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Journal of Emerging Knowledge on Emerging Markets
Volume 2
November 2010

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This paper has two parts. In the first part, I articulate the short but significant evolution of the study of fractals. In the second part, I discuss the application of fractals to business models and business analytics.

"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line"(Mandelbrot, 1982, pg. 1). Thus began the study of fractals on a large scale. Fractals in their most common sense are nothing but interesting images. To scientists, however, they represent a fairly untrodden field in mathematics. Until the 1970s, fractals had been identified but not studied due to the lack of adequate technology. With the invention of the computer, however, fractals have been gaining increasing popularity among everyone from artists to businesspeople.

Background Information: What is a Fractal?

A fractal is an object whose topological dimension differs from its capacity dimension, where the topological dimension is defined as the minimum value of n, such that any open cover[1] has a refinement[2] in which no point is included in more than $n+1$ elements

[1] If Y is a subset of X and if C is a collection of subsets of X whose union contains Y, then C is said to be a cover of Y. If the set X is a topological space, we say that C is an open cover if each of its

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DOI: 10.7885/1946-651X.1020

(Weisstein, 2006). Fractals are found in a number of places in nature – from clouds to snowflakes to vascular systems. Complex adaptive systems are subject to the laws of nature, which themselves rest on the fundamental physical laws of matter and the universe. Moreover, of all the physical situations permitted by those laws, only specific conditions permit complex adaptive systems to exist (Gouyet, 1996). So, what are the regularities (values approaching either infinity or zero), and where do accidents and the arbitrary (values at critical region) enter in?

To study this new field of science, the manner in which scientists studied various phenomena had to change – i.e., the methods used had to be updated. Previously, the major practice of study was through the reduction of a whole into its composite parts, but the increasingly popular means of discovery and invention is through complexity, the antithesis of reduction. The idea of complexity is represented in DeMarco's Law: The complexity of a document is proportional to the number of fingers that you need to read it. That is, simple events can generate complex outcomes that could not be predicted by the study of the individual components of the whole. For example, by examining a single human cell, the scientist would not be able to tell that it would be able to operate with other human cells to create the human body. Complexity is thought of as the place between order and randomness – i.e., what is on the edge of chaos. Complexity is the region where sufficient chaos for novelty and creativity exists but where adequate order for consistency and patterns also exists. The arbitrary exists in this area of complexity. Complexity is related to fractals in the sense that fractals create forms that possess regions on "the edge of chaos." Complexity is illustrated below in Figure 1 and will be discussed further in relation to fractals later.

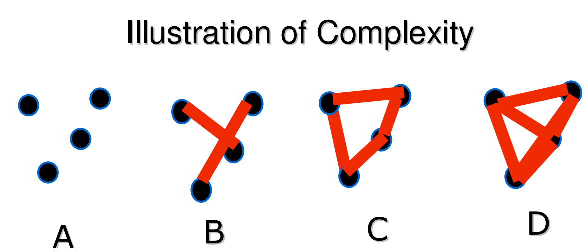

Figure 1 (created by Priya Roy): Looking at images A-D, which of the images is the most complex? *A* has no connections, *B* has two, *C* has four, and *D* has all connections. Thus, *A* & *D* and *B* & *C* are complements. Most will agree that *A* is less complex than *B*, but is *C* more complex than *B* because it has more lines? Is *D* really complex because it all possible connections, or is it just as simple as *A*, which has none?

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members is an open set (i.e. each U_α is contained in T, where T is the topology on X) (Weisstein, 2006).

[2] A refinement of a cover C of X is a new cover D of X such that every set in D is contained in some set in C (Weisstein, 2006).

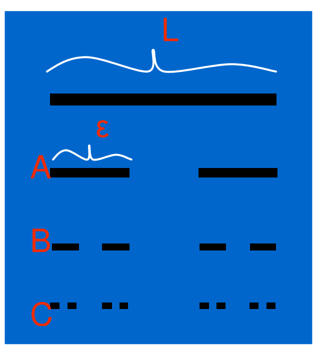
Among the various properties of fractals are the following. First, fractals must exhibit self-similarity, where self-similarity is the characteristic with which an image will look the same at whatever magnification applicable. This self-similarity need not be exact but only approximate for fractals. Secondly, fractals must have infinite complexity. For example, the coastline of Great Britain has an infinite degree of complexity because as the level of magnification increases, the degree of complexity increases as well. Assuming that infinite magnification were physically possible, the viewer would see more and more curves and crevices along the coastline. Finally, a fractal must have a power law characterization. That is, on a two-dimensional plane of log(quantity) vs. log(scale), the resulting graph would be a straight line where the slope of that line is the fractal's dimension.

A fractal's dimension, termed "fractal" or "capacity" dimension, is usually denoted by $d_{capacity}$. The real number $d_{capacity,}$ such that if n(ε) denotes the minimum number of open sets of diameter = ε, then n(ε) is proportional to ε^{-D} as ε approaches zero, equals the following:

$$d_{capacity} = - \lim_{\varepsilon \to 0} \left[ln(N) / ln(\varepsilon) \right] \quad \text{where}$$

N = number of elements forming a finite cover of the relevant metric space and
ε = size of each element used to cover set. See Figure 2 below for an illustrative example of capacity dimension.

Capacity Dimension Example: Cantor Set



$$d = - ln\,[2^{100}] / ln[1 / 3^{100}]$$
$$\approx .63$$

	N(s)	ε
A	2	L/3
B	4	L/9
C	8	L/27
D	16	L/81
N	2^N	$L / 3^N$

Figure 2 (created by Priya Roy): Assume an interval of [0,1]. To create the Cantor Set, divide a line segment of the given interval into three parts and remove the middle third. L = length of the original line segment. ε = length of each segment after each iteration. The table illustrates the relationship between the number of segments and the size of each segment. Using the aforementioned equation for capacity dimension, $d_{capacity}$ is found to be approximately .63, where the value of 100 is just an arbitrarily large number that is canceled out in the equation but inserted in this Figure for clarity. Thus, the capacity dimension of the Cantor Set is a rational one, rather than the expected whole number of non-fractal forms.

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DOI: 10.7885/1946-651X.1020

There exist a number of means for generating fractals. The three most common are as follows. First, for a fixed geometric replacement rule, Iterated Function Systems (IFSs) are used. The Cantor Set in Figure 2 is an example of an IFS. Secondly, Escape-Time Fractals can illustrate a recurrence relation at each point in space. Finally, for stochastic rather than deterministic processes, Random Fractals are generated (Elert, 2006).

For clarification, the two meanings of the term "random" need to be discussed. Using a string of bits, the first meaning is that the string is incompressible – that is, it is so irregular that no way can be found to express it in shorter form. Alternatively, this string of bits could have been generated by a random process. The former is a stochastic process, and the latter a deterministic one. If the string of bits had been generated by a random process, then it could be incompressible, somewhat incompressible, or completely compressible (i.e., not random at all). If something is produced using the latter meaning, then it is not complex. For example, television "snow" is random, but it possesses no qualities of complexity, for its random placement of pixels is rather simple – the probability of a pixel being chosen for snow is equal for all pixels and, thus, a simple process. Figure 3 below illustrates an IFS using the Sierpinski Gasket, a fractal formed quite simply using any initial shape.

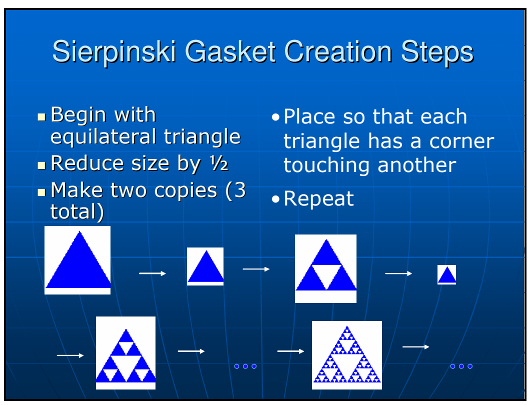
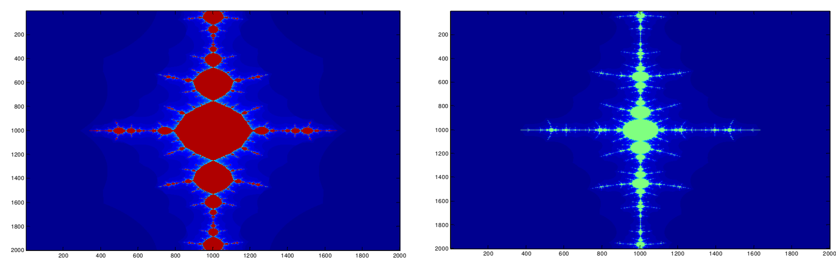


Figure 3 (created by Priya Roy): The scaling factor is ε = ½, and N(S) = 3 is the number of segments per triangle. The area of this figure approaches zero, and its $d_{capacity} \approx 1.585$. This process illustrates how a simple iteration algorithm can create an infinitely complex figure.

History: How Has Technology Helped Shaped the Study of Fractals?

Before computers, fractals had been discovered but not studied. While their existence was noted, no researcher had the means of pursuing the field further. The first major instance of fractals' existence being acknowledged was when British surveyors attempted to measure the length of Great Britain's coastline. With a map of a small magnification, the coastline appeared to be one number – say, for instance, 5000 units. With a map of a greater magnification, however, it appeared to be a greater number – e.g., 8000 units. Thus, as the magnification increased, so did the coastline's length. This characteristic illustrates the second property of fractals – namely, infinite complexity. A finite area, Great Britain, is surrounded by an infinite perimeter, the coastline.

Later, Gaston Julia (1893-1978), a French mathematician studied what the effects of a complex polynomial function would be. Using the complex plane, he iterated the function $y = x^2$ as z: $z \rightarrow z^2 + c$, where $z = x + iy$, c is a constant, and i is a complex number equaling $\sqrt{1}$ (Hypertextbook). With computers, using the number of iterations for the graph of the function to pass its orbit, a color can be assigned to each pixel (x,y). Before the computer, however, Julia had to rely on much more simple plots. See Figure 4 below for an example of a Julia Set produced using the color algorithm.



(-1.312,0) (-1.375,0)

Figure 4 (created by Priya Roy): These two images were created using the program MATLAB (which is discussed in the following paragraph). The coordinates indicate (c_1, c_2), and c is determined by grid location. The initial z-value was zero. The colors indicate how many iterations were required to approach infinity. Dark blue indicates z took ≥1000 iterations to become >2. Light blue indicates close to 1000 iterations to become >2 (along a gradient). Orange indicates relatively few iterations. Red indicates that always <2). The behavior of these two Julia Sets illustrates the chaotic behavior of fractals. The sensitivity to initial conditions is shown because the coordinates of each graph are relatively similar to one another, yet the graphs are so dissimilar.

Finally, the researcher who ultimately set the study of fractals forward was Benoit Mandelbrot (1924-), an employee at IBM. He was the first to use computers to study fractals, and he capitalized on Julia's studies earlier that century. Using the same function $y = x^2$ and the same iteration z: $z \rightarrow z^2 + c$, he set the initial z-value to zero but let c be the pixel. So, his method was slightly different from Julia's but similar enough that the Mandelbrot Set contains Julia Sets in its most complex parts (Elert, 2006). See Figure 5 below for an illustration. If the iteration yields $z = 2$, then the value stays constant. If $z > 2$, then the values go off to infinity. Finally, if $z < 2$, then the value approaches zero.

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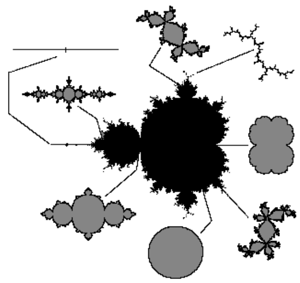
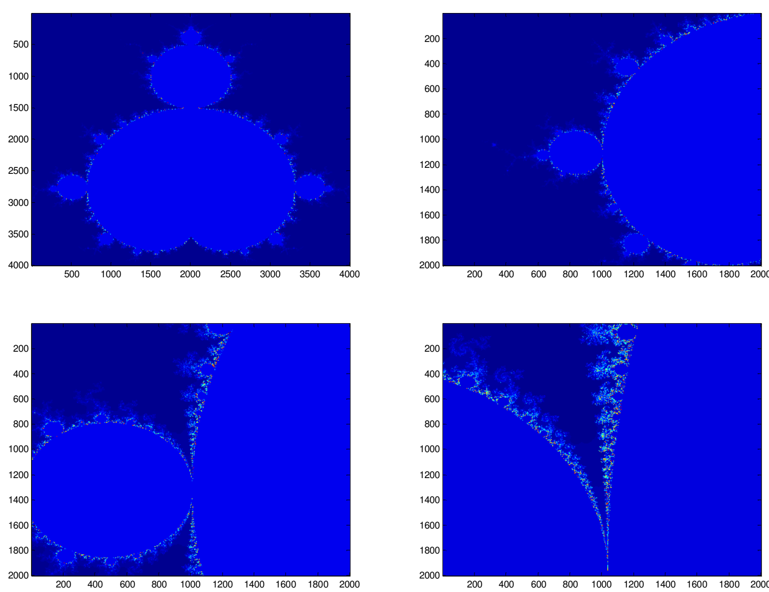


Figure 5 (from www.hypertextbook.com/choas): The Mandelbrot Set with various Julia Sets at the critical locations along it.

Due to the non-integer dimensions of many of the fractals, the visualization of fractals prior to the use of computers was rather difficult. With programs such as MATLAB and Mathematica, researchers are now able to not only calculate but also graph fractals. Specifically looking at the generation of fractals using IFSs, researchers had to do iterations by hand (or perhaps with the aid of an unsophisticated calculating device), but with the advent of computers, scientists and mathematicians are now able to calculate large numbers of iterations, usually up to a few thousand. Below is a series of zoom-ins on the Mandelbrot Set using MATLAB.



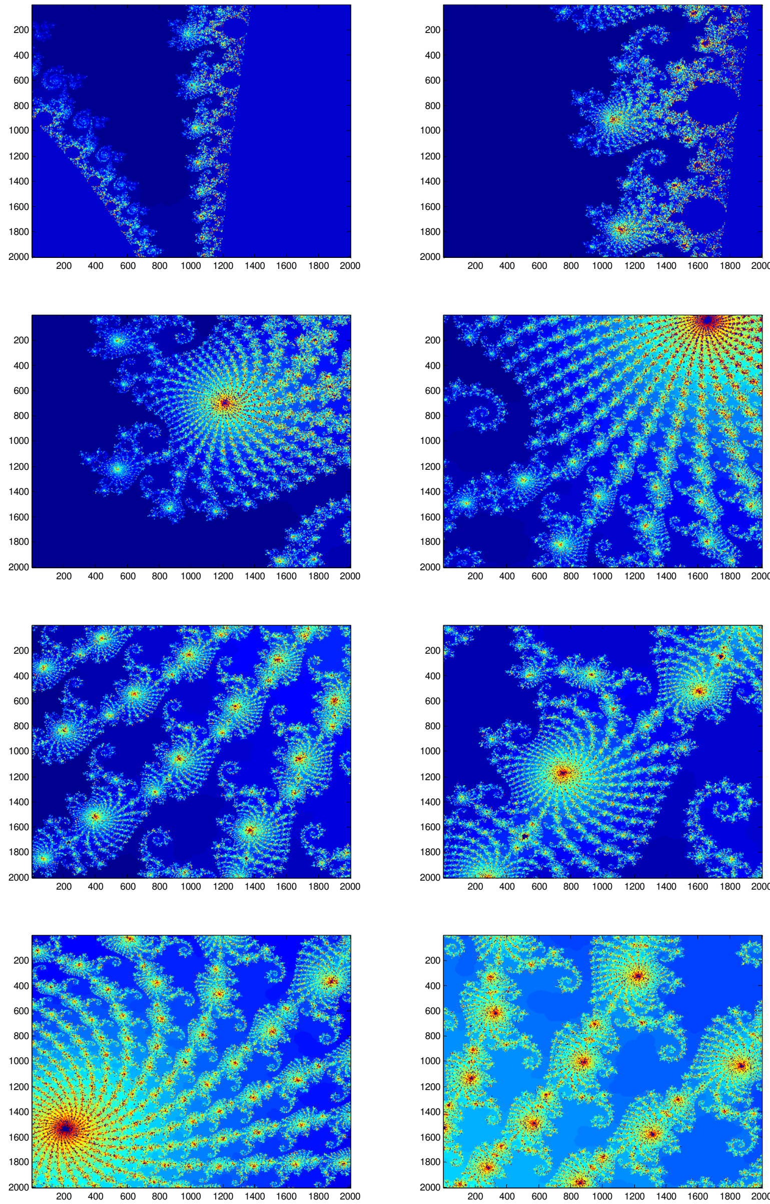


Figure 6 (created by Priya Roy): This is a generation of the Mandelbrot Set using MATLAB. The sequence goes from left to right and downwards. Not only does it illustrate the self-similarity of fractals discussed earlier, but it also shows what computers have done to facilitate the study of fractals. Now, with computers, a scientist is able to detail an IFS using various initial values. The first image has no zoom. It is merely the Mandelbrot Set with a gridsize of 2000. The zoom is 10^{power} where power is the interval [.25, 5.00] using steps of 0.5.

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Future: What are the Shortcomings of Current Computer Programs?

While the computer has offered scientists an easier, more effective, and more efficient route to the study of fractals, there remain many shortcomings. As shown in Figure 6, even with a gridsize of 2000, the clarity of the images is poor. In addition, the program that produced these images had to be left running overnight in order to have sufficient time to process and render these images. Further, the program cannot do more than a few thousand repetitions, even when left overnight, because of temporary memory issues. With all of these issues, many researchers are turning to C++ as an alternative to MATLAB or Mathematica. This transition may be a difficult one, however, since the researchers will have to learn a new programming language, rewrite all of their programs, and potentially lose years of labor, but once the transition is made, the length of time required for a program should be shorter and the programs should be able to be of greater complexity and volume.

Application of Fractals to Business Models and Business Analytics

The explanation of fractals given in this paper can be easily applied to add value to business analytics and models.

The fractal paradigm (loosely defined as a form that can be broken into smaller self-similar forms across the whole) can be used at various levels of flexible business process and software systems in an organization. Although the use of fractals in business could be found Lamarckian in style, fractals can help the organizational leadership optimize gargantuan flows of volatility, content and pattern toward the core mission of a department and/or the organization; this is because business problems contain Hamletian organized patterns as part of the larger business systems. The Cisco-Apple fractal business model (i.e., to innovate internally and scale across entire value chain) is the polar opposite of the Sony-Philips-Samsung non-fractal business model (i.e., innovations lack the ability to set standards). In the future, organizations will increasingly use fractals as a catalyst to identify/align market opportunities faster in order to adjust the opportunity cost of their business models (for mergers and acquisitions, products, customers, etc.).

Trading is another area where fractals can be profitably utilized. In trading, the amalgam of volume and price action creates proximate support and resistance with proliferation of information. Fractals, by dealing with real time, can apply the Occam's razor to reduce the Rubik's cube of complexity of the problem-trading situation. Organizations use models based on assumptions to guide their decisions in the capital markets. Profitability depends on, inter alia, how well the model aligns with market gyrations. Benoit Mandelbrot has cogently argued that the movement of market prices has much in common with Levy processes. The implications being that Black Swans are more common than conventional statistics may otherwise depict, and also that price series are similar at any given scale. The advantage of using fractal models is that by taking into account factors such as political events and expectations, such models tantamount to greater accuracy than the random walk models. That fact that an increasing number of technical analysis software envisage fractal indicators (via skew, kurtosis, and rescaled range analysis) is a testament to the escalating use of fractal geometry as an alibi in the markets of the future.

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