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Implementation of Range Autofocus for SAR Radar Imaging

Nicholas J. Testin
Kennesaw State University

Philip Davis
Kennesaw State University

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Philip Davis and Nicholas Testin

Kennesaw State University

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Abstract

The range calculation for an FMCW radar depends on accurate linear modulation. In some circumstances, linear modulation may not be available and must be corrected for. This paper describes an autofocus technique used to correct for phase error due to non-linearities in the components of a FMCW radar. Also described here is the algorithm used in calculating the phase error and application of the phase correction with triangle modulation. Known errors were calculated at certain distances and applied to correcting the phase of data taken at similar distances. The results given were generated using a SAR working outside linear ranges.

Keywords: Autofocus, FMCW, Radar, Radar imaging, Radar signal processing, Range processing, Synthetic aperture radar.

Implementation of Range Autofocus for SAR Radar Imaging

Skolnik (2001, p. 185) defines Synthetic Aperture Radar (SAR) as a radar configuration where a single transmit and receive unit are used to simulate a much larger antenna or array, allowing for the collection of high-resolution data perpendicular to the direction of movement (or crossrange). SAR possesses a narrow antenna beamwidth with high resolution and gain, and is excellent for mapping the details of a static environment, due to the lack of physical changes which occur between each data collection point, or “shot”.

A challenge with small radar modules, especially those with a SAR configuration, is to accommodate large bandwidth requirements for high resolution data without increasing transmitter power to unreasonable levels. With a continuous wave radar, a method for pulse compression must be used, where the radar signal is continuously modulated with respect to frequency or phase. By comparing the difference in frequencies between the received signal and the originally transmitted signal, it is argued, one can obtain data which can be analyzed for typical radar properties or measurements (Griffiths 1990 p. 185). Furthermore, by using a long pulse and modulating it so its bandwidth is much greater than the inverse of the pulse length, as well as applying a matched filter to the modulated signal to make the pulse width of the signal approximately equal to the inverse of the bandwidth, it is possible to give the pulse the equivalent bandwidth of a smaller pulse while maintaining the low power requirements necessary for a small module (Skolnik 2001, p. 195). This modulation scheme is known as frequency modulated continuous wave, or FM-CW.

One method of determining a target’s distance away from a radar is to utilize the change in the received signal’s phase with respect to the originally transmitted signal. In an ideal system, the phase will change only with the incrementing “range bins”, the smallest range resolution within which the radar can identify targets. With real-world radar targets, a “phase error” between the expected and actual values of the phase will be present, which will distort the data and introduce artifacts into the processing of the signal. This phase error can be corrected and its effects on measured targets can be reduced. An algorithm for phase correction has been previously described by Grosch (2016, p. 2), and it serves as the basis for the work described here.

The radar module used for these measurements consists of a SAR, modified from the design used by Charvat *et al.* (2011) at the MIT OpenCourseware website. The radar has been modified to transmit a continuous wave at 1.9 GHz, and modulate the frequency of the wave up to 3 GHz. Further revisions by Testin, Davis, *et al.* (2016) to the design include on-board high-sampling digital data logging, linear distance tracking, and automatic data storage. Data collected from this radar is processed by MATLAB code to generate images of reflector targets.

FM-CW and Phase Error

The ideal case for a signal $f(t)$ emitted by a continuous wave radar with linear modulation can be modeled by a cosine function:

$$f(t) = A\cos((\omega + m)t) \quad (1)$$

with A as the amplitude of the signal, m is the modulation index in rad/s, and ω as the frequency in radians/s. When the signal is reflected off a stationary point target, it yields a changed signal $r(t)$ modeled by:

$$r(t) = B\cos((\omega + m)(t + \tau)) \quad (2)$$

where B is the amplitude of the reflected signal (which will have some innate loss) and τ is the round trip delay:

$$\tau = \frac{2r}{c} \tag{3}$$

with r being the range to target and c being the speed of light (Grosch 2017) (Skolnik 2001, p. 195).

A basic FM-CW radar modulates its transmitted waveform by varying its frequency linearly over time: this is known as a "chirp", and the instantaneous frequency of this signal can be modeled as a function of time, given below:

$$\omega(t) = \frac{d}{dt}\theta(t) = mt + \omega_0 \text{ rad/s} \tag{4}$$

with ω_0 as the starting frequency, and m as the modulation rate in rad/s/s. When the transmitted and received signals are mixed, the difference between the frequencies of the two signals can be used to calculate the target range as a function of the two signals' beat frequency (Griffiths 1990 p. 185-186). This beat frequency, f_b, is described as:

$$f_b = \frac{4\Delta f \cdot f_m \cdot R}{c} \tag{5}$$

with Δf being the peak-to-peak frequency deviation. Since c is a constant, and both Δf and f_m are held constant by the modulation scheme, this demonstrates that the beat frequency will increase as a function of range to target (Skolnik 2001 p. 195).

Typically a mixer in an FMCW radar outputs the product of the transmit and receive signals and a low pass filter passes the difference component:

$$\text{Mixer Output} = ABcos(m\tau t) \tag{6}$$

which describes a constant sinusoidal signal of phase rotation, $m\tau$ verses time. From Equation 6, one can determine that the change in phase which occurs is a function of the range to target. When the range to target is known, the ideal phase of the waveform reflected by a target at r range can be calculated: if this ideal phase is then compared to real data, the difference between the ideal phase value and the actual phase of the signal can be referred to as the phase error. The phase error will degrade radar performance, and can result in the production of an "echo" response that will distort the image of the point target (Griffiths 1990 p. 189). An assumption we made here is that the primary source of phase error is due to nonlinearity in the VCO used for the signal's frequency modulation (Grosch 2016 p. 2).

Taking the instantaneous phase of the transmit signal from Equation 4 yields:

$$\theta_t(t) = \frac{1}{2}mt^2 + \omega_0 t + \phi \ \text{rad} \tag{7}$$

where ϕ is a constant phase offset. If a target is present at a range r, and the reflected signal is delayed by a time τ:

$$\theta_t(t) = \frac{1}{2}m(t-\tau)^2 + \omega_0(t-\tau) + \phi \ \text{rad} \tag{8}$$

If a receive mixer is acting as a phase comparator, the mixer outputs the signal as:

$$\theta_r - \theta_t = \theta_\delta = \frac{1}{2}m(2t\tau - \tau^2) + \omega_0\tau \ \text{rad} \tag{9}$$

Rearranging the equation as a function of time and delay:

$$\theta_\delta(t) = \frac{m}{2}\tau^2 + \omega_0\tau + mt\tau \ \text{rad} \tag{10}$$

And also yields the frequency of the baseband signal (Grosch 2016 p. 1-2):

$$\omega_\delta(t) = \frac{d}{dt}\theta_\delta(t) = mt \ \text{rad/s} \tag{11}$$

A nonlinear (i.e. practical) source will introduce error into the ramping frequency. If the error rate is a function of time $\varepsilon(t)$ and the average modulation rate is m_0 , the actual ramping frequency $m(t)$ is modeled by:

$$m(t) = m_0 + \varepsilon(t) = \sum_{i=0}^{\infty} m_i t^i \ \text{rad} \tag{12}$$

With error in modulation present, an error in phase will occur, which consists of the difference between a perfectly modulated signal and a modulated signal with error, expressed by the equation:

$$\theta_\varepsilon(t) = \frac{1}{2}\left[m_0\,(2t\tau - \tau^2) + \sum_{i=1}^{\infty} m_i\big((t-\tau)^{i+2} - t^{i+2}\big)\right] - \omega_0\tau \quad \text{rad} \tag{13}$$

With a constant frequency factor $m_0\, t\tau$ from Equation 10 added, and assumptions that both constant phase is ignored and the round trip delay τ is much less than the time interval t, one can simplify the equation to be:

$$\theta_\varepsilon(t) = t\sum_{i=1}^{\infty} d_i\, t^i \tau \tag{14}$$

where the factor d_k can be found by a least squares approximation. Significantly, this demonstrates that the phase error is a function of distance to target (Grosch 2016 p. 2-3).

However, there are methods of correcting this phase error. By storing the mixed “echo” signal in digital form and performing a Fast Fourier Transform (FFT) on it, the range information is converted to a series of range bins in the frequency domain. By mathematically manipulating this set of range data, it is possible to use a method such as back projection to build a two-dimensional image of the data.

Autofocus

A method to reduce the phase error of the signal created from the nonlinear voltage-to-frequency characteristics of the modulation has been proposed and evaluated by Grosch (2016, p. 2-3). This technique can focus at a given range bin by assuming a point target in that range and unwrapping the phase error at that point.

Assuming an ideal system's signal in response to an ideal point target at range d_c, is $x(t) \xrightarrow{FT} X(k)$ and the actual signal is $s(t) \xrightarrow{FT} S(k)$ then the angle of the error can be calculated by:

$$\theta_e(t) = \text{angle}\left(\frac{s(t)}{x(t)}\right) \tag{15}$$

and can then applied to produce the corrected signal $a(t)$:

$$a(t) = \frac{s(t)}{e^{i\theta_e(t)}} \tag{16}$$

To apply this to every range bin, $\theta_e(r,t)$ (which is a function of range and time) is calculated and applied to each. To generalize the correction for each range bin with a distance d we used previously calculated correction angles at a distance d_c and stepped through the distances.

$$c(t) = e^{i\theta_e(t)\frac{d}{d_c}} \tag{17}$$

$c(t)$ is the correction angles at distance d. If we are working discretely at multiple range bins, then d becomes the range resolution rr multiplied by the current range sample n or

$$c_n(t) = e^{i\theta_e(t)\frac{rr}{d_c}n} \tag{18}$$

where,

$$\text{rr} = \frac{c}{2 * BW} \tag{19}$$

d_c and θ_e are calculated beforehand from data taken for calibration at certain distances. $\theta_e(k)$ is the phase error found at that point. If the correction is applied in the same way as Equation 16:

$$a_n(t) = \frac{s_n(t)}{c_n(t)} \tag{20}$$

then the reflected energy from a target in range bin n is

$$a_n(t) \xrightarrow{FT} A_n(k) \text{ at } k = n \tag{21}$$

Assuming the error produced from the radar is consistent, then using previously generated phase errors saves significant time compared to calculating the phase error for new data.

Applying the Algorithm

The radar was used to collect data in a field with 3 trihedral reflector point targets, each placed along a line 33 feet away from, and approximately parallel with, the linear direction of radar motion. We used a triangle wave as the modulation with a ramp time of 17.23ms. The data was sampled at a rate of 100 KHz using a PSOC 4200M microcontroller, and stored on a SD card. The data was collected over 100 feet with the VCO pushed passed the linear operating range. Appendix A, Fig. 2 shows the image formed without the autofocus algorithm; Appendix A, Fig. 3 shows it with the algorithm applied to the data. A Hamming window was applied to both sets of data, and each set was imaged using RMA (Range Migration Algorithm).

The algorithm shows a noticeable reduction in the amount of clutter visible in the image. The clutter around the point targets is visibly reduced, and the targets themselves more closely resemble points. There is additionally less clutter downrange behind the targets, which, while not a focus of the data, demonstrates an additional use of the autofocusing algorithm. As an additional tool, Appendix A, Fig. 4 compares the FFT of the data before and after the phase correction. We used the typical phase error of a target at 15 meters to correct the phase of these 3 targets. The phase error was first interpolated to account for the new range sample size then for each range bin the phase known phase error is shifted using Equation 18. Appendix A, Fig. 5 shows the phase error calculated and used in the correction. Since 1680 data points were collected, approximately 1.4 million correction calculations had to be performed and applied to the data, which took a measured 41 seconds in MATLAB.

Conclusion

Using phase correction can be an effective method of removing errors induced by nonlinearity of components; however, due to the amount of calculations, generating or applying the corrections becomes a time consuming and computationally intensive task. Further development could be made to allow the algorithm to be deployed on a low-power, low-speed microcontroller for on-the-fly correction.

References

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Appendix A

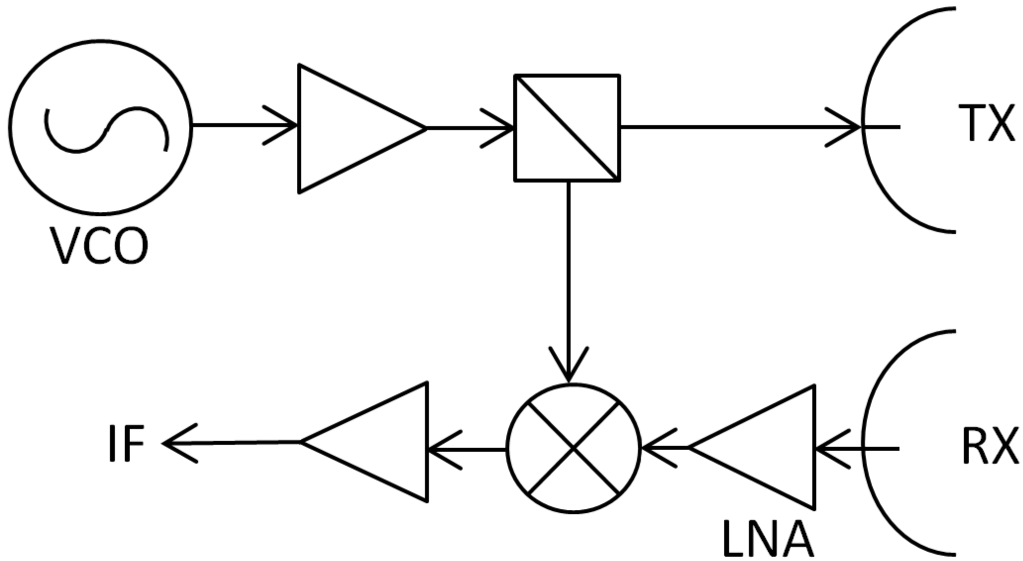

Fig. 1: Block Diagram of FM-CW Radar

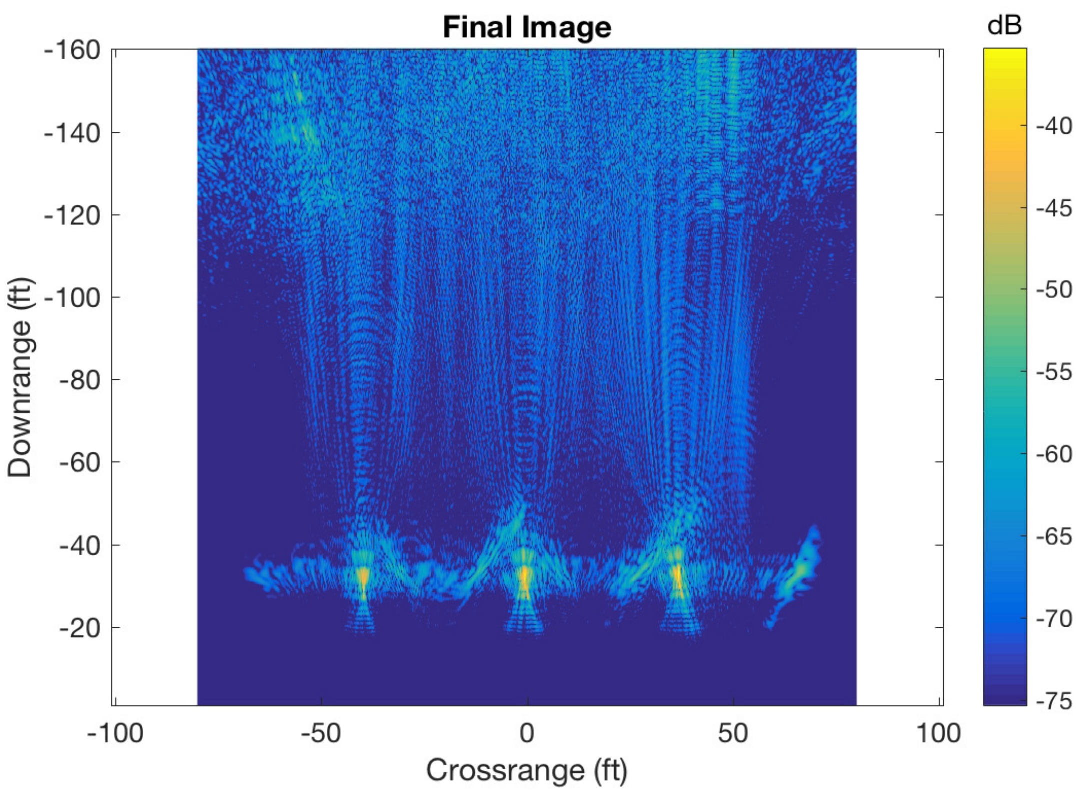

Fig. 2 Image produced with the phase uncorrected

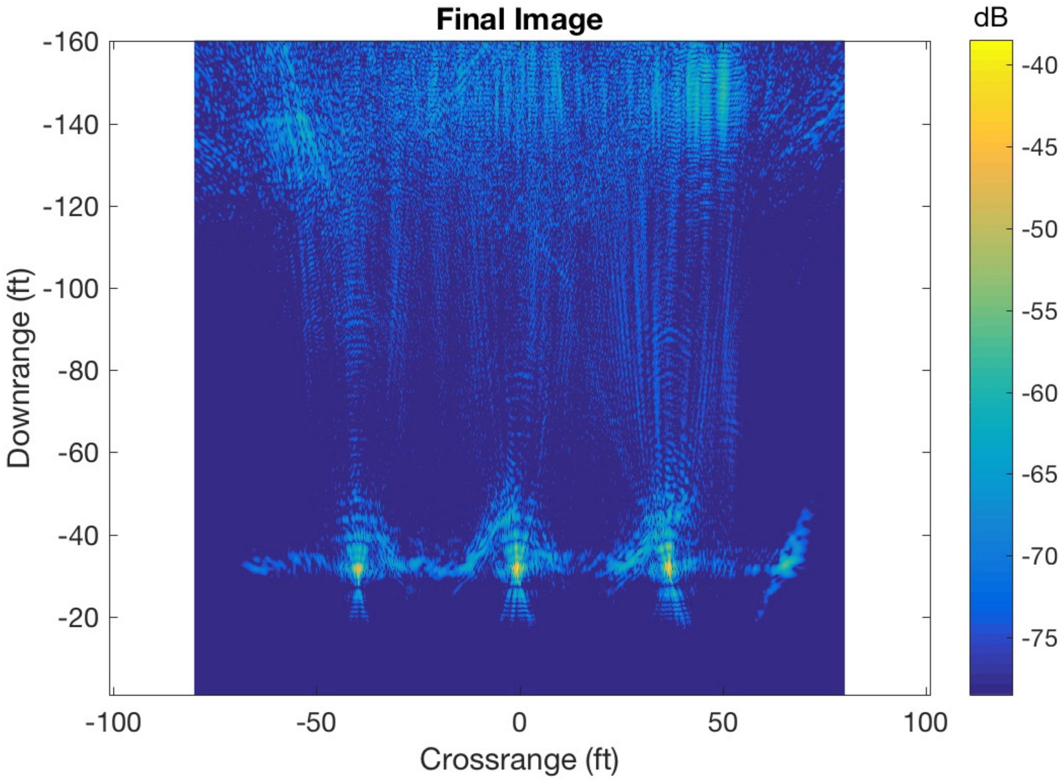

Fig. 3 Image produced with the phase corrected

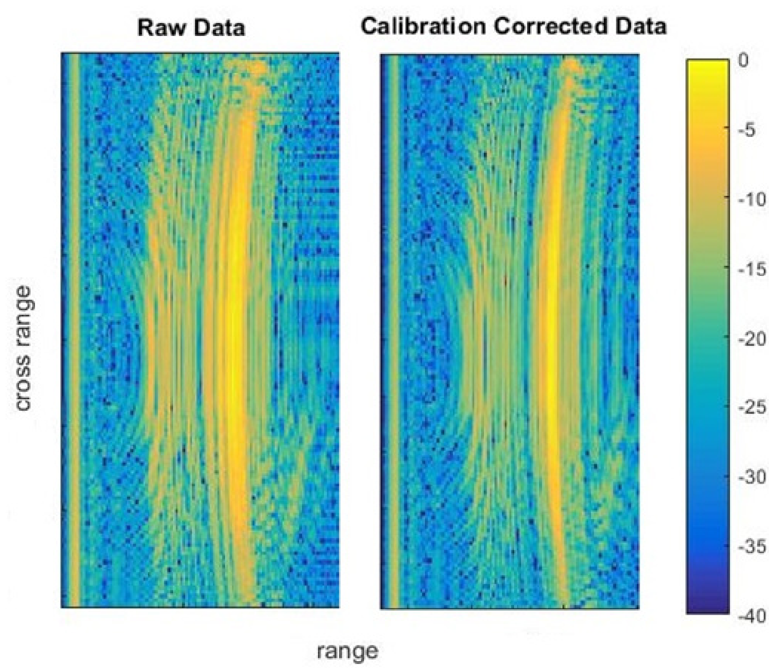

Fig. 4 FFT of the data before and after the phase correction

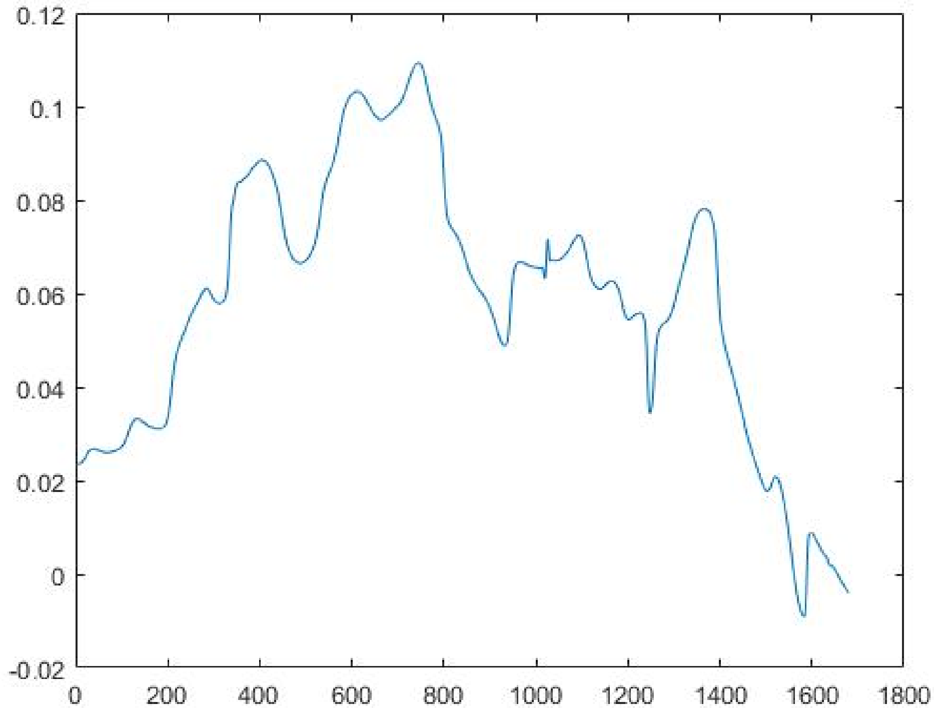

Fig. 5 Phase error at n=1 range bin

Implementation of Range Autofocus for SAR Radar Imaging

Philip Davis and Nicholas Testin, *Student Member, IEEE*

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***Index Terms*—Autofocus, FMCW, Radar, Radar imaging, Radar signal processing, Range processing, Synthetic aperture radar**

I. INTRODUCTION

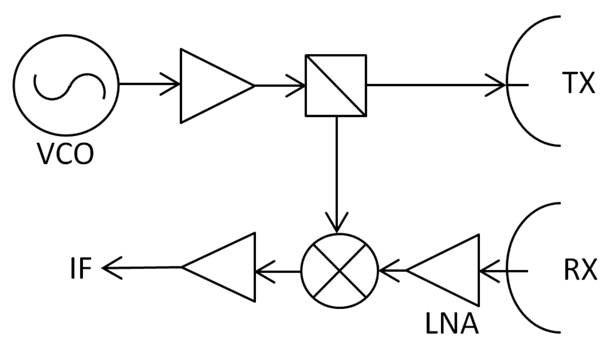
Synthetic Aperture Radar (SAR) is a radar configuration where a single transmit and receive unit are used to simulate a much larger antenna or array, allowing for the collection of high-resolution data perpendicular to the direction of movement (or crossrange). SAR possesses a narrow antenna beamwidth with high resolution and gain, and is excellent for mapping the details of a static environment, due to the lack of physical changes which occur between each data collection point, or "shot" [1].

A challenge with small radar modules, especially those with a SAR configuration, is to accommodate large bandwidth requirements for high resolution data without increasing transmitter power to unreasonable levels. With a continuous wave radar, a method for pulse compression must be used, where the radar signal is continuously modulated with respect to frequency or phase. By comparing the difference in frequencies between the received signal and the originally transmitted signal, one can obtain data which can be analyzed for typical radar properties or measurements [2]. Furthermore, by using a long pulse and modulating it so its bandwidth is much greater than the inverse of the pulse length, as well as applying a matched filter to the modulated signal to make the pulse width of the signal approximately equal to the inverse of the bandwidth, it is possible to give the pulse the equivalent bandwidth of a smaller pulse while maintaining the low power requirements necessary for a small module [1]. This modulation scheme is known as frequency modulated continuous wave, or FM-CW.

One method of determining a target's distance away from a radar is to utilize the change in the received signal's phase with respect to the originally transmitted signal. In an ideal system, the phase will change only with the incrementing "range bins", the smallest range resolution within which the radar can identify targets. With real-world radar targets, a "phase error" between the expected and actual values of the phase will be present, which will distort the data and introduce artifacts into the processing of the signal. This phase error can be corrected and its effects on measured targets can be reduced. An algorithm for phase correction has been previously described, and it serves as the basis for the work described here [3].

The radar module used for these measurements consists of a SAR, modified from the design used by Charvat *et al.* at the MIT OpenCourseware website [4]. The radar has been modified to transmit a continuous wave at 1.9 GHz, and modulate the frequency of the wave up to 3 GHz. Further revisions to the design include on-board high-sampling digital data logging, linear distance tracking, and automatic data storage [5]. Data collected from this radar is processed by MATLAB code to generate images of reflector targets.

II. FM-CW AND PHASE ERROR



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with A as the amplitude of the signal, m is the modulation index in rad/s, and ω as the frequency in radians/s. When the signal is reflected off a stationary point target, it yields a changed signal $r(t)$ modeled by:

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with Δf being the peak-to-peak frequency deviation. Since c is a constant, and both Δf and f_m are held constant by the modulation scheme, this demonstrates that the beat frequency will increase as a function of range to target [1].

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which describes a constant sinusoidal signal of phase rotation, $m\tau$ verses time. From Equation 6, one can determine that the change in phase which occurs is a function of the range to target. When the range to target is known, the ideal phase of the waveform reflected by a target at r range can be calculated: if this ideal phase is then compared to real data, the difference between the ideal phase value and the actual phase of the signal can be referred to as the phase error. The phase error will degrade radar performance, and can result in the production of an "echo" response that will distort the image of the point target [2]. An assumption we made here is that the primary source of phase error is due to nonlinearity in the VCO used for the signal's frequency modulation [3].

Taking the instantaneous phase of the transmit signal from Equation 4 yields:

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where ϕ is a constant phase offset. If a target is present at a range r, and the reflected signal is delayed by a time τ:

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If a receive mixer is acting as a phase comparator, the mixer outputs the signal as:

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Rearranging the equation as a function of time and delay:

$$\theta_\delta(t) = \frac{m}{2}\tau^2 + \omega_0\tau + mt\tau \text{ rad} \quad (10)$$

And also yields the frequency of the baseband signal [3]:

$$\omega_\delta(t) = \frac{d}{dt}\theta_\delta(t) = mt \text{ rad/s} \quad (11)$$

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With a constant frequency factor $m_0 t\tau$ from Equation 10 added, and assumptions that both constant phase is ignored and the round trip delay τ is much less than the time interval t, one can simplify the equation to be:

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However, there are methods of correcting this phase error.

By storing the mixed "echo" signal in digital form and performing a Fast Fourier Transform (FFT) on it, the range information is converted to a series of range bins in the frequency domain. By mathematically manipulating this set of range data, it is possible to use a method such as back projection to build a two-dimensional image of the data.

III. Autofocus

A method to reduce the phase error of the signal created from the nonlinear voltage-to-frequency characteristics of the modulation has been proposed and evaluated by Grosch [3]. This technique can focus at a given range bin by assuming a point target in that range and unwrapping the phase error at that point.

Assuming an ideal system's signal in response to an ideal point target at range d_c, is $x(t) \xrightarrow{FT} X(k)$ and the actual signal is $s(t) \xrightarrow{FT} S(k)$ then the angle of the error can be calculated by:

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then the reflected energy from a target in range bin n is:

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Assuming the error produced from the radar is consistent, then using previously generated phase errors saves significant time compared to calculating the phase error for new data.

IV. Applying the Algorithm

The radar was used to collect data in a field with 3 trihedral reflector point targets, each placed along a line 33 feet away from, and approximately parallel with, the linear direction of radar motion. We used a triangle wave as the modulation with a ramp time of 17.23ms. The data was sampled at a rate of 100 KHz using a PSOC 4200M microcontroller, and stored on a SD card. The data was collected over 100 feet with the VCO pushed passed the linear operating range. Fig. 2 shows the image formed without the autofocus algorithm; Fig. 3 shows it with the algorithm applied to the data. A Hamming window was applied to both sets of data, and each set was imaged using RMA (Range Migration Algorithm).

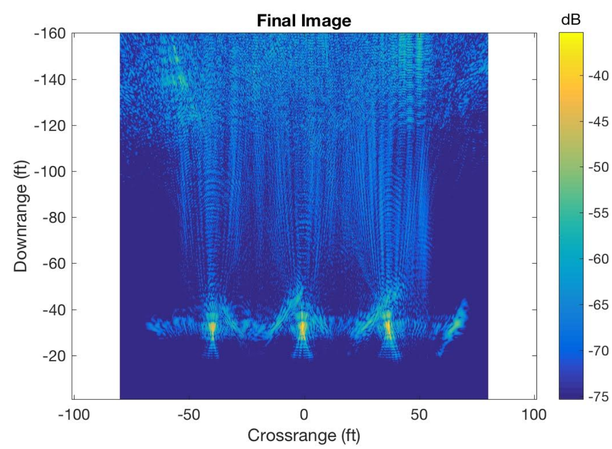

Fig. 2 Image produced with the phase uncorrected

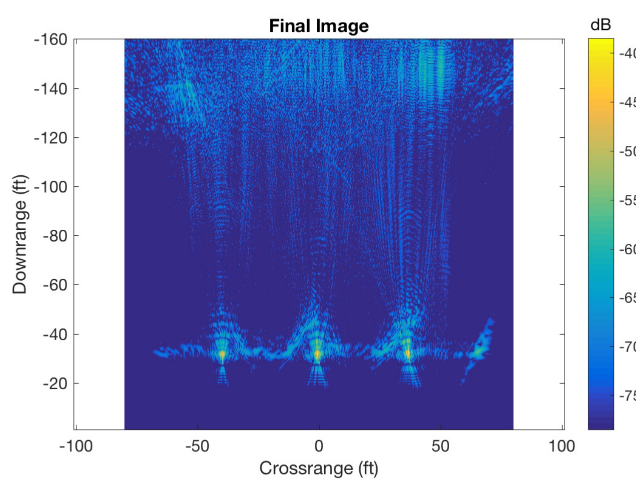

Fig. 3 Image produced with the phase corrected

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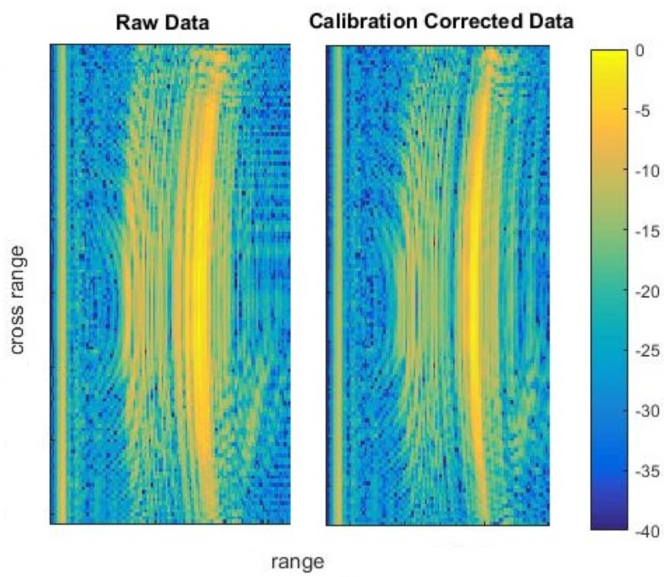

Fig. 4 FFT of the data before and after the phase correction

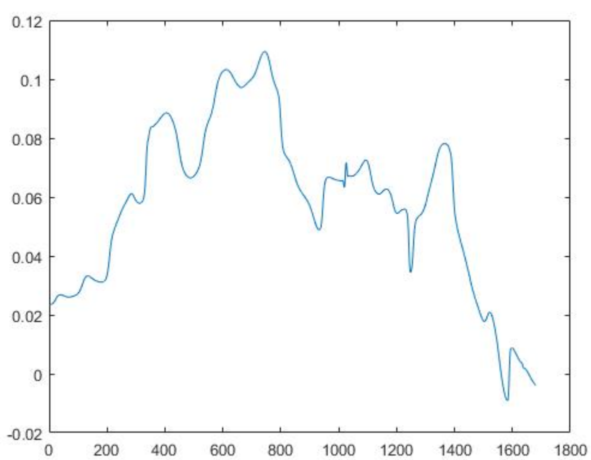

Fig. 5 Phase error at n=1 range bin

V. Conclusion

Using phase correction can be an effective method of removing errors induced by nonlinearity of components; however, due to the amount of calculations, generating or applying the corrections becomes a time consuming and computationally intensive task. Further development could be made to allow the algorithm to be deployed on a low-power, low-speed microcontroller for on-the-fly correction.

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