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DESIGNING VISUALLY INTERACTIVE LEARNING MODULES TO PROMOTE STUDENTS' CRITICAL THINKING IN MATHEMATICS

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DESIGNING VISUALLY INTERACTIVE LEARNING MODULES TO PROMOTE STUDENTS' CRITICAL THINKING IN MATHEMATICS

A Thesis Presented to

The Faculty of the Computer Science Department

by

Linda Vu

In Partial Fulfillment

of Requirements for the Degree

Master of Computer Science

Kennesaw State University 2019.06

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ABSTRACT

This Innovative Practice Full Paper presents our study of promoting critical thinking and achievement in mathematics with the use of technology via easily accessible resources. We believe that creating easy-to-use learning modules that are accessible for students everywhere is a great way to help students comprehend abstract logic process in problem solving. More specifically, we developed a series of learning modules that interactively visualize and illustrate the steps of Geometry proofs by using GeoGebra, a dynamic learning platform for mathematics [5, 10]. Survey study shows that our interactive learning modules are beneficial in promoting students' learning interests and ability of critical thinking.

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CHAPTER I: Introduction

There are times in everyone's life where they have wanted to raise their hands to ask for assistance. This happens a lot when something becomes too difficult for people to manage. Imagine, when you did not have that helping hand. Did you give up? Did you keep trying? Or did you seek other resources? According to the National Center for Children in Poverty, "…45 percent live in low-income families and approximately one in every five (22 percent) live in poor families" [12]. There are about 72 million children in the United States that live in a low social economic status (LSES) background [24]. There are several researches about how living in a low-social economic background has an impact on a student's academic success [13][27]. Focusing on the area of academic success in mathematics, a research showed that "high levels of families in LSES situations have a negative relation to the mathematics achievement" [9]. This study stated that there was a lack of resources/support for LSES students that was so essential for students to succeed. Some of the essential resources include involvement from the parent, motivation, and concrete resources such as books, internet and technology [9]. Not only do students in LSES homes have a lack of resources, but so do the schools they attend.

According to the U.S. Department of Education, "all young people should be prepared to think deeply and to think well so that they have the chance to become the innovators, educators, researchers, and leaders who can solve the most pressing challenges facing our nation and our world, both today and tomorrow" [5]. However, about one third of Americans say they are not good at math, although over ninety percent of Americans admitted that math is important to be successful [19]. The anxiety that Americans have about math is problematic. Why do people find math difficult? People find math difficult because math is hard to relate to, it is too abstract, it is too tedious, and any negative stigmatism about math pasted down by past generations.

Given that critical thinking in math is essential to success in STEM disciplines, the goal of our study is to develop widely accessible computing-based learning modules to promote problem solving skills in math. One of the most important characteristics of our learning modules is that they should be able to divide a complex problem into multiple smaller problems and to demonstrate an abstract math process in intuitive and easy-to-follow steps. In this paper, we will report our development of interactive learning modules on high school Geometry.

Another important goal that we are pursuing is that all our learning modules should be widely accessible. First, all learning modules will be able for access through Internet. However, given that there are still certain percentages of homes that don't have Internet access [18], we are also in the process of installing our learning modules on donated used tablets and mobile devices and donating those devices to public libraries so that students can check them out for access.

The rest of the paper will be organized in the following ways. The second section explains related research that have been successful in increasing student performance and assessment scores. Several of the research stated that it was able to help students and people across the country. In section 3, we describe our design of interactive learning modules on high-school Geometry in details and we report our survey study that demonstrates the effectiveness of our learning modules. Section 4 describes the research and the survey methodology. Finally, we conclude our paper and briefly mention some future work in section 5, Conclusion and Future Works.

CHAPTER II: Related Research

The trend for learning math online did not grow until YouTubers posted online math help online. One YouTuber, Sal Khan, posted math help online back in 2006. Sal's goal was to help his family across the country. This is what lead towards the development of Khan Academy. Now there are over 365 million videos and over 1.8 billion solved math problems [22].
The National Council of Teachers of Mathematics believe that, "All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding [23]." NCTM states that being able to apply and understanding mathematics is an important factor in the work force. If we want students to be able to apply and understand mathematics than to give them a free resource to discover, visualize and understand mathematics. When students start to take ownership of their learning the more successful they will be in that content. According to research in the NCTM article, students become more confident when they solve a difficult problem on their own [24]. Students would more likely feel accomplished and successful when they take ownership of their own learning. The importance of students being able to manipulate and come up with their own conclusions is the foundation to critical thinking and problem solving. We believe that students being able to have assistive interactive resources at home would benefit students with their critical thinking and problem-solving skills.

Technology in the classroom today has been a growing topic for several years. There are several countries like Lithuania, Austria, Germany, Sweden and several other countries that are trying to integrate GeoGebra into the classroom [6, 25]. Several research studies believe that students need could be met by integrating interactive learning into the classroom [10, 20, 14]. By using technology to discover and learning mathematics, students would be able to see how to

solve different problems in new ways [10]. Mathematics is abstract which makes it difficult to visualize and understand. However, by using technology to help visualize the concepts it would be able to close the gap of abstract thinking. This way students would be able to discover and learn concepts they are able to see. Being able to visualize difficult topics could cause students to discover, analyze, evaluate and question the solution for themselves. We believe that this will allow students to practice critical thinking and problem solving. Technology also gives student, the opportunity to discover and problem solve at their own pace. Just as a reminder, technology should not be a replacement for educators, they should only be used as an assistant to help facilitate learning.

GeoGebra is a dynamic mathematics software that is free to download on any device or use through an internet browser [8]. This dynamic software helps teachers create, program, and model several abstract concepts such as proofs. The benefit of using GeoGebra is that it is free, assessible, innovative, open source and user friendly. After GeoGebra has been downloaded onto a device it does not require internet, but only occasionally for additional resources such as workbooks. GeoGebra is a great program that allows students and teachers to be more creative and be able to visualize mathematics. There is also an expansion of GeoGebra where you can script some of the GeoGebra workbook. The disadvantages of using the software is creating lesson are time consuming and there will be a learning period for both teachers and students. The target of Khan academy are students that are in grade levels K through 12. Several teachers and schools have already implemented it into their classroom. With this Khan Academy focuses on providing students a better-quality product and learning experience through research and studies they conduct. In 2011, Khan Academy want to expand their videos and help sessions on fill-in any gaps that with parallel with the Common Core mathematics standards; setting a goal for Fall

of 2014. They were able to achieve this by having eight schools and over 50 teachers implement this into their classroom/school for about two years [22]. What the teachers and schools were able to find while students enriched their learning with Khan Academy was teachers were able to organize their curriculum and content more effectively, teachers wanted to be able to customize the problems and tutorials, allow students to work at their own pace, and teachers were able to filter the class by student level and assign problems accordingly. Another interesting factor was that teachers found was that the videos and the problem set were geared towards their age group, which kept them engaged in the topic and helped the students stay focused. This research found that 48% of the teachers that served in LSES communities were able to move more quickly through their curriculum. The benefits that Khan Academy had for the students are the enjoyment of interacting with the hardware, students liked the engaging features such as games, students liked the immediate feedback, hints and access to videos as they needed help, and the application giving the students a sense of ownership and control over their learning [22]. About 80%of the teachers believed that Khan Academy had a positive impact on the students conceptual understanding of math. These are all very important factors to education and learning. When students can say that they learned it themselves by something they did is when the learning truly begins.

Another online and interactive medium that has partnered with the college board and many schools is Khan Academy. Khan Academy, run by Salman Khan, has over ten million different users as of February 2014. This online resource started with helping people online with math and science concepts for students learning on their own [26]. A research study, run by Khan Academy, took this idea more ways to make learning more interactive with different learning approaches. Khan Academy's goal was to make something teachers and students

globally would be able to use. They believe that teachers need better quality information and content to provide to their students to get better results from the students in terms of learning. Khan Academy was able to run several studies for a couple years to perfect their implementation and the content they provided. They were able to take on student and teacher feedback and made modifications based on their feedback of which they thought would be successful. Khan Academy believes that everyone has a right to a good education and that is the type of product they wanted to produce. This non-profit company has provided over 3,500 videos solely on math and about over 100,000 practice math problems for people to practice. Math has been one of Khan Academy's most popular videos and concepts to look over, due to the challenges people may have on math. Truly, math is something that should be practiced constantly, otherwise, it is lost. People use Khan Academy primarily for the following: learning new math skills, filling in gaps in conceptual understanding, tracking and monitoring school work, spending more time with peer teaching, self-study, and allowing teachers more time with students [26].

SRI Education tried the study with nine different schools that all have unique needs and backgrounds. One of the schools, where about 45% of the students were eligible for free-reduced lunch went through the self-paced learning model [26]. The student was done with ninth and tenth graders for two years in math. The students had 22% of their instruction time using Khan Academy for the first year. In the second year they made sure that the students were solely self-paced and self-directed in their academics. However, they split the learning period into teacher directed instruction and the second hour to student directed learning. For the second hour teacher's set-up around the room as facilitators [26]. The results of this study were successful, it helped students develop content knowledge, academic skills, and critical non-cognitive skills. Other skills the students were able to gain was being able to set learning goals and holding

themselves accountable for their education. Khan Academy was structured enough for the students, for them to be successful in math [26]. Most of the feedback Khan Academy received was positive. Teachers liked technology being used in their class as an extension of learning, students were able to be more independent because it was more self-paced. Khan Academy provided an alternative method of learning with a lot more accountability for the students [26]. Another study, also using Khan Academy, controlled one of the groups by limited the access to internet [15]. They found that with the group that did not have internet in the classroom had been affected more than the students with internet. They found that students that did not use internet had an eight-point increase on scores and with internet only had a six-point increase. Even though this score may seem small, keep in mind that this study was only for a short period of time [15]. We believe that the most important part of this study is the resource the learning is coming from.

Overall, technology can have a positive effect on teacher instruction as well as student learning. We believe that Khan Academy has great intentions and the right idea on students becoming independent learners. Providing more content for teachers to use to enable this will help education for the future success of students globally.

CHAPTER III: Interactive Learning Modules

The core task of this project is to design interactive learning modules that visually presents critic concepts and the proof processes of high school Geometry, such that abstract math concepts and processes can be perceived on a step by step manner. We use the software GeoGebra [5, 10] to build our learning modules. The ultimate goal of this task is to cover all knowledge points that are included in the State Geometry Standards. Our design is highly modulized such that a learning module may include component modules for constructing concepts or processes and this learning module may be a building block for a bigger learning module. Figure 1 shows an example module that we built to demonstrate proof that triangle DEM and triangle ABM are similar.

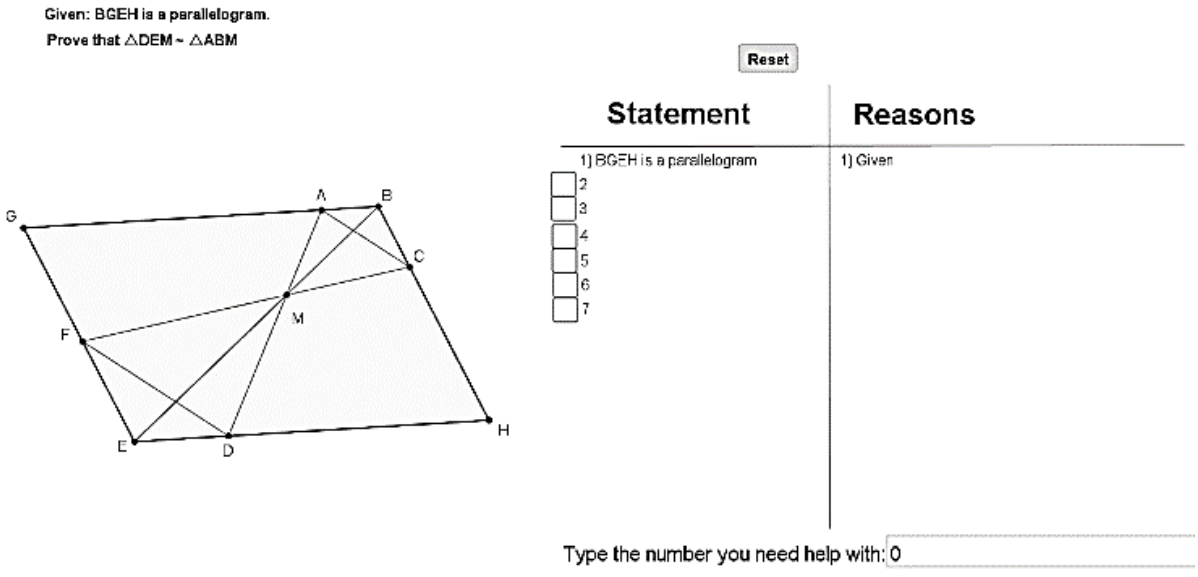

Figure 1: Triangle Similar Module 1

When working on this module, students will see a two-column proof process that demonstrates proof statements and their reasons step-by-step as shown in Figure 1(b). Students can explore

this learning module in two ways: 1) Using the check boxes to show the statement and reason for the next line, as shown in Figure 2; and 2) Getting help on any step by inputting the number of that step as shown in Figure 3

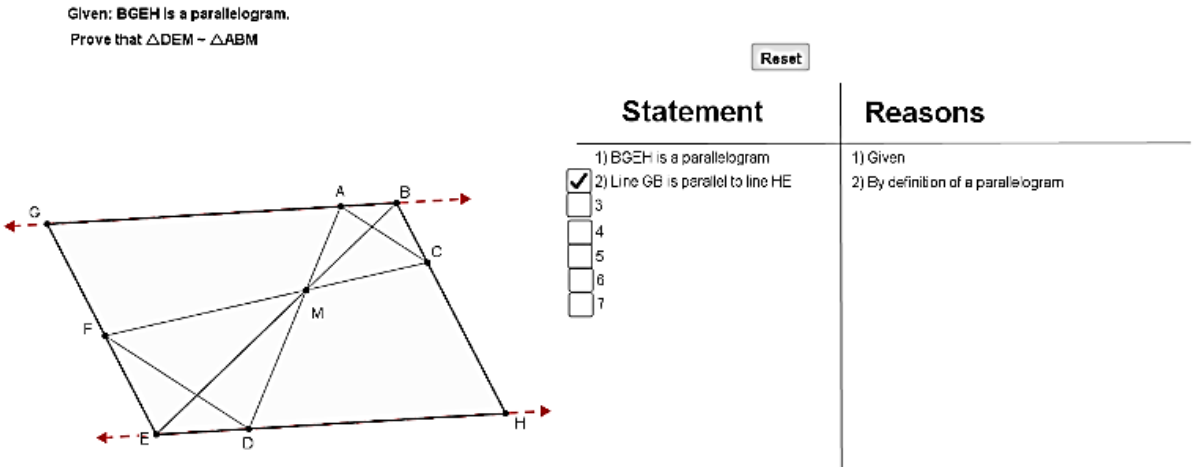

Figure 2: Interactively learn each step of the proof process; check the next step

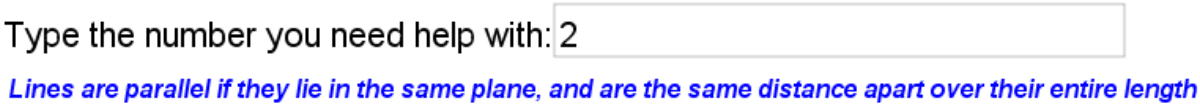

Figure 3: Interactively learn each step of the proof process; choose a step to get help

Figure 4 shows what it looks like as the student progresses through the proof. If the student wants to start over and try to do the problem again themselves, they can reset the module clicking the 'Reset' button. The great thing about the reset option is that students could retry the problem themselves until they understand or see the whole picture. After the students have worked out what was given and what could be inferred based on those givens, this would be where new concepts come in and understanding the definitions of shapes, theorems, and postulates. Figure 5, shows one way to solve for the proof.

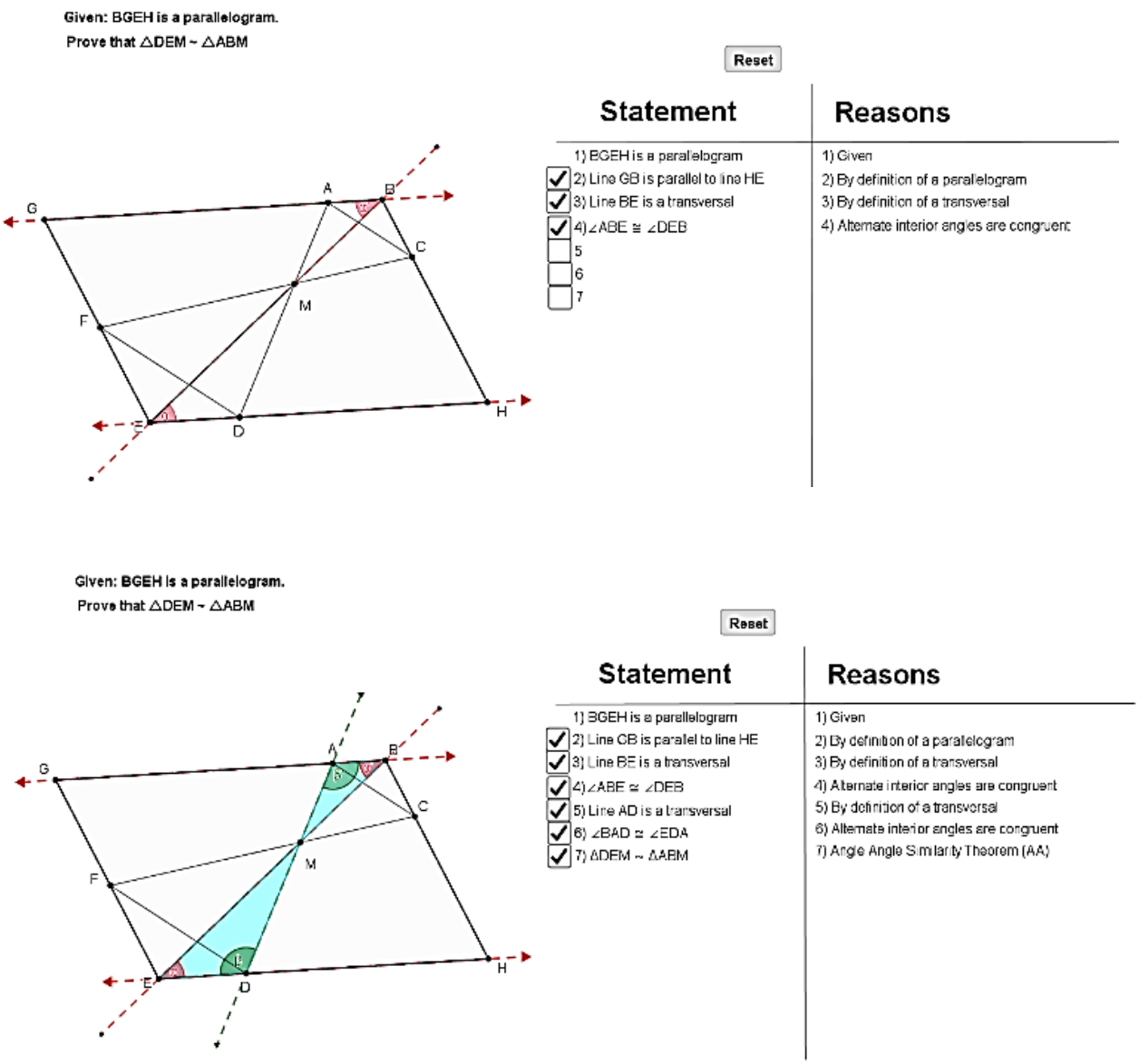

Figure 4: Progress Through the Proof

Because of the modulized design, a student who needs to review certain definitions/concepts used in the current learning module can refer to a component module to learn that specific concept. For example, if the student had a problem with understanding what a parallelogram is (the second step of the current module), he or she could open the corresponding component module on parallelogram (as shown in Figure 4) and explore this particular concept.

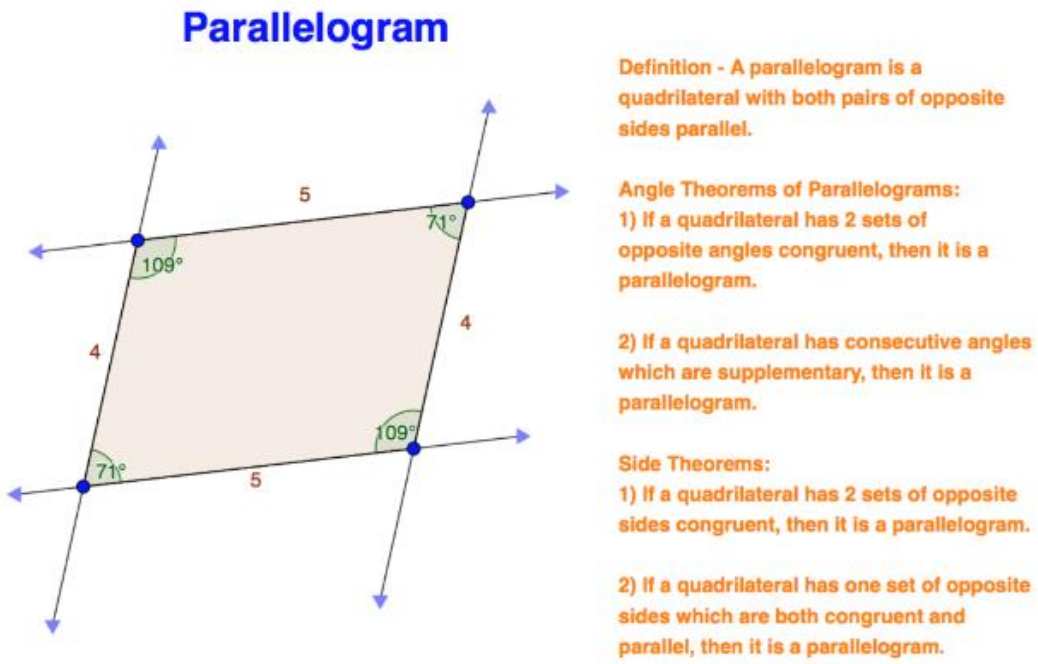

Figure 5: Component Module on Parallelogram

This current learning module that demonstrates the proof that triangle ABC and triangle DEF are similar can be further used as a building block for an extension problem. For instance, Figure 6 demonstrates an extension module where the conclusion of the current learning module (i.e., $\Delta DEM \sim \Delta ABM$) is used as the second step to prove that $\Delta ABC \sim \Delta DEF$ given BGEH is a parallelogram. Our modulized design also helps to train students using divide and conquer strategy to solve complex problems.

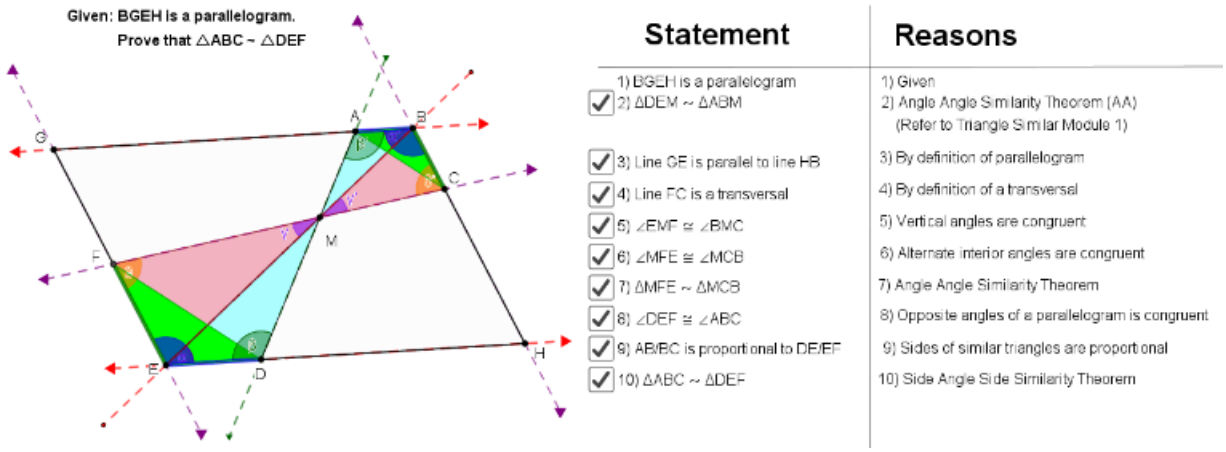

Figure 6: Triangle Similar Module 1 as One Step in a Bigger Problem

CHAPTER IV: Survey Study

The populations of our survey study are Georgia public school students in tenth-grade Geometry. The learning module of the extension problem that is described in section 2, as well as all its component modules were given to 80 students who are currently enrolled at a public high school in Georgia. These 80 students were separated by class level: Honors Geometry and Geometry. The goal of the survey study was to see how our modules would help the students in understanding the logic steps of Geometry proof without an instructor. The following survey was given to students to critique.

Lesson Survey

Date: _____ **Subject**: _____ **Period:** _____

Age: _____ **Gender (Circle one):** Female / Male **Grade Level:** _____

1. What did we learn about in this lesson? _____

_____

_____

Questions	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
I was interested in this topic BEFORE we started to study it.					
I am interested in this topic AFTER having studied it.					
I liked how this information was presented through the application.					
The application helped me visualize and understand proofs.					
Technology was a useful and important part of this lesson.					
I would be able to follow the steps of the lessons on my own.					
The lesson was easy to follow.					
The lesson was easy for me to understand.					
I will remember what I learned in this lesson.					
I would like to do more lessons using the application.					
I learn best while using technology.					
I was engaged while using the application.					
Do you own a computer or laptop at home?	Circle one: Yes or No				
Do you have access to a phone or tablet?	Circle one: Yes or No				
Do you have access to the internet at home?	Circle one: Yes or No				

6. Think about the different parts of this lesson, and answer the two questions below.

a) What is one thing that you liked about this lesson? _____

_____

_____

b) What's one thing that you did NOT like about this lesson? _____

_____

_____

7. How can I make this lesson better in the future? _____

_____

_____

8. Any other comments or questions about this lesson? _____

_____

Figure 7: Student Survey Questions

The survey data (shown in table 1) collected from students who used our learning modules shows that the modules were beneficial to a noticeable extent. For instance, the learning modules helped to promote students' interests in the subject. The data showed a shift in percentage when it came to the question of interestingness after and before the lesson. Most of the answers shifted from one rating higher than the previous under the level of interest in the topic. More specifically, the data shifted from 21 out of the 80 students strongly disagreeing to only 10 strongly disagreeing; shifted from 26 out of the 80 disagreeing to only 15 disagreeing; and shifted from 4 out of 80 agreeing to 15 agreeing.

Second, the learning modules helped to promote critical thinking in an intuitive way. Data shows that the majority 72% of the students liked how the information was presented through the applications; 79% of the students thought that the application helped them visualize and understand the proofs; and 67% of the students thought that technology was a useful and important part of this lesson.

Furthermore, students can easily follow the learning modules without extra instructions. 65% of students agreed or strongly agreed that "I was able to follow the steps of the lesson on my own", and 68% thought the modules were easy to follow. Figure 7, further presents the percentages of what the students answered from a five-point scale from strongly disagree to strongly agree.

Table 1: Aggregated Survey Data

QUESTIONS	STRONGLY DISAGREE–	DISAGREE–	NEUTRAL–	AGREE–	STRONGLY AGREE–	TOTAL number of students
Percent	27%	33%	31%	5%	4%	
I was interested in this topic BEFORE we studied it	21	26	24	4	3	78
Percent	13%	19%	45%	19%	4%	
I am interested in this topic AFTER we studied it	10	15	35	15	3	78
Percent	1%	8%	18%	49%	23%	
I liked how this information was presented through the application	1	6	14	38	18	77
Percent	3%	4%	14%	53%	26%	
The application helped me visualize and understand proofs	2	3	11	40	20	76
Percent	3%	7%	24%	39%	28%	
Technology was a useful and important part of this lesson	2	5	18	30	21	76
Percent	1%	8%	25%	51%	14%	
I was able to follow the steps of the lesson on my own	1	6	19	39	11	76
Percent	3%	12%	18%	52%	16%	
The lesson was easy to follow	2	9	14	40	12	77
Percent	1%	7%	25%	43%	24%	
The lesson was easy for me to understand	1	5	19	33	18	76
Percent	0%	9%	37%	41%	12%	
I will remember what I learned in this lesson	0	7	28	31	9	75
Percent	7%	16%	28%	39%	11%	
I will remember what I learned using the application	5	12	21	29	8	75
Percent	7%	16%	29%	40%	8%	
I learn best while using technology	5	12	22	30	6	75
Percent	4%	13%	31%	44%	8%	
I was engaged while using the application	3	9	22	31	6	71

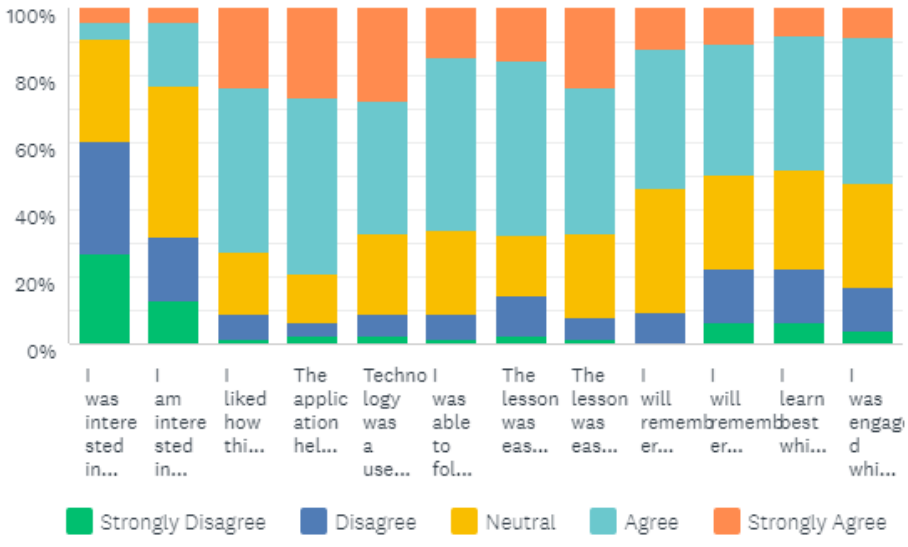

Figure 8: Bar Chart of Survey Results

CHAPTER V: Future work and Conclusion

We developed a series of interactive learning modules to promote logic thinking in high-school Geometry study by following the state standards. Our survey study shows that the learning modules are beneficial in promoting students' learning interests and enhancing students' comprehension of critical concepts and proof processes. Our future work includes completing our development of all learning modules and broadening our survey study by involving more schools with higher diversity and social economic backgrounds. Examples of diversity are low social economic with students that require free or reduced lunches, how does it affect girls and boys differently and level of math. Other future works we would like to provide are pre- and post-assessments to be able to analyze impact of using technology or not using technology. This way we would be able to have a controlled group and a non-controlled group. We also want to figure out a way to measure critical thinking skills which could be combine with daily assessments, homework or class discussions either online or in the classroom. We would also like to add on an introductory class to inform students on how to use GeoGebra to decrease the amount of confusion. Besides making our learning modules publicly available via Internet, we also plan to install our learning modules on donated used tablets and mobile devices and donate them to public libraries and schools so that students who have the need can easily access the technology.

Recently, in January 2019, we had an opportunity to observe more students in a tenth grade Geometry class at the same high school. This class was a small group Geometry class that consist of students with learning disabilities. Student in this course had a vast number of learning disabilities from processing speeds and not being able to write on their own. GeoGebra may not be an option for all students, however, we believe that it could help students with learning disabilities.

Whether it is to understand what a shape is or what it is through numbers and discovery. Students could also draw and construct the shape required by the curriculum. GeoGebra has endless possibilities for students. We believe that there should be a resource where students and teachers could easily access and use daily. Also, technology-based learning seems to be a trending topic. Just like Khan Academy, developing a way for students to be able to take ownership for their learning and be able to have control over their learning will have a positive impact on their success. Math seems to be a difficult topic for several people, but if they had a self-paced learning and discovery model. This would improve the conceptual understanding much better.

We believe that there could be more improvements in this study, in terms of the length of the study and the number of classes. With the amount of time we were given and the results that we got, we believe that GeoGebra and the modules would be a great resource for teachers and students around the world. Although, there could be additions such as more learning modules of different mathematical concepts and videos to provide an ease of use. One of the studies mentioned above on Khan Academy were able to get students to be self-paced and independent learners. If we can give students some sort of feedback to encourage the student of their success, it would encourage independent learning.

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