
<tr>
<td>2. A circle with a radius of 7 is cut into 8 parts. Find the area of 3 parts in terms of ( \pi ).</td>
<td></td>
</tr>
<tr>
<td>3. A circular hot tub will have a diameter of 8 ft. Decorative brick will be placed around the edge of the hot tub except in a 60° section, which is where the steps will be. How many feet of decorative brick are needed?</td>
<td>4. How far does the tip of a 14 cm long minute hand on a clock move in 15 minutes?</td>
</tr>
<tr>
<td>4. How far does the tip of a 14 cm long minute hand on a clock move in 15 minutes?</td>
<td>5. A landscaping company is planting flowers around the edge of a circular pond with a radius of 14’. A 60° section will be planted with yellow flowers, which will be spaced six inches apart. How many yellow flowers are needed?</td>
</tr>
<tr>
<td>5. A landscaping company is planting flowers around the edge of a circular pond with a radius of 14’. A 60° section will be planted with yellow flowers, which will be spaced six inches apart. How many yellow flowers are needed?</td>
<td>6. A circle with a radius of 9 is cut into 6 parts. Find the area of 3 parts.</td>
</tr>
<tr>
<td>6. A circle with a radius of 9 is cut into 6 parts. Find the area of 3 parts.</td>
<td>7. A flower bed that is a quarter circle needs to be covered with mulch. The flower bed has a radius of 5 feet. How much mulch is needed?</td>
</tr>
<tr>
<td>7. A flower bed that is a quarter circle needs to be covered with mulch. The flower bed has a radius of 5 feet. How much mulch is needed?</td>
<td>KEY: 2) ( \frac{147\pi}{8} ) in(^2) 3) 20.11 in(^2) 4) 7(\pi) cm 5) 29 flowers 6) 127.23 ft(^2) 7) 19.63 ft(^2)</td>
</tr>
</tbody>
</table>
### Day 4 Warm-Up

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) SHOW AS MANY WAYS AS YOU CAN TO SET UP THE FOLLOWING PROBLEM: How far does the tip of an 8 cm long hour hand on a clock move in 3 hours?</td>
<td>12.57 cm</td>
</tr>
<tr>
<td>2) A circle with a radius of 7 is cut into 8 parts. Find the area of 3 parts in terms of ( \pi ).</td>
<td>( \frac{147\pi}{8} )</td>
</tr>
</tbody>
</table>

**Key:**

1) 12.57 cm  
2) \( \frac{147\pi}{8} \)
Day 4 Classwork: Desmos Arc Length and Sector Area Activity

https://teacher.desmos.com/activitybuilder/custom/58d92ba29623f50ba8d7f2af
<table>
<thead>
<tr>
<th>Day 4 Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ms. Bates insists on eating 56.55 square inches of a pizza... she is very</td>
</tr>
<tr>
<td>particular. If the pizza has a 12-inch diameter, what angle slice should you</td>
</tr>
<tr>
<td>cut?</td>
</tr>
<tr>
<td>2. A 90° sector of a circle has an area of 25π. What is the radius of the</td>
</tr>
<tr>
<td>circle?</td>
</tr>
<tr>
<td>3. In the diagram below, the length of arc AB is given. Calculate the angle</td>
</tr>
<tr>
<td>at the center of the sector.</td>
</tr>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
</tr>
<tr>
<td>4. In the diagram below, the length of the arc is given. What is the radius?</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
<tr>
<td>5. On one lazy afternoon Superman travels 12,437.57 miles just above the</td>
</tr>
<tr>
<td>Earth’s surface. The Earth has a radius of 3,959 miles. How many degrees</td>
</tr>
<tr>
<td>did he travel? What fraction of the earth did he travel?</td>
</tr>
<tr>
<td>6. In the circle below, the length of arc AB is 6 ft. What is the radius?</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
</tr>
<tr>
<td>7. The area of a sector is 3π. If the radius is 6, what is the central angle?</td>
</tr>
<tr>
<td>8. Mr. Sizemore needs to replace a windshield wiper, but the auto parts store</td>
</tr>
<tr>
<td>can’t locate the size, and it turns out a zombie apocalypse happened so he</td>
</tr>
<tr>
<td>can’t go to any other store.... The windshield wiper travels 3.5 ft at an</td>
</tr>
<tr>
<td>angle of 120°. How long is his windshield wiper?</td>
</tr>
</tbody>
</table>
Day 5 Warm-Up

<table>
<thead>
<tr>
<th>1) In the diagram below, the length of arc AB is given. Calculate the angle at the center of the sector.</th>
<th>2) How far does the tip of an 9 cm long minute hand on a clock move in 20 minutes?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td></td>
</tr>
</tbody>
</table>

Key: 1) $40^\circ$  2) 18.85 cm
Day 5 Classwork: How Many Radians in a Circle?

Take a circle.
Find the diameter, then half it to get the radius.
Cut a pipe cleaner to the length of the radius.
Put the pipe cleaner around part of the edge of the circle.
Continue cutting more radii until no more complete ones will fit.
Day 5 Homework

No homework assigned
### Day 6 Warm-Up

1) A pendulum is 30 cm long. When the pendulum swings, it travels along the arc of a circle and covers a distance of 20.94 cm. Calculate the angle through which the pendulum travels.

2) A circle with a radius of 8 is cut into 6 parts. Find the area of 2 parts.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>A pendulum is 30 cm long. When the pendulum swings, it travels along the arc of a circle and covers a distance of 20.94 cm. Calculate the angle through which the pendulum travels.</td>
</tr>
<tr>
<td></td>
<td>![Diagram of a pendulum]</td>
</tr>
<tr>
<td>2)</td>
<td>A circle with a radius of 8 is cut into 6 parts. Find the area of 2 parts.</td>
</tr>
<tr>
<td>![Diagram of a circle cut into 6 parts]</td>
<td></td>
</tr>
</tbody>
</table>

Key: 1) 40°  2) 67.02
Day 6-7 Classwork: How Many Radians in a Circle? (continued)

Finish your pipe-cleaner circle from yesterday.

(https://teacher.desmos.com/activitybuilder/custom/5e34c62714cc15a6215ebe4c adapted from https://teacher.desmos.com/activitybuilder/custom/58435d82bdad03af056a4c2)

Draw a segment from the center of the circle to the end of each radius. These angles are each one radian.
Day 6 Homework

No Homework
Day 7 Warm-Up

No Warm-Up
Day 7 Classwork: Converting Between Radians and Degrees

Since you know $360^\circ$ and $2\pi$ radians are equal, how can you convert between radians and degrees?

For starters, try:

- Convert $180^\circ$ to radians
- Convert $90^\circ$ to radians

What is your method for converting degrees to radians?

Convert $\pi/4$ radians to degree
Convert $1$ radian to degrees

What is your method for converting radians to degrees?
Day 7 Homework: DeltaMath Practice

8 questions converting between radians and degrees like this:

Convert the angle \( \frac{13\pi}{4} \) radians to degrees.

Answer: _____  
Submit Answer

2 clock problems like these:

What is the positive radian measure of the smaller angle formed by the hands of a clock at 4 o’clock? Reduce your answer completely.

Answer: _____  
Submit Answer
### Day 8 Warm-Up

<table>
<thead>
<tr>
<th>1) Convert to Radians</th>
<th>2) Convert to degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>$360^\circ$</td>
<td>$\pi$</td>
</tr>
</tbody>
</table>

Key: 1) $2\pi$  2) $180^\circ$
Day 8 Classwork: Desmos Arc Length and Sector Area Review

https://teacher.desmos.com/activitybuilder/custom/5e34de9b6679e20515b996b4/edit
Day 8 Homework

1. Find the length of the indicated arc.
   ![Diagram of a circle with a 240° arc and a radius of 8 ft]

2. Find the area of the indicated sector.
   ![Diagram of a circle with a 135° sector and a radius of 11 m]

3. Find the area of the shaded region using a proportion.
   ![Diagram of a circle with a 7 in sector]

4. Find the area of the shaded region using a proportion.
   ![Diagram of a circle with a sector divided into six parts]

5. A landscape company is planting flowers in a circular flowerbed with a diameter of 10’. The border of a 120° sector will be planted with purple flowers. What is the length of the arc of that will be planted in purple flowers?

6. A lawn sprinkler sprays water in a circular pattern with a radius of 30 feet. If the sprinkler is set to rotate in a 280° arc, how many square feet will it not water within its range?

7. Convert 270° to radians. Be sure to show your work.

8. Convert 3π/4 to degrees. Be sure to show your work.

9. Define radians. Describe how they relate radius to arc length and circumference.

Key: 1) \( \frac{32\pi}{3} \text{ ft} \) 2) \( \frac{363\pi}{8} \text{ m}^2 \) 3) \( \frac{98\pi}{5} \text{ in}^2 \) 4) \( \frac{27\pi}{2} \text{ cm}^2 \) 5) \( \frac{10\pi}{3} \text{ ft} \) 6) \( 200\pi \text{ ft}^2 \) 7) \( \frac{3\pi}{2} \) 8) \( 135° \)
Appendix K

EXPLICIT LESSON PLANS FOR ARC LENGTH/SECTOR AREA
Day 1 Warm-Up

Find the distance around the circle.

Find the space inside the circle.
Day 1 Classwork and Homework: Arc Length and Sector Area

Definitions
Arc Length: the distance around part of the edge of a circle
Units for Arc Length: It’s distance, so units will be in, cm, yd, etc.

Formulas:
\[
\text{Arc Length} = \frac{2\pi r \theta}{360^\circ}
\]
\[
\text{Arc Length} = 2\pi r \left( \frac{\text{part}}{\text{whole}} \right)
\]
\[
\frac{\text{Arc Length}}{\text{Circumference}} = \frac{\text{part}}{\text{whole}}
\]

Sector: pie-shaped piece of a circle whose vertex is at the center of the circle
Sector Area: amount of space inside a sector
Units for Sector Area: It’s area, so units will be in\(^2\), cm\(^2\), yd\(^2\), etc.

Formulas:
\[
\text{Sector Area} = \frac{\pi r^2 \theta}{360^\circ}
\]
\[
\text{Sector Area} = \pi r^2 \left( \frac{\text{part}}{\text{whole}} \right)
\]
\[
\frac{\text{Sector Area}}{\text{Circle Area}} = \frac{\text{part}}{\text{whole}}
\]

Examples:

1. Find the arc length.

2. Find the area of the sector.
### DIY Section

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>3. Find the length of the indicated arc.</td>
<td>4. Find the length of the indicated arc.</td>
<td></td>
</tr>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>5. Find the area of the indicated sector.</td>
<td>6. Find the area of the indicated sector.</td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td><img src="image4.png" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>7. Find the area of the shaded regions.</td>
<td>8. Find the length of the indicated arc.</td>
<td></td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram" /></td>
<td><img src="image6.png" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>9. What is the area of the shaded region?</td>
<td>10. Find the area of the shaded region.</td>
<td></td>
</tr>
<tr>
<td><img src="image7.png" alt="Diagram" /></td>
<td><img src="image8.png" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>11. Find the length of the indicated arc.</td>
<td>Key</td>
<td></td>
</tr>
<tr>
<td><img src="image9.png" alt="Diagram" /></td>
<td>1. (\frac{15\pi}{4} \text{yd}) 2. (\frac{81\pi}{4} \text{m}^2) 3. (\frac{49\pi}{2} \text{km}) 4. (\frac{55\pi}{6} \text{cm})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5. (\frac{605\pi}{8} \text{in}^2) 6. (\frac{320\pi}{3} \text{yd}^2) 7. (\frac{75\pi}{8} \text{cm}^2) 8. (\frac{10\pi}{3} \text{cm})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9. (\frac{625\pi}{36} \text{cm}^2) 10. (\frac{125\pi}{8} \text{cm}^2) 11. (5\pi \text{ cm})</td>
<td></td>
</tr>
</tbody>
</table>
### Day 2 Warm-Up

1) Find the length of the indicated arc.

![Diagram of a circle with a dashed arc of length 4 cm.]

2) Find the area of the indicated sector.

![Diagram of a sector of a circle with a radius of 18 km and a central angle of 150°.]

**Key:**

1) \( \frac{8\pi}{3} \)  
2) \( 135\pi \text{ km}^2 \)
### Day 2 Classwork and Homework: Using Arc Length and Sector Area

<table>
<thead>
<tr>
<th>Formula</th>
<th>Arc Length</th>
<th>Sector Area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{Arc length} = 2\pi r \left( \frac{\text{part}}{\text{whole}} \right)$</td>
<td>$\text{Sector area} = \pi r^2 \left( \frac{\text{part}}{\text{whole}} \right)$</td>
</tr>
<tr>
<td>Meaning</td>
<td>Distance around part of the edge</td>
<td>A pie-shaped piece of the inside of a circle</td>
</tr>
<tr>
<td>Phrases to Look For</td>
<td>Crust, edge, border, distance, length Feet, inches, yards</td>
<td>Amount of pizza you eat Amount of grass you water Amount of pie you eat Area Sq Ft, Sq. In. Sq. Yards</td>
</tr>
<tr>
<td>Example Problems</td>
<td>Ex 1. A hula-hoop has a diameter of 3’. How long is a 60° section?</td>
<td>Ex 2. A lawn sprinkler sprays water in a circular pattern with a radius of 25 feet. If the sprinkler is set to rotate in a 300° arc, how many square feet will it water?</td>
</tr>
</tbody>
</table>

### Questions to ask yourself as you solve these problems:
1. Length or Area?  
2. Diameter or Radius?  
3. Degrees in the problem, or the rest of the circle?

### Your Turn!

3. A circle has a diameter of 10”. What is the length of a 270° arc?  
4. Rose makes lemon pies in pans that have an 8” diameter. If she cuts me a 60° piece, how many square inches is my slice?  

5. A gardener is putting border around a circular flower bed. The radius of the flower bed is 5 feet. He runs out of border, leaving a 45° section without border. How much of the flower bed has no border?  
6. A pendulum 30 cm long swings through an angle of 25°. How far does the pendulum travel on each swing?
7. A circular hot tub will have a diameter of 10 ft. Decorative brick will be placed around the edge of the hot tub except in a 60° section, which is where the steps will be. How many feet of decorative brick are needed?

8. Shaq and his friends order a pizza. While everyone else is watching the game, Shaq eats 210° of the extra-large pizza, which had a radius of 11 in. How much pizza is left for his friends?

9. A deli sells sandwiches on circular slices of bread which are 6" across. After you have eaten a 45° section of the bread crust, how much crust have you eaten?

10. Find the perimeter of the shaded region.

11. A landscape company is planting flowers in a circular flowerbed with a diameter of 10'. A 120° sector will be planted with purple flowers, and the rest of the circle will be planted with orange flowers. What is the area of the flowerbed that will be planted in orange flowers?

12. A bicycle wheel with 18 spokes has a diameter of 22". How far does the bicycle travel as the wheel goes from one spoke pointing directly at the ground to the next?

13. A garden is in the shape of a semicircle with a diameter of 40 m. What is the area of the garden?

14. A lawn sprinkler sprays water in a circular pattern with a radius of 20 feet. If the sprinkler is set to rotate in a 250° arc, how many square feet will it water?

**KEY:**

<p>| | | | | | |</p>
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<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>( \frac{\pi}{2} ) ft or 1.57 ft</td>
<td>2)</td>
<td>1636.25 ft²</td>
<td>3)</td>
<td>( \frac{15\pi}{2} )</td>
</tr>
<tr>
<td>4)</td>
<td>( \frac{8}{3} ) ( \pi ) in²</td>
<td>5)</td>
<td>3.93 ft</td>
<td>6)</td>
<td>( \frac{25\pi}{6} ) in</td>
</tr>
<tr>
<td>7)</td>
<td>26.18 ft</td>
<td>8)</td>
<td>158.39 in²</td>
<td>9)</td>
<td>2.36&quot;</td>
</tr>
<tr>
<td>10)</td>
<td>21.42</td>
<td>11)</td>
<td>( \frac{50}{3} ) ( \pi )</td>
<td>12)</td>
<td>3.84&quot;</td>
</tr>
<tr>
<td>13)</td>
<td>200( \pi )</td>
<td>14)</td>
<td>872.66 ft²</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Day 3 Warm-Up

1) Find the perimeter of the shaded region in terms of $\pi$.

![Diagram of a circle with a shaded region]

Key: $4\pi + 10$

2) A circular hot tub will have a diameter of 12 ft. Decorative brick will be placed around the edge of the hot tub except in an $80^\circ$ section, which is where the steps will be. How many feet of decorative brick are needed?

Key: 2) $8.38$ ft.
Day 3 Classwork: Foldable

Before the day’s lesson, students filled in a foldable using their notes.

Arc Length

Sector Area
Day 3 Classwork and Homework: Other Ways to Set Up Arc Length and Sector Area Problems

Since arc length and sector area are just fractional parts of a circle, there are a lot of ways you can describe the fractional part other than degrees/360.

<table>
<thead>
<tr>
<th>If you’re talking about...</th>
<th>... there are this many parts in the whole.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degrees in a circle</td>
<td></td>
</tr>
<tr>
<td>Minutes on a clock</td>
<td></td>
</tr>
<tr>
<td>Hours on a clock</td>
<td></td>
</tr>
<tr>
<td>Pie cut into slices</td>
<td></td>
</tr>
</tbody>
</table>

Ex. 1
A 12” diameter pizza has a 60° slice cut out. What is the area of the slice?

\[
\text{Sector Area} = \pi \left( \frac{60°}{360°} \right)
\]

Ex. 2
How far does the tip of an 8 cm long minute hand on a clock move in 10 minutes?

\[
\frac{\text{Arc Length}}{\text{Circumference}} = \frac{\text{part}}{\text{whole}}
\]

\[
\frac{\text{Arc Length}}{2\pi \times 8} = \frac{10}{60}
\]

Ex. 3
A circle with a radius of 5 is cut into 6 parts. Find the area of 1 part.

\[
6 \times \text{Sector Area} = \pi r^2
\]

Arc Length and Sector Area Practice (C.5)
Try to solve these problems using a method other than the formula you have been using.

4. How far does the tip of a 12 cm long minute hand on a clock move in 45 minutes?

5. A circle with a radius of 7 is cut into 8 parts. Find the area of 3 parts in terms of \(\pi\).

6. A pizza delivery store in New York (\textit{because that is where the best pizza is made}) receives an order for a 16” diameter pizza with \(\frac{1}{10}\) of the pizza topped with pepperoni (New Yorkers are very demanding about their pizzas). How much of the pizza will have pepperoni?
7. A circular hot tub will have a diameter of 8 ft. Decorative brick will be placed around the edge of the hot tub except in a 60° section, which is where the steps will be. How many feet of decorative brick are needed?

8. How far does the tip of a 14 cm long minute hand on a clock move in 15 minutes?

9. A landscaping company is planting flowers around the edge of a circular pond with a radius of 14'. A 60° section will be planted with yellow flowers, which will be spaced six inches apart. How many yellow flowers are needed?

10. A circle with a radius of 9 is cut into 6 parts. Find the area of 3 parts.

11. A flower bed that is a quarter circle needs to be covered with mulch. The flower bed has a radius of 5 feet. How much mulch is needed?

12. A circle is divided into 5 equal sections. The diameter of the circle is 12 inches. What is the area of the shaded section?

**KEY:**

1) $6\pi \text{ in}^2$ or 18.85 $\text{ in}^2$
2) $\frac{8\pi}{3} \text{ cm}^2$ or 8.38 cm
3) 13.09 or $\frac{25\pi}{6}$
4) $18\pi$
5) $\frac{147\pi}{8}$
6) 20.11 $\text{ in}^2$
7) 20.94 ft
8) $7\pi$ cm
9) 29 flowers
10) 127.23
11) 19.63 $\text{ ft}^2$
12) $\frac{36\pi}{5}$ $\text{ in}^2$
### Day 4 Warm-Up

<table>
<thead>
<tr>
<th>1) SHOW AS MANY WAYS A YOU CAN TO SET UP THE FOLLOWING PROBLEM: How far does the tip of an 8 cm long hour hand on a clock move in 3 hours?</th>
<th>2) A circle with a radius of 7 is cut into 8 parts. Find the area of 3 parts in terms of $\pi$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key: 1) 12.57 cm 2) $\frac{147\pi}{8}$</td>
<td></td>
</tr>
</tbody>
</table>

![Diagram of a circle divided into 8 parts with 3 parts shaded]
Day 4 Classwork and Homework: Solving for the Radius or Angle in ALSA Problems

Example 1. Ms. Bates insists on eating 56.55 square inches of a pizza... she is very particular. If the pizza has a 12-inch diameter, what angle slice should you cut?

\[
\frac{\text{Sector area}}{\pi 6^2} = \frac{56.55}{\pi 6^2} = \frac{56.55}{\pi} = \frac{56.55}{180} \text{ degrees}
\]

\[
\frac{X}{2 \pi (180)} = \frac{0.50}{2 \pi (180)} = \frac{0.50}{360}
\]

\[180 \degree = X\]

Example 2. A 90° sector of a circle has an area of $25\pi$. What is the radius of the circle?

\[
\frac{25\pi}{\pi r^2} = \frac{25}{r^2}
\]

\[100 = r^2 \]

\[r = 10\]

3. In the diagram below, the length of arc AB is given. Calculate the angle at the center of the sector.

4. In the diagram below, the length of the arc is given. What is the radius?

5. On one lazy afternoon Superman travels 12,437.57 miles just above the Earth’s surface. The Earth has a radius of 3,959 miles. How many degrees did he travel? What fraction of the earth did he travel?

6. In the circle below, the length of arc AB is 6 ft. What is the radius?

7. The area of a sector is $3\pi$. If the radius is 6, what is the central angle?

8. A pendulum is 45 cm long. When the pendulum swings, it travels along the arc of a circle and covers a distance of 27.5 cm. Calculate the angle through which the pendulum travels.
9. A water sprinkler waters 523.6 square feet of grass. If the radius is 20 feet, what angle is the sprinkler set to?

10. A water sprinkler waters 981.75 square feet of grass. If it’s set to water in a half-circle, what is the radius?

11. In the diagram below, the length of arc AB is given. Calculate the angle at the center of the sector.

\[ \text{Arc length} = \theta \cdot r \]

\[ \frac{5\pi}{3} \text{ cm} \]

\[ 10 \text{ cm} \]

12. Mr. Sizemore needs to replace a windshield wiper, but the auto parts store can’t locate the size, and it turns out a zombie apocalypse happened so he can’t go to any other store…. The windshield wiper travels 3.5 ft at an angle of 120°. How long is his windshield wiper?

13. Find the area of the shaded region. (Hint: Find the area of the rectangle first.)

14. Find the perimeter of the shaded region.

15. How far does the tip of a 12 cm long minute hand on a clock move in 20 minutes?

16. A circle is divided into 5 equal sections. The diameter of the circle is 8 inches. What is the area of the shaded section?

**KEY:**

1) 180°  
2) 10  
3) 60°  
4) 20  
5) 180°, half  
6) 4.58 ft  
7) 30°  
8) 35°  
9) 150°  
10) 25 ft  
11) 30°  
12) 1.67 ft  
13) 90 – 18π  
14) 12 + 3π  
15) 8π  
16) \( \frac{32\pi}{5} \)
## Day 5 Warm-Up

1) In the diagram below, the length of arc AB is given. Calculate the angle at the center of the sector.

![Diagram of a sector with AB as the arc and A and B as endpoints.](image)

2) How far does the tip of an 9 cm long minute hand on a clock move in 20 minutes?

| Key: 1) $40^\circ$ 2) 18.85 cm |  |  |
Day 5 Classwork and Homework: What Are Radians?

**Degrees** are one way to measure angles.

**Radians** are another way to measure angles.

You can convert between radians and degrees using the following formulas:

\[
\text{Radians} = \text{Degrees} \times \frac{\pi}{180} \quad \text{Degrees} = \text{Radians} \times \frac{180}{\pi}
\]

**Definition of a radian:** A radian is the central angle formed when the radius of a circle is equal to the length of the intercepted arc.

Angle α is 1 radian.

Angle β is 2 radians.

<table>
<thead>
<tr>
<th>Ex 1) Convert to degrees.</th>
<th>Ex 2) Convert to radians.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{13\pi}{12} )</td>
<td>125°</td>
</tr>
<tr>
<td>3) Convert to degrees.</td>
<td>4) Convert to degrees</td>
</tr>
<tr>
<td>( \frac{3\pi}{2} )</td>
<td>( \frac{7\pi}{12} )</td>
</tr>
<tr>
<td>5) Convert to radians.</td>
<td>6) Convert to radians.</td>
</tr>
<tr>
<td>200°</td>
<td>190°</td>
</tr>
<tr>
<td>7) Convert to radians.</td>
<td>8) Convert to radians.</td>
</tr>
<tr>
<td>135°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9) Convert to degrees.</td>
</tr>
<tr>
<td>---</td>
<td>-----------------------</td>
</tr>
<tr>
<td></td>
<td>( \frac{3\pi}{4} )</td>
</tr>
<tr>
<td></td>
<td>12) Convert to degrees.</td>
</tr>
<tr>
<td></td>
<td>( \pi )</td>
</tr>
</tbody>
</table>
Day 6 Warm-Up

1) A pendulum is 30 cm long. When the pendulum swings, it travels along the arc of a circle and covers a distance of 20.94 cm. Calculate the angle through which the pendulum travels.

2) A circle with a radius of 8 is cut into 6 parts. Find the area of 2 parts.

Key: 1) $40^\circ$  2) 67.02
Day 6 Classwork: Foldable

Converting to Radians

Converting to Degrees
Converting to Radians

\[ \text{Degrees} \cdot \frac{\pi}{180^\circ} = \text{Radians} \]

Example

Converting to Degree

\[ \text{Radians} \cdot \frac{180^\circ}{\pi} = \text{Degrees} \]

Example

What is a radian?

Draw a picture showing a radian.
Day 6 Classwork and Homework: DeltaMath Practice

16 questions converting between radians and degrees similar to this:

Mr. Sizemore
Converting Radians to Degrees
Dec 07, 9:04:53 PM

Watch help video  Problem types

Convert the angle $\frac{13\pi}{4}$ radians to degrees.

Answer: $\text{Submit Answer}$

2 clock problems similar to this:

Radians on the Clock
Dec 27, 5:24:41 PM

What is the positive radian measure of the smaller angle formed by the hands of a clock at 4 o’clock? Reduce your answer completely.

Answer: $\pi$  
$\text{Submit Answer}$
Plus a timed activity in which students tried to convert common angle measures in under 100 seconds similar to this:

**Degrees to Radians (Common Angles)**
Dec 27, 5:25:12 PM

---

**Time to beat: 100 seconds**

- **Current Time:** 19.0
- **Best Time:** ---
- **Score:** 0/10

---

60°

- \(\frac{\pi}{4}\)
- \(\frac{\pi}{3}\)
- \(\frac{\pi}{2}\)
- \(\frac{\pi}{6}\)
## Day 7 Warm-Up

<table>
<thead>
<tr>
<th>1) Convert to radians.</th>
<th>2) Convert to degrees.</th>
</tr>
</thead>
<tbody>
<tr>
<td>200°</td>
<td>( \frac{5\pi}{18} )</td>
</tr>
</tbody>
</table>

Key: 1) \( \frac{10\pi}{9} \) 2) 50°
Day 7 Classwork and Homework: Converting Between Radians and Degrees Puzzle

Day 7: Converting Radians and Degrees Puzzle

Name: ______________________

Instructions: Cut out the puzzle pieces on the other page. Assemble the puzzle in the space below so that equal values touch. (Values with π are in radians.)
<table>
<thead>
<tr>
<th>Puzzle Pieces:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varnothing$ 228°</td>
</tr>
<tr>
<td>$\pi/2$</td>
</tr>
<tr>
<td>$1^\circ$</td>
</tr>
<tr>
<td>$16^\circ$</td>
</tr>
<tr>
<td>$0^\circ$</td>
</tr>
<tr>
<td>$270^\circ$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>$2\pi$</td>
</tr>
<tr>
<td>$330^\circ$</td>
</tr>
<tr>
<td>Key:</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>330°</td>
</tr>
<tr>
<td>11/12</td>
</tr>
<tr>
<td>2/3</td>
</tr>
<tr>
<td>150°</td>
</tr>
<tr>
<td>9/12</td>
</tr>
<tr>
<td>3/13</td>
</tr>
<tr>
<td>180°</td>
</tr>
<tr>
<td>9/11</td>
</tr>
<tr>
<td>2/11</td>
</tr>
<tr>
<td>45°</td>
</tr>
<tr>
<td>7/12</td>
</tr>
<tr>
<td>225°</td>
</tr>
<tr>
<td>60°</td>
</tr>
<tr>
<td>3 1/2</td>
</tr>
<tr>
<td>270°</td>
</tr>
<tr>
<td>121°</td>
</tr>
<tr>
<td>135°</td>
</tr>
<tr>
<td>165°</td>
</tr>
<tr>
<td>13 11/12</td>
</tr>
</tbody>
</table>

191
**Day 8 Warm-Up**

<table>
<thead>
<tr>
<th>1) Convert to Radians 360°</th>
<th>2) Convert to degrees $\pi$</th>
</tr>
</thead>
</table>

Key: 1) $2\pi$ 2) $180^\circ$
<table>
<thead>
<tr>
<th><strong>Arc Length and Sector Area Review</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How are arc length and sector area applications of similarity?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. Find the length of the indicated arc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 1](8 cm 210°)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. Find the length of the indicated arc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 2](15 cm 225°)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. Find the length of the indicated arc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 3](90° 16 km)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5. Find the area of the indicated sector.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 4](8 ft 240°)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6. Find the area of the indicated sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 5](13 mi)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>7. Find the area of the indicated sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 6](11 m 135°)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8. Find the area of the shaded region using a proportion.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 7](7 in)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9. Find the area of the shaded region using a proportion.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 8](6 cm)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>10. Find the area of the shaded region using a proportion.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 9](5 in)</td>
</tr>
</tbody>
</table>
11. A landscape company is planting flowers in a circular flowerbed with a diameter of 10\(\text{'}\). The border of a 120° sector will be planted with purple flowers. What is the length of the arc of that will be planted in purple flowers?

12. A lawn sprinkler sprays water in a circular pattern with a radius of 30 feet. If the sprinkler is set to rotate in a 280° arc, how many square feet will it not water within its range?

13. The circle is divided into four congruent sections. Find the length of \(AB\).

![Diagram](https://via.placeholder.com/150)

**Radians Review**

14. Define radians. Describe how they relate radius to arc length and circumference.

15. Convert to radians. Be sure to show your work.

\[225°\]

16. Convert to radians. Be sure to show your work.

\[150°\]

17. Convert to radians. Be sure to show your work.

\[100°\]

18. Convert to degrees. Be sure to show your work.

\[\frac{7\pi}{4}\]

19. Convert to degrees. Be sure to show your work.

\[\frac{31\pi}{36}\]

20. Convert to degrees. Be sure to show your work.

\[\frac{7\pi}{6}\]

**Key:**

2) \(\frac{28\pi}{3}\) cm

3) \(\frac{75\pi}{4}\) cm

4) 8\(\pi\) km

5) \(\frac{128\pi}{3}\) ft\(^2\)

6) \(\frac{1183\pi}{12}\) mi\(^2\)

7) \(\frac{363\pi}{8}\) m\(^2\)

8) \(\frac{98\pi}{5}\) in\(^2\)

9) \(\frac{27\pi}{2}\) cm\(^2\)

10) \(\frac{25\pi}{4}\) in\(^2\)

11) \(\frac{10\pi}{3}\)

12) 200\(\pi\)

13) \(\frac{5\pi}{2}\)

14) \(\frac{5\pi}{4}\)

15) \(\frac{5\pi}{6}\)

16) \(\frac{5\pi}{9}\)

17) 315°
Appendix L

INQUIRY-BASED LESSON PLANS FOR GRAPHING CIRCLES
Day 1 Warm-Up

Solve for x.

Solve for x.

\[
\begin{align*}
5 \text{ mi} & \quad 12 \text{ mi} \\
x & \\
\end{align*}
\]

\[
\begin{align*}
12 \text{ cm} & \quad x \\
9 \text{ cm} & \\
\end{align*}
\]
Day 1 Classwork: Equations of Circles

On the first graph, show all the points 5 units from the origin.
Prove that at least 12 of those points are 5 units from the origin.

On the second graph, show all the points 5 units from the point (1, 2).
Prove that at least 12 of those points are 5 units from the origin.

On the third graph, show all the points 5 units from the point (−2, 5).
Prove that at least 12 of those points are 5 units from the origin.
Day 1 Homework

No Homework Assigned
Day 2 Warm-Up

1. What’s a circle? Be specific.

2. Solve for x.

   8 mi
   6 mi
   x
Day 2 Classwork: Equations of Circles (continued)

Pick a figure below. How long are \(a\), \(b\), and \(c\)?

**Figure 1**

- \((1, 5)\)
- \((-3, 2)\)

**Figure 2**

- \((9, 7)\)
- \((-3, 2)\)

Write an equation showing all the \((x, y)\) points 5 units from the point \((-3, 2)\).
Identify as many relationships as you can between the Pythagorean Theorem and your equation for a circle.
Equation showing all the (x, y) points 5 units from the point (–3, 2):
Day 2 Homework

No Homework Assigned
Day 3 Warm-Up

1a. Find a in the figure.

1b. Find b in the figure.

1c. Find c in the figure.
Equation showing all the \((x, y)\) points 5 units from the point \((-3, 2)\):

The equation we came up with was

\[
(x - (-3))^2 + (y - 2)^2 = 5^2
\]
Identify as many relationships as you can between the Pythagorean Theorem and our equation for the circle.

\[(x - (-3))^2 + (y - 2)^2 = 5^2\]

The equation can be rewritten as:

\[(x + 3)^2 + (y - 2)^2 = 25\]
Day 3 Homework

No Homework Assigned
Day 4 Warm-Up

1. Graph
   
   \[(x - 4)^2 + (y + 3)^2 = 4\]

2. Graph
   
   \[(x - 4)^2 + y^2 = 1\]
Day 4 Classwork: If You’re Given Points on a Circle

What’s the equation for this circle?

What’s the equation for this circle?
### Day 4 Homework

**Distance Formula:** \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

**Midpoint Formula:** \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \)

<table>
<thead>
<tr>
<th>Write the equations of these circles in standard form:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Center: (-5,10) and point on circle: (-14,10)</td>
</tr>
<tr>
<td>2. Endpoints of diameter: (-10,5) and (16,-1)</td>
</tr>
<tr>
<td>3. Center: (12,4) and point on circle: (13,5)</td>
</tr>
<tr>
<td>4. Endpoints of diameter: (6,3) and (-12,3)</td>
</tr>
<tr>
<td>5. Center: (9,-15) and point on circle: (10,-18)</td>
</tr>
<tr>
<td>6. Endpoints of diameter: (-4, -4) and (12, -16)</td>
</tr>
</tbody>
</table>

**Answers:**

1) \((x + 5)^2 + (y - 10)^2 = 81\)  
2) \((x - 3)^2 + (y - 2)^2 = 178\)  
3) \((x - 12)^2 + (y - 4)^2 = 2\)  
4) \((x + 3)^2 + (y - 3)^2 = 81\)  
5) \((x - 9)^2 + (y + 15)^2 = 10\)  
6) \((x - 4)^2 + (y + 10)^2 = 100\)
<table>
<thead>
<tr>
<th></th>
<th>Day 5 Warm-Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>What is the radius of this circle?</td>
</tr>
<tr>
<td></td>
<td>Center: $(-16, 9)$</td>
</tr>
<tr>
<td></td>
<td>Point on Circle: $(-18, 11)$</td>
</tr>
<tr>
<td>2</td>
<td>What is the radius of this circle?</td>
</tr>
<tr>
<td></td>
<td>Ends of a diameter: $(12, 1)$ and $(2, -7)$</td>
</tr>
</tbody>
</table>
In the equation below, why are \((x - 9)\) and \((y + 3)\) squared? And why do you have to take the square root of 16 to find the radius?

\[
(x - 9)^2 + (y + 3)^2 = 16
\]

This equation can be written other ways. For example, you can square both terms on the left and combine like terms. Do that now.

Now, subtract 16 from both sides. That puts the equation in “general form”.

214
### Day 5 Homework

Convert the following equations from standard form to general conic form.

1. \[
\begin{align*}
(x - 11)^2 + (y + 5)^2 &= 49 \\
(x - 11)(x - 11) + (y + 5)(y + 5) &= 49 \\
x^2 - 11x - 11x + 121 + ___ + ___ + ___ + ___ &= 49 \\
x^2 - ___ + y^2 + ___ + 146 &= 49 \\
x^2 + y^2 - 22x + 10y + ____ &= 0
\end{align*}
\]

2. \[(x - 14)^2 + (y - 15)^2 = 9\]

3. \[(x - 2)^2 + (y + 12)^2 = 16\]

4. \[(x - 14)^2 + (y + 1)^2 = 13\]

5. \[(x + 1)^2 + (y - 10)^2 = 25\]

**Answers:**

1. \[x^2 + y^2 - 22x + 10y + 97 = 0\]
2. \[x^2 + y^2 - 28x - 30y + 412 = 0\]
3. \[x^2 + y^2 - 4x + 24y + 132 = 0\]
4. \[x^2 + y^2 - 28x + 2y + 184 = 0\]
5. \[x^2 + y^2 + 2x - 20y + 76 = 0\]
<table>
<thead>
<tr>
<th>Day 6 Warm-Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. This is a question from 9th grade! Factor: ( x^2 + 8x + 15 ) ((x + _____)(x + _____))</td>
</tr>
<tr>
<td>2. This is a question from Friday. Convert to general conic form. ( (x + 10)^2 + (y - 15)^2 = 9 )</td>
</tr>
</tbody>
</table>
Day 6-7 Classwork: Completing the Square

Fill in as much as you can showing what the process from Friday would look like if we did it backwards.

\[ x^2 + y^2 - 22x + 10y + 97 = 0 \]

Simplify these:
\[(x + 9)^2\]
\[(x - 3)^2\]
\[(x + 5)^2\]
\[(x - 8)^2\]

What patterns do you notice?
What would need to go in the blanks to create perfect squares?

\[ x^2 + 20x + \_ \_ \_ \_ \]

\[ x^2 - 12x + \_ \_ \_ \_ \]

\[ x^2 + 14x + \_ \_ \_ \_ \]

\[ x^2 + 34x + \_ \_ \_ \_ \]

Factor these perfect squares:

\[ x^2 + 8x + 16 \]

\[ x^2 - 2x + 1 \]

\[ x^2 + 12x + 36 \]
\[ x^2 + y^2 - 6x + 12y + 20 = 0 \]

\[ (x^2 - 6x ) + (y^2 + 12y) = -20 \]

\[ (x^2 - 6x + \____) + (y^2 + 12y + \____) = -20 + \____ + \____ \]

Center (___,___)  \( r = \____ \)
Work with a partner to convert to standard form:

\[ x^2 + y^2 - 8x - 10y + 5 = 0 \]
\[(x^2 - 8x + ___) + (y^2 - 10y + ___) = -5 + ___ + ___\]

Work with a partner to convert to standard form:

\[ x^2 + y^2 + 6x + 20y + 93 = 0 \]
\[(x^2 + 6x + ___) + (y^2 + 20y + ___) = -93 + ___ + ___\]
Day 6 Homework

No Homework Assigned
Day 7 Warm-Up

1. Convert to general conic form.

\[(x + 7)^2 + (y + 13)^2 = 25\]
Day 7 Homework

1. Rewrite in standard form:
\[ x^2 + y^2 + 2x + 6y = 26 \]
\[ (x^2 + 2x + ____) + (y^2 + 6y + ____) = 26 + ____ + ____ \]
\[ (x + ____)(x + ____) + (y + ____)(y + ____) = ____ \]
\[ (x + ____)^2 + (y + ____)^2 = 36 \]

2. Find the center:
\[ x^2 + y^2 + 10x - 16y + 88 = 0 \]
\[ (x^2 + 10x + ____) + (y^2 - 16y + ____) = -88 + ____ + ____ \]
\[ (x + ____)(x + ____) + (y - ____)(y - ____) = ____ \]
\[ (x + ____)^2 + (y - ____)^2 = ____ \]
Center: (____, ____)

3. Convert to standard form:
\[ x^2 + y^2 - 6x + 20y + 28 = 0 \]
\[ (x^2 - 6x + ____) + (y^2 + 20y + ____) = -28 + ____ + ____ \]
\[ (x - ____)(x - ____) + (y + ____)(y + ____) = ____ \]
\[ (x - ____)^2 + (y + ____)^2 = ____ \]

4. Rewrite in standard form:
\[ x^2 + y^2 - 4x - 12y + 31 = 0 \]
5. Find the center:

\[ x^2 + y^2 - 14x + 26y + 217 = 0 \]

Center: (___, ___)

6. Convert to standard form:

\[ x^2 + y^2 + 6x + 8y - 96 = 0 \]

7. Find the center:

\[ x^2 + y^2 + 30y + 209 = 0 \]

Answers:

1. \((x + 1)^2 + (y + 3)^2 = 36\)  
2. \((-5, 8)\)  
3. \((x - 3)^2 + (y + 10)^2 = 81\)  
4. \((x - 2)^2 + (y - 6)^2 = 9\)  
5. \((7, -13)\)  
6. \((x + 3)^2 + (y + 4)^2 = 121\)  
7. \(x^2 + (y + 15)^2 = 16\)
Day 8 Warm-Up

1. Convert to standard form.
   \[ x^2 + y^2 - 20x + 22y + 205 = 0 \]

2. Convert to standard form.
   \[ x^2 + y^2 + 14x - 4y + 17 = 0 \]
Day 8 Classwork: Test Review

1. Which circle is bigger?
\[(x - 2)^2 + (y - 1)^2 = 25\] \[(x + 500)^2 + (y - 837)^2 = 16\]

2. Which circle is bigger?
\[x^2 + y^2 + 4x - 32y + 259 = 0\] \[x^2 + y^2 - 8x - 10y + 25 = 0\]

3. Which circle is bigger?
Center: \((0, -2)\)
Point on Circle: \((6, -8)\)
Ends of a diameter: \((-4, 15)\) and \((-16, 3)\)
4. Make any circle on the graph below and show how its equation is related to the Pythagorean Theorem. (Hint to make it easier for you: If you’ll use a radius of 5, 10, or 13, some coordinates on the circle will land on the gridlines.)
Day 8 Homework

1. Graph the circle.
   \[(x - 4)^2 + (y + 4)^2 = 9\]

2. Convert to standard form.
   \[x^2 + y^2 + 22x + 8y + 121 = 0\]

3. Write the equation in standard form.
   \[(x - 2)^2 + y^2 = 16\]

4. Write the equation in standard form.
   Center: \((-9, 12)\)
   Point on Circle: \((-9, 5)\)

5. Write the equation in standard form.
   Ends of a diameter: \((10, 17)\) and \((18, 9)\)

6. Convert to general conic form.
   \[(x + 2)^2 + (y - 11)^2 = 36\]

Key:
1. Center is \((4, -4)\) and radius is 3
2. \((x + 11)^2 + (y + 4)^2 = 16\)
3. \((x - 2)^2 + y^2 = 16\)
4. \((x + 9)^2 + (y - 12)^2 = 49\)
5. \((x - 14)^2 + (y - 13)^2 = 32\)
6. \(x^2 + y^2 + 4x - 22y + 89 = 0\)
Appendix M

EXPLICIT LESSON PLANS FOR GRAPHING CIRCLES
Day 1 Warm-Up

Solve for x.

\[
\begin{align*}
5 \text{ mi} & \quad 12 \text{ mi} \\
x & \\
\end{align*}
\]

Solve for x.

\[
\begin{align*}
12 \text{ cm} & \quad x \\
9 \text{ cm} &
\end{align*}
\]
Day 1 Classwork and Homework: Graphing Circles

Equations of circles are based on the Pythagorean Theorem (or the Distance Formula, which is also based on the Pythagorean Theorem).

- \( (x - h) \) or \( (1 - (-3)) \) is the length of the horizontal leg of the triangle and is equal to \( a \).
- \( (y - k) \) or \( (5 - 2) \) is the length of the vertical leg of the triangle and is equal to \( b \).
- The radius of the circle is the hypotenuse of the triangle and is equal to \( c \).

Since \( x - h \) and \( y - k \) are the length of the legs, that means we have to change the signs to get \((h,k)\) which is the center of the circle.

Since the number at the end of the equation is \( c^2 \), we have to take the square root of it to find \( c \), which is the radius of the circle.

Ex 1. Graph.
\[
(x - 2)^2 + (y + 1)^2 = 25
\]

Ex 2. Write the equation of the circle.
Your Turn 3. Graph
\[(x + 3)^2 + (y - 4)^2 = 1\]

Your Turn 4. The equation of a circle is
\[(x - 2)^2 + (y - 2)^2 = 4\]. Tell whether each point is on the circle, in the interior of the circle, or in the exterior of the circle.

- a) (1, 2)
- b) (1, 4)
- c) (2, 0)
- d) (4, 2)
- e) (4, 4)
- f) (3, 2)

5. Graph
\[(x - 1)^2 + (y + 3)^2 = 25\]

6. Graph
\[(x + 3)^2 + (y - 4)^2 = 16\]
7. Graph \( x^2 + (y - 4)^2 = 9 \)

8. Graph \( x^2 + y^2 = 16 \)

9. Indicate which set of coordinates are on the circle.
\( (x - 2)^2 + (y - 2)^2 = 9 \)

- (3,2) 
- (5,1) 
- (-1,1) 
- (5,6)

10. Write the equation for the circle shown below.

11. Write the equation for the circle shown below.

12. Write the equation for the circle shown below.

Answers:  
1. Center=(2, -1) Radius=5  
2. Center=(3, 3) Radius=4  
3. Center=(−3,4) Radius=1  
4. Center=(2, 2) Radius=2  
5. Center=(1, −3), Radius=5  
6. Center=(−3, 4) Radius=4  
7. Center=(0, 4) Radius=3  
8. Center=(0, 0) Radius=4  
9. Center=(2, 2) Radius=3  
10. Center=(2, 3) Radius=2  
11. Center=(−3, −4) Radius=2  
12. Center=(3, 1), Radius=4
Day 2 Warm-Up

1a. Find $a$ in the figure below.
1b. Find $b$ in the figure below.
1c. Find $c$ in the figure below.

2. Graph

$$x^2 + (y - 4)^2 = 4$$
Day 2 Classwork and Homework: If You’re Given Points on a Circle

Distance Formula: \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)

Find the distance between (3, 0) and (-5, 6).

Midpoint Formula: \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \)

Find the midpoint of the segment whose endpoints are (3, 0) and (-5, 6).

Ex 1. If you’re given THE CENTER AND A POINT …

\[ (13, -10) \quad \text{and} \quad (16, -6) \]

1. Find the radius using distance formula
2. Write the equation using center and radius

Ex 2. If you’re given TWO POINTS…

\[ (-7, 0) \quad \text{and} \quad (5, 16) \]

1. Find the center using midpoint formula
2. Find the radius using distance formula
3. Write the equation using center and radius

Write the equations of these circles in standard form.

Your Turn 3. Center: (-5,10) and point on circle: (-14,10)

Your Turn 4. Endpoints of diameter: (-10,5) and (16,-1)
5. Center: (12,4) and point on circle: (13,5)  
6. Endpoints of diameter: (6,3) and (-12,3)

7. Center: (9,-15) and point on circle: (10,-18)  
8. Endpoints of diameter: (-4, -4) and (12, -16)

9. Center: (-17, 15) and point on circle: (-15, 15)  
10. Endpoints of diameter: (-9, -7) and (-1, -1)

11. Center: (8, 3) and point on circle: (3, 11)  
12. Endpoints of diameter: (-15, 10) and (-13, 8)

13. Center: (-6, 0) and point on circle: (1, 9)  
14. Endpoints of diameter: (-10, -12) and (10, -4)

**Answers:**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.</td>
<td>(x + 5)^2 + (y - 10)^2 = 81</td>
<td>4.</td>
<td>(x - 3)^2 + (y - 2)^2 = 178</td>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
<td>(x + 3)^2 + (y - 3)^2 = 81</td>
<td>7.</td>
<td>(x - 9)^2 + (y + 15)^2 = 10</td>
<td>8.</td>
</tr>
<tr>
<td>9.</td>
<td>(x + 17)^2 + (y - 15)^2 = 4</td>
<td>10.</td>
<td>(x + 5)^2 + (y + 4)^2 = 25</td>
<td>11.</td>
</tr>
<tr>
<td>12.</td>
<td>(x + 14)^2 + (y - 9)^2 = 2</td>
<td>13.</td>
<td>(x + 6)^2 + y^2 = 130</td>
<td>14.</td>
</tr>
</tbody>
</table>
Day 3 Warm-Up

1a. Find a in the figure below.
1b. Find b in the figure below.
1c. Find c in the figure below.

2. Graph

\[(x + 4)^2 + (y - 1)^2 = 4\]
Day 3 Classwork: Graphing Circles Foldable

How to Write Equations for Circles

- **Given the Center and Radius**
- **Given the Center and a Point on the Circle**
- **Given the Ends of the Diameter**
<table>
<thead>
<tr>
<th>The only step:</th>
<th>Example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>The two steps:</td>
<td>Example:</td>
</tr>
<tr>
<td>The three steps:</td>
<td>Example:</td>
</tr>
</tbody>
</table>
Day 3  Classwork and Homework: In-Class Practice

Write the equations for these circles:

1. Center: (3, 2)
   Radius: 2

2. Center: (10, 6)
   Point on Circle: (3, 4)

3. Ends of Diameter: (0, –8) and (14, 6)
### Day 4 Warm-Up

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. What is the radius of this circle? Center: ((-16, 9))</td>
<td></td>
</tr>
<tr>
<td>Point on Circle: ((-18, 11))</td>
<td></td>
</tr>
<tr>
<td>2. What is the radius of this circle? Ends of a diameter: ((12, 1)) and ((2, -7))</td>
<td></td>
</tr>
</tbody>
</table>
Day 4 Classwork: DeltaMath Practice

Determine the equation of the circle graphed below.

![Circle 1 Graph]

Answer: 

Determine the equation of the circle graphed below.

![Circle 2 Graph]

Answer:
Day 5 Warm-Up

<table>
<thead>
<tr>
<th>1. What is the radius of this circle?</th>
<th>2. What is the radius of this circle?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center: ((-12, 11))</td>
<td>Ends of a diameter: ((16, 4)) and ((12, 14))</td>
</tr>
<tr>
<td>Point on Circle: ((-6, 8))</td>
<td></td>
</tr>
</tbody>
</table>
Day 5 Classwork and Homework: Converting from Standard to General Form

Ex 1. Convert this equation from standard form to general conic form.

\[(x - 11)^2 + (y + 5)^2 = 49\]
\[(x - 11)(x - 11) + (y + 5)(y + 5) = 49\]
\[x^2 - 11x - 11x + 121 + ___ + ___ + ___ + ___ = 49\]
Distribute
\[x^2 - ___ + y^2 + ___ + 146 = 49\]
Combine like terms
\[x^2 + y^2 - 22x + 10y + ___ = 0\]
Subtract 49 from both sides

Your Turn 2. Write in general conic form.

\[(x - 14)^2 + (y - 15)^2 = 9\]
\[(x - 14)(x - 14) + (y - 15)(y - 15) = 9\]
\[x^2 - 14x - ___ + ___ + y^2 - ___ - ___ + ___ = 9\]
\[x^2 - ___ + y^2 - ___ + ___ = 9\]
\[x^2 + y^2 - ___ - ___ + ___ = 0\]

Convert the following equations from standard form to general conic form.

3. \[(x - 2)^2 + (y + 12)^2 = 16\]
4. \[(x - 14)^2 + (y + 1)^2 = 13\]

5. \[(x + 1)^2 + (y - 10)^2 = 25\]
6. \[(x - 6)^2 + (y - 15)^2 = 16\]
### Answers:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(x^2 + y^2 - 22x + 10y + 97 = 0)</td>
</tr>
<tr>
<td>2.</td>
<td>(x^2 + y^2 - 28x - 30y + 412 = 0)</td>
</tr>
<tr>
<td>3.</td>
<td>(x^2 + y^2 - 4x + 24y + 132 = 0)</td>
</tr>
<tr>
<td>4.</td>
<td>(x^2 + y^2 - 28x + 2y + 184 = 0)</td>
</tr>
<tr>
<td>5.</td>
<td>(x^2 + y^2 + 2x - 20y + 76 = 0)</td>
</tr>
<tr>
<td>6.</td>
<td>(x^2 + y^2 - 12x - 30y + 245 = 0)</td>
</tr>
<tr>
<td>7.</td>
<td>(x^2 + y^2 - 8x + 32y + 270 = 0)</td>
</tr>
<tr>
<td>8.</td>
<td>(x^2 + y^2 - 24y + 128 = 0)</td>
</tr>
<tr>
<td>9.</td>
<td>(x^2 + y^2 - 16x + 14y + 49 = 0)</td>
</tr>
<tr>
<td>10.</td>
<td>(x^2 + y^2 + 8x - 26y + 169 = 0)</td>
</tr>
</tbody>
</table>
# Day 6 Warm-Up

<table>
<thead>
<tr>
<th>1. This is a question from 9th grade! Factor: $x^2 + 8x + 15$</th>
<th>2. This is a question from Friday. Convert to general conic form. $$(x + 10)^2 + (y - 15)^2 = 9$$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x + ____)(x + ____)$</td>
<td></td>
</tr>
</tbody>
</table>
Day 6 Classwork and Homework: Completing the Square

The goal is to create a perfect square.

\[ x^2 + 18x + 81 \] is called a perfect square because it factors into

\[ (\underline{\quad})(\underline{\quad}) = (\underline{\quad})^2 \]

Notice the 2\textsuperscript{nd} term (18) divided by 2 and then squared gives you the 3\textsuperscript{rd} term.

\[ \left(\frac{18}{2}\right)^2 = 9^2 = 81 \]

Fill in the blank to make perfect squares using the pattern \( \left(\frac{2\text{nd term}}{2}\right)^2 \)

1. \( x^2 + 20x + \underline{\quad} \)
2. \( x^2 - 12x + \underline{\quad} \)
3. \( x^2 + 14x + \underline{\quad} \)
4. \( x^2 + 34x + \underline{\quad} \)

<table>
<thead>
<tr>
<th>Standard Form of a Circle</th>
<th>General Conic Form of a Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x - h)^2 + (y - k)^2 = r^2)</td>
<td>(x^2 + y^2 + Dx + Ey + F = 0)</td>
</tr>
</tbody>
</table>

Ex 1. Rewrite the equation in standard form.

\[ x^2 + y^2 - 6x + 12y + 20 = 0 \]

\[ (x^2 - 6x + \underline{\quad}) + (y^2 + 12y + \underline{\quad}) = -20 + \underline{\quad} + \underline{\quad} \]

\[ (x - \underline{\quad})(x - \underline{\quad}) + (y + \underline{\quad})(y + \underline{\quad}) = \underline{\quad} \]

\[ (x - \underline{\quad})^2 + (y + \underline{\quad})^2 = \underline{\quad} \]

Center \((\underline{\quad}, \underline{\quad})\) \(r = \underline{\quad}\)

2. Rewrite in standard form:

\[ x^2 + y^2 + 2x + 6y = 26 \]

\[ (x^2 + 2x + 1) + (y^2 + 6y + \underline{\quad}) = 26 + 1 + \underline{\quad} \]

\[ (x + \underline{\quad})(x + \underline{\quad}) + (y + \underline{\quad})(y + \underline{\quad}) = \underline{\quad} \]

\[ (x + \underline{\quad})^2 + (y + \underline{\quad})^2 = 36 \]
3. Find the center:

\[ x^2 + y^2 + 10x - 16y + 88 = 0 \]

\[
(x^2 + 10x + \_\_) + (y^2 - 16y + \_\_) = -88 + 25 + 64
\]

\[
(x + \_\_)(x + \_\_) + (y - \_\_)(y - \_\_) = \_
\]

\[
(x + \_\_)^2 + (y - \_\_)^2 = \_
\]

Center: (\_, \_)

4. Convert to standard form:

\[ x^2 + y^2 - 6x + 20y + 28 = 0 \]

\[
(x^2 - \_\_x + \_\_) + (y^2 + \_\_y + \_\_) = -28 + \_\_ + \_
\]

\[
(x - \_\_)(x - \_\_) + (y + \_\_)(y + \_\_) = \_
\]

\[
(x - \_\_)^2 + (y + \_\_)^2 = \_
\]

5. Rewrite in standard form:

\[ x^2 + y^2 - 4x - 12y + 31 = 0 \]

6. Find the center:

\[ x^2 + y^2 - 14x + 26y + 217 = 0 \]

7. Convert to standard form:

\[ x^2 + y^2 + 6x + 8y - 96 = 0 \]
8. Rewrite in standard form:
   \[ x^2 + y^2 - 10x + 20y + 109 = 0 \]

9. Find the center:
   \[ x^2 + y^2 + 30y + 209 = 0 \]

10. Convert to standard form:
    \[ x^2 + y^2 - 14x + 2y - 14 = 0 \]

11. Find the radius:
    \[ x^2 + y^2 + 10x + 8y - 80 = 0 \]

**Answers:**

2. \((x + 1)^2 + (y + 3)^2 = 36\)

3. \((-5, 8)\)

4. \((x - 3)^2 + (y + 10)^2 = 81\)

5. \((x - 2)^2 + (y - 6)^2 = 9\)

6. \((7, -13)\)

7. \((x + 3)^2 + (y + 4)^2 = 121\)

8. \((x - 5)^2 + (y + 10)^2 = 16\)

9. \((0, -15)\)

10. \((x - 7)^2 + (y + 1)^2 = 64\)

11. \((x + 5)^2 + (y + 4)^2 = 121\)
Day 7 Warm-Up

1. Convert to general conic form.

\[(x + 7)^2 + (y + 13)^2 = 25\]
Day 7 Classwork and Homework: DeltaMath Practice

Determine the center and radius of the following circle equation:

\[ x^2 + y^2 - 20x - 10y + 44 = 0 \]

Center: (\underline{\hspace{2cm}}, \underline{\hspace{2cm}})

Radius: \underline{\hspace{2cm}}
## Day 8 Warm-Up

1. Convert to standard form.
   \[ x^2 + y^2 - 20x + 22y + 205 = 0 \]

2. Convert to standard form.
   \[ x^2 + y^2 + 14x - 4y + 17 = 0 \]
Day 8 Classwork: Circles Review on Mini-Dry Erase Boards

Identify the center and radius of each. Then sketch the graph.

1) \((x - 4)^2 + (y + 1)^2 = 4\)
   
   ![Graph of circle with center (4, -1) and radius 2](image)

2) \(x^2 + y^2 + 2x + 4y + 4 = 0\)
   
   ![Graph of circle with center (-1, -2) and radius 1](image)

Use the information provided to write the standard form equation of each circle.

3) \(x^2 + y^2 - 16x + 16y + 79 = 0\)
   \((x - 8)^2 + (y + 8)^2 = 49\)

4) \(x^2 + y^2 - 30x - 32y + 472 = 0\)
   \((x - 15)^2 + (y - 16)^2 = 9\)

5) \((x + 4)^2 + (y + 2)^2 = 1\)

6) Center: \((-4, -14)\)
   Point on Circle: \((-6, -17)\)
   \((x + 4)^2 + (y + 14)^2 = 13\)
7) Center: \((-3, 1)\)
   Point on Circle: \((6, 6)\)
   
   \[(x + 3)^2 + (y - 1)^2 = 106\]

8) Ends of a diameter: \((4, 6)\) and \((6, 16)\)

   \[(x - 5)^2 + (y - 11)^2 = 26\]

9) Ends of a diameter: \((-11, -4)\) and \((-7, -10)\)

   \[(x + 9)^2 + (y + 7)^2 = 13\]
Day 8 Homework: Test Review

1. Graph the circle.
   
   \[(x - 4)^2 + (y + 4)^2 = 9\]

2. Graph the circle.
   
   \[x^2 + y^2 + 2x + 6y + 6 = 0\]

3. Convert to standard form.
   
   \[x^2 + y^2 + 22x + 8y + 121 = 0\]

4. Convert to standard form.
   
   \[x^2 + y^2 - 10x + 30y + 246 = 0\]

5. Write the equation in standard form.

6. Write the equation in standard form.

   Center: \((-9, 12)\)
   
   Point on Circle: \((-9, 5)\)
7. Write the equation in standard form.
   Ends of a diameter: (10, 17) and (18, 9)

8. Convert to general conic form.
   \((x + 2)^2 + (y - 11)^2 = 36\)

9. Explain how the equation for a circle is related to the Pythagorean Theorem.
   Including a diagram will be helpful.

Key: 1. Center is (4, -4) and radius is 3  
   2. Center is (-1, -3) and radius is 2  
   3. \((x + 11)^2 + (y + 4)^2 = 16\)  
   4. \((x - 5)^2 + (y + 15)^2 = 4\)  
   5. \((x - 2)^2 + y^2 = 16\)  
   6. \((x + 9)^2 + (y - 12)^2 = 49\)  
   7. \((x - 14)^2 + (y - 13)^2 = 32\)  
   8. \(x^2 + y^2 + 4x - 22y + 89 = 0\)
Appendix N

STUDENT SURVEY OPEN-ITEM RESPONSES
Responses from the Inquiry-Based Group on Arc Length and Sector Area

“no”
“they where [sic] good”
“they helped some”
“Most group activities made my thinking process less limited, as it allowed me to hear direct input from another persons [sic] cognitive process of each question and assignment.”
“3-5 practice problems IN CLASS, no homework!”
“small group activities “
“homework did not help much because it was easy work and i got it in class enough.”
“i liked the group assignments”
“I did not like switching partners.”
“No During the unit we had formulas on the board and we were not provided them for the test.”
“The Circle and Radians helped a lot”
“Well, we did do a crafting activity and lots of practice work. I still need help in some areas, but these things kinda [sic] helped.”
“All the activities that i [sic] was here for helped a lot”
“They helped a lot!”
“Some of the worksheets on solving perimeter and area helped.”
“The group assignments barely helped me.”
“The ones where we had to come up with as many possible equations helped a ton.”
“Nope”
“I dont [sic] understand some of the work”
“the group work really helped a lot”
“The circle plate, it helped me”
“They helped a lil [sic]”

“Working with other students helped”

“hands on activities”

“Doing projects, like the one with the plates and pipe cleaners to help us understand how radii makes up a circle and how that connects to the radian, helped me a lot. Both in remembering what a radian is, and how to find it.”

“The circle plates and working in small groups helped”

“They were fun but i didnt [sic] understand”

“They really help me, understand the situation better, and it really helped me improve your skill better”

“we did the sector of length and area the learned about the clock a little we also did the pizza idea many of times”

“Some of the activities really helped put the standard into my head while others I just found pointless, though doing a variety of activities is good as it helps all of the students in the class to learn.”

“When we were able to just talk among the people around us about problems and work more in smaller groups rather than just following along with the teacher, it was so much better.

“I like doing the worksheets and activities, and then getting personal help with a teacher to help me work them out/figure out why I'm wrong after we start trying to do it on our own. And the part where we try to work it before instruction is nice because it really gets the mind working.”

“The circle with the radius and the different colors around it. It didn't help me”

“I sort of remember key ideas”

“Group activities helped”

“the activities we really helped me with understand”

“It's way too all over the place and I dont [sic] understand it”
Responses from the Explicit Group on Arc Length and Sector Area

“I believe having the teacher go over a few practice problems in class helped me understand the work better.”

“I dont [sic] like groups, so the worksheets and teacher explanation was much more useful. But, I haven’t done the 2nd part of the experiment so we shall see.”

“we just did worksheets”

“Writing down the formulas really helped me”

“yes [sic] they helped me and him going over every second he could also helped.”

“talk ab it”

“Well I didn’t know the formulas or how to set up or even solve a proportion with it, also what the heck is this thing: θ”

“Yeah the helped me by burning it in to [sic] my skull”

“there hard.”

“it helped me”

“A great deal”

“Y’all did great activities. I just don’t think we should do computer work like the delta math thing. USA test prep is fine tho [sic]”

“They were fun and eventful. I liked it and it helped me to understand the formulas and how to do easier formulas then on the milestone formula key.”

“They did”

“They Helped”

“Where we changed radians and degrees was a helpful way of learning the beginning of the problems”

“watching it be done and explained”

“foldables helped to look back at”

“We did stuff that helped me.”
“Nothing”
“I liked working with partners to find out the different ways it could work or not.”
“I got alot of practise [sic] from the activities that we did and it helpd [sic] me a lot”
“ummmm i [sic] felt confident”
“the way the tout the class”
“foldables. they personally helped me”
“it helped a lot”
“a circle is 360”
Responses from the Inquiry-Based Group on Graphing Circles

“they helped me memerize [sic] the problems better”

“We worked in groups and it really helped me understand other peoples [sic] ideas on the problems in math.”

“Working in groups helped a lot”

“They were literally just ‘discuss with your partner’”

“Working with a partner helped”

“They helped me remember them”

“Yes they helped me a lot”

“Yes he wrote them on the board an [sic] it helped me remember them more.”

“The actual going through of the problems helped a lot but the groups of students didn’t really do much for me personally, i’m [sic] sure it worked out for other students maybe.”

“no activities”

“Teaching other students how to do the things helped me memorize them.”

“When we worked with a partner to solve a problem.”

“We got little to no help.”

“They helped, but I did it a little different but got the same answer a simpler way.”

“examples”

“We did a lot of group assignments that helped me remember.”

“i [sic] liked being able to reuse papers everyday to look back on.”

“When you give us a paper to do and i can see what i did wrong when you go over it”

“Yes, Doing certain [sic] activities helped me remember important things I needed to know and it was helpful when other students explained how they done there [sic] work to see another way of how to do the work insteat [sic] of how we were taught.”

“i liked working in groups or with the person beside me”
“its [sic] better for me now”

“Solving circles / graphing them”

“we did a lot of practice questions and the teachers explained to us how to do it”

“Partners went over some other students work”

“Didn't Help.”

“i like working with students that are smarter than me”

“Working with other students helped”

“Yes they helped”

“No Comment.”

“Learning how to solve double problems”
Responses from the Explicit Group on Graphing Circles

“group”

“Everything we did was a great help to me and for my peers.”

"learning different methods helped alot [sic]”

“The teacher tell what to do and then have a small practice sheet for class not at home no one likes home work [sic] and it doesn't get done it's a waste of time so have your students donut n class!!!”

“All of them helped”

“The Groups weren't as helpful to me but if i got to choose who I was with I think it would help more”

“no”

“I like working independently.”

“I don't know”

“The teacher stood up and explained it all. This did help me to remember some key ideas.”

“The circle project helped me remember”

“Yeah it was ok.”

“Doing the class work helped a bunch and having people to work with During the arc lengths helped too”

“working together”

“working on my own helped me the most.”

“Group work did not help me at all.”

“Yep, namely the practice work drilled the equations into my brain.”

“no comments”

“I liked working in groups because it made me do my work and listen to my friends tell me how to do it”
“Explaining the warm ups was alot [sic] of help”
“Circle graphing”
“extra problems to do on my own.”
“Lots of homework wasnt [sic] helpful. more time in class and groups were better for me.”
“they helped a lot”
“the work papers”
“They helped a lot”

“When we started, we were shown the equation for graphing a circle with no context what-so-ever, but then Mr. Sizemore broke down and explained the parts of the equation for graphing a circle. That what made it easy for me to understand. There was also a lot of practice work, but that made it easy for me to memorize the formulas for finding the center, radius, converting to standard form, general conic form, etc.”
“didnt really help. i [sic] need like 1 on 1 help because i feel dumb when talk infront [sic] of people and get it wrong.”
“idk”
“Doing worksheets and the foldables”
“Distance fourmula [sic] converting equations”
“They somewhat gave a basic understanding of the unit.”
“the activities we did really helped”
“Without being shown enough on how to do it like there wasn't enough examples shown for me to really understand well enough.”
“they were very helpful”