NEW ALGORITHMS FOR COMPRESSED SENSING OF MRI: WTWTS, DWTS, WDWTS

Srivarna Settisara Janney

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NEW ALGORITHMS FOR COMPRESSED SENSING OF MRI:
WTWTS, DWTS, WDWTS

An Abstract of

A Thesis Presented to

The Faculty of Department of Computer Science

by

Srivarna Settisara Janney

In Partial Fulfillment

of Requirements for the Degree

Master of Science

Kennesaw State University

May 2018
ABSTRACT

Magnetic resonance imaging (MRI) is one of the most accurate imaging techniques that can be used to detect several diseases, where other imaging methodologies fail. MRI data takes a longer time to capture. This is a pain taking process for the patients to remain still while the data is being captured. This is also hard for the doctor as well because if the images are not captured correctly then it will lead to wrong diagnoses of illness that might put the patients lives in danger. Since long scanning time is one of most serious drawback of the MRI modality, reducing acquisition time for MRI acquisition is a crucial challenge for many imaging techniques. Compressed Sensing (CS) theory is an appealing framework to address this issue since it provides theoretical guarantees on the reconstruction of sparse signals while projection on a low dimensional linear subspace. Further enhancements have extended the CS framework by performing Variable Density Sampling (VDS) or using wavelet domain as sparsity basis generator. Recent work in this approach considers parent-child relations in the wavelet levels.

This paper further extends the prior approach by utilizing the entire wavelet tree structure as an argument for coefficient correlation and also considers the directionality of wavelet coefficients using Hybrid Directional Wavelets (HDW). Incorporating coefficient thresholding in both wavelet tree structure as well as directional wavelet tree structure, the experiments reveal higher Signal to Noise ratio (SNR), Peak Signal to Noise ratio (PSNR) and lower Mean Square Error (MSE) for the CS based image reconstruction approach. Exploiting the sparsity of wavelet tree using the above-mentioned techniques achieves further lessening for data needed for the reconstruction, while improving the reconstruction result. These techniques are applied on a variety of images including both MRI and non-MRI data. The results show the efficacy of our techniques.
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Advisors: Dr. Sumit Chakravarty and Dr. Chih-Cheng Hung

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This thesis is dedicated to my husband who has been my inspiration and my strongest supporter. I would also thank my parents, in-laws, brother & his family, and sister-in-law & her family.
PREFACE

The thesis is submitted in partial fulfillment of the requirements for the Master of Science degree program at Kennesaw State University, Kennesaw.

The research project has been conducted under the supervision of Dr. Sumit Chakravarty from Department of Electrical Engineering and Dr. Chih-Cheng Hung from Department of Computer Science, during the years 2017-2018. Financing for the work has been provided in the form of scholarship from the Graduate College of Kennesaw State University through the Graduate Research Assistantship.
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QUOTES

“Arise, Awake, and stop not until the Goal is achieved”
- Swami Vivekananda

“The important thing is to not stop questioning. Curiosity has its own reason for existing.”
- Albert Einstein

“We all need people who will give us feedback. That’s how we improve.”
- Bill Gates
Table of Contents

Chapter 1 .................................................................................................................. 14
  1.1 Introduction........................................................................................................... 14
    1.1.1 Magnetic Resonance Imaging (MRI) .......................................................... 15
    1.1.2 Basic Concepts of Compressed Sensing .................................................... 16

Chapter 2 .................................................................................................................. 21
  2.1 Literature survey and problem statement ......................................................... 21
    2.1.1 Compressed Sensing for MRI ................................................................. 21
    2.1.2 Traditional Compressed Sensing Methodologies in MRI ......................... 24
    2.1.3 Drawbacks ................................................................................................. 28
    2.1.4 Problem Statement ................................................................................... 28

Chapter 3 .................................................................................................................. 29
  3.1 Proposed Solutions ........................................................................................... 29
    3.1.1 Weighted Threshold Wavelet Tree Structure (WTWTS) ......................... 30
    3.1.2 Directional Wavelet Tree Structure (DWTS) ............................................ 32
    3.1.3 Weighted Directional Wavelet Tree Structure (WDWTS) ....................... 35
  3.2 Complexity Analysis .......................................................................................... 37

Chapter 4 .................................................................................................................. 38
  4.1 Experiments and results .................................................................................... 38
    4.1.1 Test Environment Specification ............................................................... 38
    4.1.2 Data Set ..................................................................................................... 38
    4.1.3 Hyper-Parameters .................................................................................... 41
    4.1.4 Parameters ............................................................................................... 42
    4.1.5 Preliminary Results .................................................................................. 42
    4.1.6 Results and Evaluation ............................................................................. 52
    4.1.7 Comparison between WTWTS, DWTS, WDWTS ..................................... 58

Chapter 5 .................................................................................................................. 59
  Future Work ............................................................................................................ 59

Chapter 6 .................................................................................................................. 63
  Conclusion ............................................................................................................... 63

Bibliography ............................................................................................................. 64

Appendix .................................................................................................................... 72
  List of Algorithms .................................................................................................. 72
List of Figures

Figure 1.1: MRI scanning .................................................................14
Figure 1.2: MRI scanning machine [2] .............................................16
Figure 1.3: Compressed Sensing vs Image Compression Framework [3] .........................................................17
Figure 1.4: Basic Compressed Sensing matrix [3] ................................18
Figure 1.5: Comparison between (a) l1 is sparse and (b) l2 norms is not sparse ..........19
Figure 1.6: (a) kspace data (b) kspace data with noise like pattern cause by incoherence .................................................................19
Figure 2.1: Image Reconstruction from k-space to pixel image [40] .........................22
Figure 2.2: Represents the original image and its reconstructed image using SVD method........................................................................................................25
Figure 2.3: (a) Original Image (b) CS with PCA, red circle means the area has higher SNR compared to previous methods........................................................................26
Figure 2.4: (a) Wavelet Decomposition (b) Decomposed Image direction ................27
Figure 3.1: Wavelet Quad-Tree Structure (a) A cardiac MR image (b) Corresponding Wavelet Tree Group of Hierarchy GP-P-C Structure ........................................30
Figure 3.2: Directional filter bank frequency partitioning using eight directions ..........32
Figure 3.3: (a) Vertical directional filter banks (b) Horizontal directional filter banks ....32
Figure 3.4: Example of Directional filter bank on representation ..................................33
Figure 3.5: Eight Directional filter banks represent using classic Barbara Image ..........33
Figure 4.1: MRI Data ........................................................................39
Figure 4.2: Sampling Mask ..................................................................39
Figure 4.3: (a) T1 image and (b) T2 image ..............................................40
Figure 4.4: Portrait Data .......................................................................40
Figure 4.5: Crowd Data ........................................................................41
Figure 4.6: Landscape Data .................................................................41
Figure 4.7: Variation of number of zero trees based on GP-P-C Hierarchy for one brain image .........................................................................................43
Figure 4.8: Variation of number of zero tree based on GP-P-C Hierarchy for T1 images.43
Figure 4.9: Variation of number of zero trees based on GP-P-C hierarchy for T2 image .44
Figure 4.10: Variation of number of zero trees based on GP-P-C hierarchy level for entire 500 brain images ..................................................................................44
Figure 4.11: 3D Graph of range of threshold sets against Entropy of data at Parent-Child Level .........................................................................................45
Figure 4.12: Comparison of different thresholding sets against SNR for different resolution sizes for Brain Images (Red line indicate reference SNR of WatMRI algorithm) ..................................................................................46
Figure 4.13: Comparison of different thresholding sets against SNR for different resolution sizes for Heart Images (Red line indicate reference SNR of WatMRI algorithm) ..................................................................................46
Figure 4.14: Comparison of different thresholding sets against SNR for different resolution sizes of Shoulder Images (Red line indicate reference SNR of WatMRI algorithm) ..................................................................................46
Figure 4.15: Comparison of different thresholding sets against SNR for different resolution sizes for Chest Images (Red line indicate reference SNR of WatMRI algorithm) 47

Figure 4.16: SNR comparison between WatMRI (Green color) and WTWTS with Set 2 (Red color) and 3 (Blue color) approaches 47

Figure 4.17: Comparison the SNR for Portrait, Crowd and Landscape images for thresholding set 2 and 3 (Red line indicates reference SNR of WatMRI algorithm) 48

Figure 4.18: Histogram of 8 Directions at Parent and Child level for Brain Image 48

Figure 4.19: Histogram of 8 Directions at Parent and Child level for Heart Image 49

Figure 4.20: Histogram of 8 Directions at Parent and Child level for Shoulder Image 49

Figure 4.21: Histogram of 8 Directions at Parent and Child level for Chest Image 50

Figure 4.22: Comparison of Portrait image against original data using WTWTS and WDWTS algorithms with chosen threshold set 51

Figure 4.23: Comparison of Crowd image against original data using WTWTS and WDWTS algorithms with chosen threshold set 51

Figure 4.24: Comparison of Landscape image against original data using WTWTS and WDWTS algorithms with chosen threshold set 52

Figure 4.25: Brain image reconstruction using the 3 algorithms compared against WatMRI 53

Figure 4.26: Heart image reconstruction using the 3 algorithms compared against WatMRI 53

Figure 4.27: Shoulder image reconstruction using the 3 algorithms compared against WatMRI 54

Figure 4.28: Chest image reconstruction using the 3 algorithms compared against WatMRI 54

Figure 4.29: Portrait image reconstruction using the 3 algorithms compared against WatMRI 55

Figure 4.30: Crowd image reconstruction using the 3 algorithms compared against WatMRI 55

Figure 4.31: Landscape image reconstruction using the 3 algorithms compared against WatMRI 56

Figure 5.1: Overview of Deep Neural Network for CS in MRI 59

Figure 5.2: Reconstruction Error flow chart 60

Figure 5.3: Multi-Slice of Brain MRI [38] 60

Figure 5.4: 3D Brain MRI displaying each dimension [40] 61

Figure 5.5: Dynamic MRI represented in Tensor [46] 61

Figure 5.6: Visualization of Higher Order Single Value Decomposition (HOSVD) (a) 3D Tensor (b) 4D Tensor [45] 62
List of Tables

Table 3.1: Pseudo Code of WTWTS .................................................................31
Table 3.2: Pseudo Code of DWTS .................................................................34
Table 3.3: Pseudo Code of WDWTS ..............................................................36
Table 4.1: SNR and MSE comparison on MRI data .......................................56
Table 4.2: CPU Time comparison on MRI data ..............................................57
Table 4.3: SNR and MSE comparison on Non-MRI data ...............................57
Table 4.4: CPU Time comparison on Non-MRI data ....................................58
List of Acronyms

BMP: Basis Matching Pursuit...........................................................................22
C: Child..............................................................................................................29
CS: Compressed Sensing..................................................................................14
DFB: Directional Filter Banks..........................................................................32
DWTS: Directional Wavelet Tree Structure......................................................30, 32
FCSA: Fast Composite Splitting Algorithm.........................................................23
FISTA: Fast Iterative Shrinkage Thresholding Algorithm....................................27
GP: Grandparent...............................................................................................29
HDW: Hybrid Directional Wavelets..................................................................2
MRI: Magnetic Resonance Imaging....................................................................14
MSE: Mean Square Error....................................................................................2
OMP: Orthogonal Matching Pursuit..................................................................22
P: Parent.............................................................................................................29
PSNR: Peak Signal to Noise Ratio.................................................................2
SNR: Signal to Noise Ratio...............................................................................2
TVCMRI: Total Variation Compressed Sensing MRI...........................................23
VDS: Variable Density Sampling.................................................................2
WaTMRI: Wavelet Tree MRI.............................................................................27
WDWTS: Weighted Directional Wavelet Tree Structure.................................30, 35
WTWTS: Weighted Threshold Wavelet Tree Structure......................................30
Chapter 1

1.1 Introduction

One of the core challenges of Magnetic Resonance Imaging (MRI) reconstruction is the significant time duration required to acquire the images. One of the constraints is that patients have to remain stationary for the duration of scan that typically takes more than an hour. Another constraint is that the resolution of the reconstructed images should be of significantly high grade. In the case of MRI, considerable attention has been paid to Compressed Sensing (CS) that is known as one of the most successful methods for image reconstruction with significantly sparse amount of coefficients. Figure 1.1 shows the MRI scanning process that patients have to undergo while getting a MRI scan.

![Figure 1.1: MRI scanning](image)

Initially the patients are given an injection containing “Gadolinium” which is a heavy metal or also called as “contrast agent”. They wait for 10-15mins for the injected content to spread throughout the area that need to be scanned. Patient is then asked to lie down on a sliding table that enters the huge magnetic coil drum that captures the scan data in space known as k-space data. Compressed Sensing of wavelet is used to accurately reconstruct the image that known as MRI scanning image. The whole process typically takes around two hours of patient’s time. If there is any movement by the patient,
then inaccurate reconstruction takes place and process needs to be repeated for recapture. This is a challenging situation for both patients and radiologist.

This research paper developed three new algorithms namely Weighted Threshold Wavelet Tree Structure (WTWTS), Directional Wavelet Tree Structure (DWTS) and Weighted Directional Wavelet Tree Structure (WDWTS), using multiple MRI data including Brain, Chest, Shoulder, and Heart type of datasets. Developed algorithms were also tested on Non-MRI data (Portrait, Landscape, Crowd Images) to generalize the algorithm to any image data.

1.1.1 Magnetic Resonance Imaging (MRI)

Magnetic Resonance Imaging (MRI) scan is a technique using by radiologists that uses magnetism of the huge magnetic coil drum and algorithms are used to reconstruct image of body structures. For the procedures, a contrast agent, such as gadolinium, which is a heavy metal, is injected into the patient’s body part that needs to scan. The MRI scanner is a giant circular drum of magnet and a sliding table as shown in Figure 1.2. The patient is placed on a moveable bed that is inserted into the magnet. The magnet creates a strong magnetic field that aligns the protons of hydrogen atoms. This spins the various protons of the body, and they produce a signal that is detected during the MRI scanner. A computer processes the receiver information, and after compressed sensing using wavelet to reconstruct the final image, known as MRI is produced.

The image and resolution produced by MRI needs to be detailed and should detect tiny changes of structures within the body. The radiologist will able be accurately diagnose diseases or structural abnormalities based on the quality of image that is reconstructed from the machine. If the patient moves then the data will not be accurately captured and the radiologist has to redo the scanning.
Patients who have any kind of materials that contain metal in their bodies will create a problem during scanning. Normally they are asked to remove things like jewelry, clips etc. Most of the internal metal materials that are inside the patient’s body added as a part of medical procedures such as artificial joints, pacemaker, metallic bone plates, or prosthetic limbs etc. could distort MRI. Scanning cannot be performed on these patients. Alternative methods will be used for diagnosis [1].

Since some patients can be claustrophobic who experience fear of closed places, the radiologist may recommend a mild sedative prior to the MRI scanning. The rotating of magnetic coil generates huge noise, so earplugs might be given to patients who request them.

1.1.2 Basic Concepts of Compressed Sensing
Compressed Sensing should not be confused with traditional image compression. Figure 1.3 shows the framework of the two concepts where \( X \) is the original input signal and \( X_c \) is the reconstructed signal with sample \( N \) number of samples while \( M \) represents the sparse number of sample that is required for reconstruction.

**Figure 1.3: Compressed Sensing vs Image Compression Framework** [3]

Compressed sensing (also known as compressive sensing, sparse sampling or compressive sampling, CS) is a signal processing technique for efficiently collecting signal and accurately reconstructing the signal. Based on the principle of sparsity of the signal found, these signals have much fewer samples than that required by the Shannon-Nyquist sampling theorem [4], [5], [6], [7].

CS has gained a lot of popularity various area over last few decades. This concept was introduced by Candes, Romberg, and Tao [12], and Donoho [14]. CS helps to recover the sparse signals using only few samples in contrast to Nyquist sampling theory, where sampling of the signal is performed at a rate larger than highest frequency present in the signal. An overview of compressed sensing theory can be found in [8], [9], [10], [11].

Consider \( b \) to be the actual signal having dimensions \( N \times 1 \) where \( b \in \mathbb{R} \).
represents the sampling matrix with MxN dimensions and y is the final compressed vector with dimension Mx1 which is much smaller than N. i.e M<<N
\[ y = \Phi b \] (1)

According to Compressed Sensing, the equation looks like
\[ y = \Phi \Psi x = Ax \] (2)

where b is the non-sparse signal represented as \( b = \Psi x \), x = sparse representation in transform domain \( \Psi \). A = \( \Phi \Psi \) which represents dimension MxN sensing matrix having k sparse measurement required for reconstruction of image. The aim is to minimize coefficients to get sparse signal called \( \min_x \). Figure 1.4 shows the fundamental concepts in CS matrix representation that is shown in equation (2).

![Figure 1.4: Basic Compressed Sensing matrix [3]](image)

There are two fundamental concepts in Compressed Sensing. First one is sparsity of the signal; that is, the signal needs to sparse in transform domain. The second one is incoherence; that is, applied through the Restricted Isometric Property (RIP) that is sufficient for sparse signals.

The above reconstruction problem is non-deterministic polynomial-time
hard (NP-hard). Due to the non-convex nature of $l^0$ minimization, the problem is ill conditioned and difficult to solve as it requires an exhaustive search to find the most sparse solution (i.e. combinatorial problem) [16]. Since the unique sparse solution can be found exactly using $l^1$ minimization. $l^2$ norm is known for least square solution, a penalty will be imposed on small non-zero coefficients where it tends to energy to large number of entries and the result is not sparse [13]. Comparison between the norms is as shown in Figure 1.5 and the incoherence is shown in Figure 1.6.

![Comparison between (a) l1 is sparse and (b) l2 norms is not sparse](image)

**Figure 1.5:** Comparison between (a) l1 is sparse and (b) l2 norms is not sparse

![kspace data (a) kspace data with noise like pattern cause by incoherence](image)

**Figure 1.6:** (a) kspace data (b) kspace data with noise like pattern cause by incoherence
In addition to sparsity, the minimum acceptable number of samples for reconstruction also depends upon the incoherence between $\Phi$ and $\Psi$, such that there is no correlation between the rows of $\Phi$ and the columns of $\Psi$. To achieve this, “A” as explained in the equation (2) should satisfy the Restricted Isometry Property (RIP) [17].

1.1.2.1 Disadvantages

Early iterations may find inaccurate sample estimates, however this method will down-sample these at a later stage to give more weight to the smaller non-zero signal estimates. One of the disadvantages is the need for defining a valid starting point, as a global minimum might not be obtained every time due to the concavity of the function. Another disadvantage is that this method tends to uniformly penalize the image gradient irrespective of the underlying image structures. This causes over-smoothing of edges, especially those of low contrast regions, subsequently leading to loss of low contrast information [3].

1.1.2.2 Advantages

The advantages of this method include: reduction of the sampling rate for sparse signals; reconstruction of the image while being robust to the removal of noise and other artifacts; and use of very few iterations. This can also help in recovering images with sparse gradients [3].
Chapter 2

2.1 Literature survey and problem statement

This section is related to previous researches that have been carried out. Each sub-section showcases the idea, equation and the scope of improvement that have been brought about through the findings. These concepts mark a baseline for this research is built over some of these ideas.

2.1.1 Compressed Sensing for MRI

Compressed Sensing (CS) is one of the most successful methods of image reconstruction with significantly sparse amount of coefficients. According to CS theory [15] [19], only $O(K + K \log n)$ sampling measurements are enough to recover $K$-sparse data with length $n$, the problem in overcoming the challenge associated with imaging without a significant degradation in image quality. The key approach of compressed sensing MRI applications is to utilize separate domains. As MRI images are formed in K-space (Fourier Domain) it becomes the primary domain for monitoring reconstruction error as shown in the Figure 2.1. It has been well establish via the research of Lustig and Donoho [19] to implement sparsity that needs to operate in a sparsifying domain like wavelets. Finally as the end consumers of the image data, the need is to visualize the result in the spatial sample domain.
The non-linear conjugate gradient (NLCG [18]) was applied to solve the unconstrained problem in the Lagrangian form to get equation (3):

$$\min_x \{ F(x) = \frac{1}{2} \| Ax - b \|^2 + \beta \| \phi_x \|_1 \}$$

(3)

Where $A$ = sampling matrix obtained from partial Fourier transform of $x = M \times N$ image to be reconstructed, $\beta$ = positive parameter, $b$ = original signal data and $\phi$ = sparsifying domain,

Equation (3) attempts to minimize the mean square error in the frequency domain while choosing the sparsest set of coefficients in the wavelet domain with $L_2$ norm. This optimization can be performed via various techniques like OMP (Orthogonal Matching Pursuit) [21] or Basis Matching Pursuit (BMP) [22] algorithms.

Several studies have been done in CS - MRI domain. Much of the work deals
with manipulating the constraint term in equation (3). Techniques in the
gamut of signal processing like total variation smoothening, optical flow of
coefficients, [23] variable density sampling [24] and singular value
decomposition [25], conjugate gradient [26] and partial Fourier (RecPF) [57]
are some of the proposed approaches used in current literature. The next
section will present related work on which our current work is based.

Wavelets have been extensively used as a compressive domain in the
field of CS. Apart from the previously listed techniques; the following
methods deserve special attention. In [24] a method called total variation
compressed sensing MRI (TVCMRI) is presented. Use of total variation helps
regularize the spatial variability of the wavelet coefficient domain. It uses an
operator-splitting algorithm to solve the problem of reconstruction called
the fast composite splitting algorithm, (FCSA) [53]. This approach extends
equation (3) with the additional term appended at the end of equation (4).

\[
\min_x \{ F(x) = \frac{1}{2} \|Ax - b\|^2_2 + \alpha \|x\|_{TV} + \beta \|\phi_x\|_1 \} \tag{4}
\]

where \( \alpha \|x\|_{TV} = \sum_i \sum_j \sqrt{(\nabla^1_{ij})^2 + (\nabla^2_{ij})^2} \) \( \tag{5} \)

\( \alpha = \) positive parameter, where the term inside the root is summation
of the result of finite difference operators on the first and second coordinates
respectively, \( \beta = \) positive parameter, \( \nabla \) denotes the forward finite difference
operator for first and second co-ordinates which is represented in terms of \( x \)
as

\[
\nabla^1_{ij} = x_{i-1,j} - x_{ij} \\
\nabla^2_{ij} = x_{i,j-1} - x_{ij}
\]

Signal to Noise Ratio measures the quality of reconstructed image show in
equation (6).

\[ \text{SNR} = \frac{\mu^2}{\sigma} \]  \hfill (6)

Where \( \mu^2 \) = average root mean of signal and \( \sigma \) is the standard deviation

### 2.1.2 Traditional Compressed Sensing Methodologies in MRI

There are few well-known or conventional compressed sensing algorithms that are commonly used in MRI. Some of them are mentioned below.

1) **CS of MRI using Singular Value Decomposition (SVD)**

According to authors of [43], the reconstructed image \( M \) can be represented using the Singular value Decomposition formula

\[ M = U\Sigma V^T \]  \hfill (7)

where \( U \) and \( V \) are orthogonal matrices and \( \Sigma \) is diagonal matrix with non-zero singular values.

When equation (7) is used in inverse Fourier domain then we get

\[ \Psi(m) = U_n^* m V_n \]  \hfill (8)

where \( U_n \) and \( V_n \) are left and right unitary matrix that are able to provide a sparse representation for the desired image \( M \) when projected on \( U_n \) and \( V_n \) for \( n \) number of samples. For better results, matrices \( U \) and \( V \) are iterated with SVD using the equation (3). Figure 2.2 represents the original image and its reconstructed image using SVD method.
In Figure 2.2, the top row shows the process of generating the sparsity basis from left to right: the image I, the unitary matrix U, the diagonal matrix and unitary matrix V. The bottom row shows the result of SVD with sparse representation of desired image from left to right: the sparse image M, the unitary matrix U, the desired image M and unitary matrix V.

2) CS of MRI using Principle Component Analysis (PCA)

In [24] a method called total variation compressed sensing MRI (TVCMRI) is presented. Use of total variation helps regularize the spatial variability of the wavelet coefficient domain. It uses an operator- splitting algorithm to solve the problem of reconstruction called the fast composite splitting algorithm (FCSA) [53].

\[
\text{min}\_m \{ F(m) = \| \Phi_F(m) - y \|^2 + \beta \text{TV}(m) + \lambda \| \Psi(m) \|_1 \} \quad (9)
\]

Where \(\beta\) in equation (9) represents the weight of the total variation (TV) that is the added term for denoising.

Another method for CS based on PCA analysis in which the weight of norm
L I of the image was adaptively updated at each iteration with respect to reduction of dimensions. In [39] method, PCA adaptive approach as the weight of sparsity, the constrained optimization is converted into the minimization of the following equation:

$$\min_m \{ F(m) = \| \Phi F(m) - y \|_2^2 + \beta TV(m) + \| u \times s \times v \|_1 \| \Psi(m) \|_1 \}$$  \hspace{1cm} (10)

Matrices $u$, $s$, and $v$, are the PCA elements ($[u, s, v]=$PCA ($m$)). Efficiency and accuracy of PCA is claimed to be 15% higher than the SVD method. Result comparison is shown in Figure 2.3.

![Figure 2.3: (a) Original Image (b) CS with PCA, red circle means the area has higher SNR compared to previous methods](image)

3) CS of MRI using Wavelet Tree Structure (WaTMRI)

Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. Wavelet analysis is used to approximate functions that are contained neatly in finite domains. Wavelets are best suited for approximating data with sharp discontinuities. Wavelet applications are used in areas such as image compression, computer
vision, prediction of earthquake prediction, etc. When moved to frequency domain from time domain then Discrete Wavelet Transform (DWT) is used and Discrete Daubchies Wavelet Transform is used to obtain wavelet coefficients. The wavelet decomposition consists of four components called Approximation, Horizontal, Vertical and Diagonal details as shown in the Figure 2.4.

![Wavelet Decomposition](image_url)

**Figure 2.4:** (a) Wavelet Decomposition (b) Decomposed Image direction

This has further been applied in WaTMRI [29] in addition to using total variation, parent child relationships of wavelets are considered. The authors group all parent child coefficients. The resultant optimization equation (11) is

\[
\min_x \{ F(x) = \frac{1}{2} \|Ax - b\|_2^2 + \alpha \|x\|_{TV} + \beta \left( \|\phi_x\|_1 + \sum_{g \in C} \| (\phi_x)_g \|_2 \right) \} \quad (11)
\]

The addition of the “parent-child” group (g) term enforces the above optimization to consider coefficients in parent child pairs and thereby aims to discard from the result, weak or underperforming coefficient groups. The equation (11) is solved using Fast Iterative Shrinkage Thresholding Algorithm (FISTA) [26], [27].
2.1.3 Drawbacks

Although the WaTMRI approach considers correlation among the wavelet coefficients, it is rudimentary. It also suffers from the disadvantages of conventional wavelet transform: sensitive to shifting [31], poor directionality [32] and dearth of phase information [33].

2.1.4 Problem Statement

Problem to be resolved is image reconstruction with minimal error as well as to speed up the run time of the algorithm. The directionality of wavelet is taken into consideration. Since the target is to generalize the algorithm to cater to general case of image reconstruction this is a challenge in itself since there are traditional image processing algorithms that works quiet efficiently as well. Consideration of Non-MRI data type provides a generalization of different data distributed that could be used to simulate other MRI data set that are hard to obtain.
Chapter 3

3.1 Proposed Solutions

In this paper, a new model is proposed with wavelet tree sparsity taken into consideration along with entire wavelet “group-coefficients”. The tree structure shown in Figure 3.1, is modeled as an overlapping group regularization, with groups sharing Grandparent (GP)-Parent (P)-Child (C) coefficients. The original problem of 20% sampling rate can be decomposed to two-sub problems and solved very efficiently by using the FISTA [26], [27]. Equation (12) presents our approach in terms of the optimization terms.

\[
\min_x \{ F(x) = \frac{1}{2} \| Ax - b \|_2^2 + \alpha \| x \|_{TV} + \beta (\| \phi_x \|_1 + \sum_{t \in T} \| (\phi_x)_t \|_2) \}, \quad \text{s.t.} \quad \tau > 12
\]

where \( \tau \) is prescribed co-efficient tree threshold for each level of hierarchy. Subset \( t \in T \) is where \( T \) is the number of hierarchical tree groups.

The equation (12) has three terms, which specify the three components of the algorithm. The first term ensures reconstruction error is within range. The second terms enforces smoothness on the result. The final term enforces sparse selection only from the groups of tree coefficient greater than prescribed threshold value.
We proposed three new algorithms, which would exploit the hierarchical approach.

- Weighted Threshold Wavelet Tree Structure (WTWTS)
- Directional Wavelet Tree Structure (DWTS)
- Weighted Directional Wavelet Tree Structure (WDWTS)

### 3.1.1 Weighted Threshold Wavelet Tree Structure (WTWTS)

The hierarchy-based approach is combined with the weighted thresholding to provide a baseline for this research. The other two algorithms are based on this concept and build on it with different techniques used in the shrinkage of embedded zero tree algorithm ($z_t$) as shown in equation (13) which is considered in pseudo code in Table 3.1.

$$z_t = \text{shrinktree} \left( (T\phi_x)_t \frac{\beta}{\alpha}, \tau \right)$$  \hspace{1cm} (13)

where the $z_t$ can be further simplified to

$$z_t = \max \left( (T\phi_x)_t - \frac{\beta}{\alpha}, 0 \right)$$
and where $\phi_x \geq \tau$ and $\phi_x < \tau$ for $z_t=0$ such that $t$ has the following threshold range

Child (C) = 25% < $\tau \leq$ 50% of C co-efficient

Parent (P) = 25% < $\tau \leq$ 75% of P co-efficient

GrandParent (GP) = 75% of GP co-efficient

<table>
<thead>
<tr>
<th>Table 3.1: Pseudo Code of WTWTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input: $r^n = x_0^n$, $t^1 = 1, \alpha, \beta, \lambda, \tau$</td>
</tr>
<tr>
<td>for $n = 1$ to MaxIteration=50 do</td>
</tr>
<tr>
<td>1. Calculate $z_t = \text{shrinktree} \left( (T\phi_x)_t, \frac{\beta}{\alpha}, \tau \right)$</td>
</tr>
<tr>
<td>2. $x_t = r^n - \text{argmin}_{x_t} { \frac{1}{2} | Ar^n - b |_2^2 + \frac{\lambda}{2} | x_t - (T\phi_x)_t |_2 }$</td>
</tr>
<tr>
<td>3. $x_1 = g_1(x) + \alpha \parallel x_{TV} - x_t \parallel_2^2$</td>
</tr>
<tr>
<td>4. $x_2 = g_2(x) + \beta \parallel \phi x - x_t \parallel_2^2$</td>
</tr>
<tr>
<td>5. $x^n = \frac{(x_1 + x_2)}{2}$</td>
</tr>
<tr>
<td>6. $t^{n+1} = \frac{\sqrt{1+4(t^n)^2} - 1}{2}$</td>
</tr>
<tr>
<td>7. $r^n + 1 = x^n + \frac{t^{n-1}}{t^{n+1}}(x^n - x^{n-1})$</td>
</tr>
</tbody>
</table>

where hyper-parameters and parameters are explained below:

$\alpha = 0.001$

$\beta = 0.035$ = positive parameter

$\lambda = 0.2 \times \beta = 0.007$ = multiplier (used in all convex models)

$n = \text{iteration number}$

$\tau = \text{Weighted threshold condition mentioned below}$

Child (C) = 25% < $\tau \leq$ 50% of C co-efficient

Parent (P) = 25% < $\tau \leq$ 75% of P co-efficient

Grand Parent (GP) = 75% of GP co-efficient

$(T\phi_x)_t = \text{Non- overlapping hierarchical tree in sparse domain}$
\[ r^n = x^n_t \] each signal coefficient value from based iteration
\[ g_1(x) = \alpha ||x||_{TV} \]
\[ g_2(x) = \beta (||\phi_x||_1) \]
\[ t^1 = 1, \alpha, \beta, \lambda, \tau \] all the value which is initialized for the first iteration and later on based on the threshold value they are iteratively updated for n samples.
The steps 5, 6, 7 in the Table 3.1 are referred from WatMRI [29]. Results show the efficacy of our resent approach.

### 3.1.2 Directional Wavelet Tree Structure (DWTS)

The second approach worth highlighting in this paper is the concept of wavelet directionality as expressed in Directional Filter Banks Transform (DFB) [33] as shown in Figure 3.2 and Figure 3.3. The Directional filter banks are realized using iterated quincunx filter banks. The directional filter bank concept is extended to multi-resolution wavelet sub band decomposition as by adding the feature of directionality in an appropriate manner improvement may occur in the nonlinear approximation of wavelets as shown in Figure 3.4.

**Figure 3.2:** Directional filter bank frequency partitioning using eight directions

**Figure 3.3:** (a) Vertical directional filter banks (b) Horizontal directional filter banks
The second method utilizes the prior discussed directional wavelet transforms. Directionality increases the efficiency of the wavelet transforms. Hence wavelet transforms can be replaced with directional wavelets. This enables better consolidation of the ‘wavelet trees’ and thereby enables better sparse optimized image reconstruction. Figure 3.5 shows the eight directional filter bank on an image.
The algorithm is similar to the proposed algorithm WTWTS using Hybrid Wavelets and Directional Filter Banks [33], as both use thresholding to reduce the potential number of “candidate group of Trees” for sparse processing. The reduction of candidate groups, by thresholding groups with low scores, enables sparse selection from richly populated groups. Shrinkage algorithm uses the eight Directional filter bank wavelet as shown in Table 3.2.

Table 3.2: Pseudo Code of DWTS

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Calculate $z_t = \text{shrinktree} ((T\phi_C)_r, \frac{\beta}{\alpha}, \tau)$</td>
</tr>
<tr>
<td>2</td>
<td>$x_t = x^n - \mathop{\arg\min}_{x^n} {\frac{1}{2}</td>
</tr>
<tr>
<td>3</td>
<td>$x_1 = g_1(x) + \alpha \parallel x_{TV} - x_t \parallel^2$</td>
</tr>
<tr>
<td>4</td>
<td>$x_2 = g_2(x) + \beta \parallel \phi x - x_t \parallel^2$</td>
</tr>
<tr>
<td>5</td>
<td>$x^n = \frac{(x_1 + x_2)}{2}$</td>
</tr>
<tr>
<td>6</td>
<td>$t^{n+1} = \frac{1 + \sqrt{1 + 4(t^n)^2}}{2}$</td>
</tr>
<tr>
<td>7</td>
<td>$r^{n+1} = x^n + \frac{t^{n-1}}{t^{n+1}} (x^n - x^{n-1})$</td>
</tr>
</tbody>
</table>

where Directional Wavelet is considered. Hyper-parameters and parameters are explained below:

- $\alpha = 0.001$
- $\beta = 0.035$ = positive parameter
- $\lambda = 0.2 \times \beta = 0.007$ = multiplier (used in all convex models)
- $n$ = iteration number
- $\tau$ = Weighted threshold condition mentioned below

Child (C) = $25\% < \tau \leq 50\%$ of C co-efficient
Parent (P) = 25% < τ ≤ 75% of P co-efficient

Grand Parent (GP) = 75% of GP co-efficient

(T\Phi_x)_t = Non-overlapping hierarchical tree in sparse domain

r^n = x^n each signal coefficient value from based iteration

\text{g}_1 (x) = \alpha \|x\|_{TV}

\text{g}_2 (x) = \beta (\|\Phi_x\|_1)

t^1 = 1, \alpha, \beta, \lambda, \tau all the value which is initialized for the first iteration and later on based on the threshold value they are iteratively updated for n samples.

The steps 5, 6, 7 in the Table 3.2 are referred from WatMRI [29]. Results show the efficacy of our resented approach.

### 3.1.3 Weighted Directional Wavelet Tree Structure (WDWTS)

In the final approach, in addition to the directional wavelet, we enforce threshold constraint on the directionality. In addition to wavelet thresholding, newest algorithm also incorporates directional wavelet thresholding (DWTS). As seen in the last term of equation (12), both thresholding of wavelet coefficient group’s directionality and energy are considered. The shrinkage algorithm in equation (13) is modified with weighted directional condition. Table 3.3 shows the pseudo code of WDWTS along with the condition below:
Table 3.3: Pseudo Code of WDWTS

<table>
<thead>
<tr>
<th>Step</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$z_t = \text{shrinktree} \left((T\phi_x)_t \frac{\beta}{n}, \tau\right)$</td>
</tr>
<tr>
<td>2.</td>
<td>$x_t = r^n - \text{argmin}_{n} \left{ \frac{1}{2} | Ar^n - b |_2^2 + \frac{\lambda}{2} | z_t - (T\phi_x)_t | \right}$</td>
</tr>
<tr>
<td>3.</td>
<td>$x_1 = g_1(x) + \alpha | x_{TV} - x_t |_2^2$</td>
</tr>
<tr>
<td>4.</td>
<td>$x_2 = g_2(x) + \beta | \phi x - x_t |_2^2$</td>
</tr>
<tr>
<td>5.</td>
<td>$x^n = \frac{(x_1 + x_2)}{2}$</td>
</tr>
<tr>
<td>6.</td>
<td>$t^{n+1} = \frac{| x_t |_2^2 + \lambda + \eta}{\eta}$</td>
</tr>
<tr>
<td>7.</td>
<td>$r^n + 1 = x^n + \frac{t^{n+1}}{\tau}(x^n - x^{n-1})$</td>
</tr>
</tbody>
</table>

where hyper-parameters and parameters are explained below:

- $\alpha = 0.001$
- $\beta = 0.035$ = positive parameter
- $\lambda = 0.2 \times \beta = 0.007$ = multiplier (used in all convex models)
- $n =$ iteration number
- $\tau =$ Weighted threshold condition mentioned below

$$s.\ t.\ t\ [2, 3, 6, 7] \text{Inner Direction} = 75\%\ of\ co-efficient$$
$$[1, 4, 5, 8] \text{Outer Direction} = 25\%\ of\ co-efficient$$

$P =$ Parent
$C =$ Child
$(T\phi_x)_t =$ Non-overlapping hierarchical tree in sparse domain
$r^n = x^n_t$ each signal coefficient value from based iteration
$g_1(x) = \alpha \| x \|_{TV}$
$g_2(x) = \beta (\| \phi x \|_1)$

$t^1 = 1, \alpha, \beta, \lambda, \tau$ all the value which is initialized for the first iteration and later on based on the threshold value they are iteratively updated for n samples.
3.2 Complexity Analysis

Suppose we represent $x$ as an image with $n$ pixels, over all complexity would be $O(n' + n \log n)$ where $n'$ is number of non-zero hierarchical tree element. So that $T$ is non-overlapping hierarchical tree would have $O(n')$. Below is a complexity of each step in three algorithm namely WTWTS, DWTS, WDWTS.

Step 1: $O(n' + n \log n)$

Step 1 shows the complexity for WTWTS, DWTS and WDTWTS. Note that we considered 8 directions for DWTS, WDWTS it would be $O(n' + 8n \log n) \approx O(n' + n \log n)$.

Step 2: $O(n \log n)$
Step 3: $O(n)$
Step 4: $O(n \log n)$
Step 5: $O(n)$
Step 6: $O(n)$
Step 7: $O(n)$

Results in the next section compare the three proposed techniques and the current benchmark technique.
Chapter 4

4.1 Experiments and results

Numerous varieties of experiments have been conducted for different parameters and hyper parameters selection. This section also shows final result of the proposed algorithms performance compared with the WatMRI [29]. In the MR imaging problem, A is the partial Fourier transform with m rows and n columns. This study considered 20% sampling. MR scanning time will be reduced if less measurement is sampled. All measurements contain 0.01Gaussian White Noise. Signal To Noise Ratio (SNR) and Mean Square Error (MSE) are considered for evaluation.

4.1.1 Test Environment Specification

Detailed description of the test environment that was used for the thesis is mentioned below:

- Hardware Specification:
  - System: Mac OS v10.13.1
  - Processor: 2.4 GHz Intel Core i5
  - Memory: 8 GB 1333 MHz DDR3 (Double Data Rate Synchronous Dynamic Random-Access Memory)
  - Graphics: Intel HD Graphics 3000 512 MB

- Programming Language: MATLAB R2017a (A proprietary programming language developed by MathWorks)

4.1.2 Data Set

Both MRI and Non-MRI data types are considered to validate this
approach. 500 sets of each data types with different size from 16x16, 32x32, 64x64, 128x128, and 256x256 resolutions are considered.

**MRI Data:**

MRI data was obtained from the reference paper [29]. Brain, heart, shoulder, chest images are considered as shown in the Figure 4.1.

![Figure 4.1: MRI Data](image)

The sampling mask that is applied for spatial filter is as shown in Figure 4.2.

![Figure 4.2: Sampling Mask](image)

The two basic types of MRI images are T1-weighted and T2-weighted images, often referred to as T1 and T2 images.

**T1 images** – highlight fat tissue within the body as shown in Figure 4.3 (a)

**T2 images** – highlight fat tissue and water content within the body as shown in Figure 4.3 (b)
Experiments also considered 50 images of T1, T2, which represent different details in the brain data to identify if there is any variation based on the direction of the data. The dataset was obtained from github link as shown below: https://github.com/muschellij2/Neurohacking_data/tree/master/BRAINIX/DICOM

**Non MRI Data:**

Non-MRI data types were picked up from Google Image based on following criteria such as

- Portrait Images – This data type represents images that have both combinations of low and high pass signals that mean the only few of the data can be sparsely represented. Files are named as Lena, Obama, Einstein, and Cameraman that is JPG files as shown in Figure 4.4.
• Crowd Images – This data type represents images that have majorly high pass signal that means there are lots of variations in the data. Files are named as Crowd1, Crowd2, Crowd3 and Crowd-rice images that are JPG files as shown in Figure 4.5.

![Crowd Data](image)

**Figure 4.5: Crowd Data**

• Landscape Images – This data type represents images that have majority of the low pass signals that indicates that some of the data are redundant which could be sparsely represented. Files are named as Landscape1, Landscape2, and Landscape3 images as shown in Figure 4.6.

![Landscape Data](image)

**Figure 4.6: Landscape Data**

### 4.1.3 Hyper-Parameters

Each simulation on the data is carried out with 50 iterations with approximate 20% sampling. Wavelet Tree Structure is configured to choose the Grandparent,
parent and child relationship that are generalized for Compressed Sensing in MRI. Different thresholding set is also considered as hyper-parameters that are configured.

4.1.4 Parameters

Wavelet Tree Grouping is custom made based on which of the wavelet co-efficient contributes towards accurate image reconstruction. Wavelet Co-efficient changes are based on the thresholding sets that need to be iteratively updated to provide co-efficients to improve the reconstruction of final images.

4.1.5 Preliminary Results

4.1.5.1 Selection of Threshold set, Image size and threshold based on Wavelet Directions

Number of zero-tree (zero value co-efficient at all level) varied against range of threshold, there was no particular one value. It has been different for each data of MRI and non-MRI data.

Figures 4.7 to 4.10 shows variations of the number of zero trees occurrence (eliminated during shrinkage) at each hierarchy level with different sets of data displays a specific range of threshold $\tau$ which normally is between 0-100.
Figure 4.7: Variation of number of zero trees based on GP-P-C Hierarchy for one brain image

Figure 4.8: Variation of number of zero tree based on GP-P-C Hierarchy for T1 images
Figure 4.9: Variation of number of zero trees based on GP-P-C hierarchy for T2 image

Figure 4.10: Variation of number of zero trees based on GP-P-C hierarchy level for entire 500 brain images
The variations of number of zero trees occurrence in the data were attributed to variation in the image. To compare the entropy distribution of the image based on the level would give a better understand of distribution pattern in each level irrespective of the type of images as shown in the Figure 4.11.

![Figure 4.11: 3D Graph of range of threshold sets against Entropy of data at Parent-Child Level](image)

The Parent and Child level distribution led to using the concepts of weighted thresholding since there is no one threshold value that satisfies all the data type. There should be different range of threshold set to cater to different distribution in level. Hence weighted tree structure of different combination of ratio of threshold is being considered. (GP, P, C) threshold in percentages are:

- 75,75,50 – Set 1
- 75,75,25 – Set 2
- 75,50,50 – Set 3
- 75,50,25 – Set 4
• 75,25,25 – Set 5

Figures 4.12 to 4.15 shows comparison of the 5 sets of threshold values against 5 sets of different resolution size (16, 32, 64, 128, 256).

Figure 4.12: Comparison of different thresholding sets against SNR for different resolution sizes for Brain Images (Red line indicate reference SNR of WatMRI algorithm)

Figure 4.13: Comparison of different thresholding sets against SNR for different resolution sizes for Heart Images (Red line indicate reference SNR of WatMRI algorithm)

Figure 4.14: Comparison of different thresholding sets against SNR for different resolution sizes of Shoulder Images (Red line indicate reference SNR of WatMRI algorithm)
It was found that 256x256 was the most stable compared to others. In regards to thresholding, it was found that Set 2 and 3 gave the best and most reliable results with higher SNR value compared toWatMRI as shown in the Figure 4.16.

Figure 4.15: Comparison of different thresholding sets against SNR for different resolution sizes for Chest Images (Red line indicate reference SNR of WatMRI algorithm)

Figure 4.16: SNR comparison between WatMRI (Green color) and WTWTS with Set 2 (Red color) and 3 (Blue color) approaches
Figure 4.17 shows the behavior of non-MRI data against the thresholding set 2 and 3 when compared to the reference algorithm.

Figure 4.17: Comparison the SNR for Portrait, Crowd and Landscape images for thresholding set 2 and 3 (Red line indicates reference SNR of WatMRI algorithm)

Figures 4.18 to 4.21 shows the variation of hybrid wavelet coefficients in the 8 directions for each MRI data. Direction 2,3,6, and 7 that is considered as inner direction has higher coefficients than direction 1,4,5 and 8 that is considered as outer direction. This is the condition for the algorithm WDWTS to choose the weighted directional thresholding in the Parent and Child level while all values of GrandParent level will be considered since all the coefficients contribute towards the image reconstruction.

Figure 4.18: Histogram of 8 Directions at Parent and Child level for Brain Image
Figure 4.19: Histogram of 8 Directions at Parent and Child level for Heart Image

Figure 4.20: Histogram of 8 Directions at Parent and Child level for Shoulder Image
Figure 4.21: Histogram of 8 Directions at Parent and Child level for Chest Image

Above results shows that experiments were carried out to select threshold sets, image size and threshold for directional wavelet on MRI data. These experiments were carried out non-MRI data obtaining good result that could be generalized for any type of images. Figures 4.22 to 4.24 show the Portrait, Crowd and Landscape images results based on the selection made for threshold set, image size and threshold on wavelet direction.
Figure 4.22: Comparison of Portrait image against original data using WTWTS and WDWTS algorithms with chosen threshold set

Figure 4.23: Comparison of Crowd image against original data using WTWTS and WDWTS algorithms with chosen threshold set
4.1.6 Results and Evaluation

This section displays the evaluation of proposed algorithms incorporating the finding of hyper-parameters such as image size, threshold sets, threshold for weighted direction etc.

Figures 4.25 to 4.31 below show the result comparison of the three new algorithms developed as part of this thesis against reference paper WatMRI. Each data type has its Signal to Noise Ratio (SNR) values and Mean Square Error (MSE)
Figure 4.25: Brain image reconstruction using the 3 algorithms compared against WatMRI

Figure 4.26: Heart image reconstruction using the 3 algorithms compared against WatMRI
Figure 4.27: Shoulder image reconstruction using the 3 algorithms compared against WatMRI

Figure 4.28: Chest image reconstruction using the 3 algorithms compared against WatMRI
Figure 4.29: Portrait image reconstruction using the 3 algorithms compared against WatMRI

Figure 4.30: Crowd image reconstruction using the 3 algorithms compared against WatMRI
Table 4.1 shows the SNR and MSE comparison of MRI data for all the three algorithms against WatMRI.

Table 4.1: SNR and MSE comparison on MRI data

<table>
<thead>
<tr>
<th>Method</th>
<th>MRI Data Types</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Brain</td>
</tr>
<tr>
<td>SNR</td>
<td>MSE</td>
</tr>
<tr>
<td>WatMRI</td>
<td>19.73</td>
</tr>
<tr>
<td>WTWTS</td>
<td>19.98</td>
</tr>
<tr>
<td>WDWTS</td>
<td>20.83</td>
</tr>
</tbody>
</table>
Table 4.2 show the CPU time comparison on MRI data sets for all the three algorithms against WatMRI.

<table>
<thead>
<tr>
<th>Method</th>
<th>MRI Data Types</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Brain</td>
<td>Heart</td>
<td>Shoulder</td>
<td>Chest</td>
</tr>
<tr>
<td></td>
<td>CPU Time (s)</td>
<td>CPU Time (s)</td>
<td>CPU Time (s)</td>
<td>CPU Time (s)</td>
</tr>
<tr>
<td>WatMRI</td>
<td>1.50 ± 0.03</td>
<td>1.53 ± 0.05</td>
<td>1.60 ± 0.04</td>
<td>1.59 ± 0.03</td>
</tr>
<tr>
<td>WTWTS</td>
<td>1.38 ± 0.09</td>
<td>1.35 ± 0.1</td>
<td>1.43 ± 0.11</td>
<td>1.41 ± 0.09</td>
</tr>
<tr>
<td>DWTS</td>
<td>1.40 ± 0.08</td>
<td>1.43 ± 0.09</td>
<td>1.50 ± 0.07</td>
<td>1.47 ± 0.05</td>
</tr>
<tr>
<td>WDWTS</td>
<td>1.42 ± 0.08</td>
<td>1.66 ± 0.05</td>
<td>1.70 ± 0.6</td>
<td>1.51 ± 0.11</td>
</tr>
</tbody>
</table>

Table 4.3 show the SNR and MSE comparison on Non-MRI data sets for all the three algorithms against WatMRI.

<table>
<thead>
<tr>
<th>Method</th>
<th>Non MRI Data Types</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Portrait</td>
<td>Crowd</td>
<td>Landscape</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SNR</td>
<td>MSE</td>
<td>SNR</td>
<td>MSE</td>
</tr>
<tr>
<td>WatMRI</td>
<td>23.40</td>
<td>80.10</td>
<td>14.26</td>
<td>420.3</td>
</tr>
<tr>
<td>WTWTS</td>
<td>24.19</td>
<td>66.90</td>
<td>14.46</td>
<td>401.56</td>
</tr>
<tr>
<td>DWTS</td>
<td>24.37</td>
<td>64.15</td>
<td>14.46</td>
<td>401.07</td>
</tr>
<tr>
<td>WDWTS</td>
<td>24.67</td>
<td>59.88</td>
<td>14.70</td>
<td>379.26</td>
</tr>
</tbody>
</table>
Table 4.4 show the CPU time comparison on Non-MRI data sets for all the three algorithms against WatMRI.

### Table 4.4: CPU Time comparison on Non-MRI data

<table>
<thead>
<tr>
<th>Method</th>
<th>Portrait CPU Time (s)</th>
<th>Crowd CPU Time (s)</th>
<th>Landscape CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WatMRI</td>
<td>1.50 ± 0.01</td>
<td>1.60 ± 0.03</td>
<td>1.55 ± 0.04</td>
</tr>
<tr>
<td>WTWTS</td>
<td>1.45 ± 0.09</td>
<td>1.50 ± 0.05</td>
<td>1.42 ± 0.08</td>
</tr>
<tr>
<td>DWTS</td>
<td>1.49 ± 0.07</td>
<td>1.53 ± 0.06</td>
<td>1.47 ± 0.04</td>
</tr>
<tr>
<td>WDWTS</td>
<td>1.55 ± 0.12</td>
<td>1.61 ± 0.05</td>
<td>1.49 ± 0.08</td>
</tr>
</tbody>
</table>

#### 4.1.7 Comparison between WTWTS, DWTS, WDWTS

1. All three algorithms namely WTWTS, DWTS, WDWTS have shown better quality of reconstructed image represented by SNR value and reduction in MSE value than the reference paper (WatMRI [29]) result.
2. WTWTS produced ≈4% image quality with ≈8% reduction in algorithm run time of the algorithm. (≈ symbol represents approximation)
3. DWTS produced ≈10% quality with reduction of ≈4% in run time.
4. DWTS also consumed more memory space for storing directional information in the tree structure
5. WDWTS produced ≈20% better image quality with ≈6% reduction in run time of the algorithm.
6. Tradeoff of WDWTS is the memory space consumption for weighted directional tree structure.
Chapter 5

Future Work

This research can be taken forward to improve the results. Ideas are as follows:

- Use Deep Neural Network to train machine to learn the algorithm without any assistance from radiologists and train the reconstruction error back into the network to minimize the error. The overview is as shown Figure 5.1.

![Figure 5.1: Overview of Deep Neural Network for CS in MRI](image)

Train Reconstruction Error: Flow chart of the training reconstruction error is shown in Figure 5.2.
Update algorithm for Multi-slice 3D MRI – Multiple slice of Data for one directional image can be considered or even 3D MRI data can be considered as shown in Figure 5.3. The three cross sectional directional of the brain image is represented in Figure 5.4.

Figure 5.3: Multi-Slice of Brain MRI [38]
Dynamic MRI with Tensor will be used to work on 3D MRI data set. Each of the dimensions is considered to be 2D data set along with time. Figure 5.5 shows the overview of 3D MRI to Tensor (3 Directional axis or equivalent of volume data structure)[37, 38].

Figure 5.4: 3D Brain MRI displaying each dimension [40]

Figure 5.5: Dynamic MRI represented in Tensor [46]
The formula for Tensor is similar to CS with SVD but the data is 3D[45]. So the need is to consider Higher Order Single Value Decomposition (HOSVD) as shown in equation below

\[ M = USV^H = S \times_1 U_1 \times_2 V^H = S \times_1 U_1 \times_2 U_2 \]  

(14)

where U & V = unitary matrices, while S = pseudo-diagonal matrix which is represent in equation (15)

\[ S = A_0 \times_1 U_1^H \times_2 U_2^H \times_3 \ldots \times_n U_N^H \]  

(15)

where \( A_0 \) = nth order tensor unfolded matrix as shown below in equation (16):

\[(i_{n+1} - 1)i_{n+2}i_{n+3} \ldots \ldots I_N I_1 I_2 \ldots I_{n-1} + \]
\[(i_{n+2} - 1)i_{n+3}i_{n+4} \ldots \ldots I_1 I_2 \ldots I_{n-1} + \ldots + \]
\[(i_{N} - 1)i_{1}i_2 \ldots \ldots I_{n-1} + (i_2 - 1)i_3i_4 \ldots \ldots I_{n-1} + \ldots + i_{n-1} \]  

(16)

Figure 5.6 represents the 3D tensor and 4D tensor used for SVD calculation. This is commonly used in \( \kappa \)-t FOCUSS [42, 43] method in medical community.

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**Figure 5.6**: Visualization of Higher Order Single Value Decomposition (HOSVD) (a) 3D Tensor (b) 4D Tensor [45]
Chapter 6

Conclusion

The clear impetus of choosing Compressive Sensing (CS) approach is to utilize significant less sampling while generating a high quality reconstruction. Recent methods can reconstruct MR images with good quality from approximate 20% sampling. These results have also shown such trends as well. The proposed methods show improvement over the current benchmark algorithm in this field. Although some algorithms have been proposed to improve standard CS recovery by utilizing the tree structure of wavelet coefficients [10] [11] [12] [13], none of them exploit the tree sparsity to any significant instant and only a few of them conduct experiments on MR images to validate the practical benefit for accelerated MRI [51][52].

This research has future focus to be combined with Deep Neural Network and Multi-slicing of 3D MRI to that would incorporate multi-dimensional data into Compressed Sensing for MRI. This surely would contribute to the existing research.
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Appendix

List of Algorithms

As part of this thesis, some of the well-known algorithms are used and some also experimented with. Here are the list of algorithms names and their pseudo code. All these pseudo code is reference from [54].

1) Iterative Shrinkage-Thresholding Algorithm (ISTA) [26]

Algorithm 1. ISTA

Input: \(\rho = 1/L_f, x_0\)

repeat
   for \(k = 1\) to \(K\) do
      \(x^k = \text{prox}_\rho(g)(x^{k-1} - \rho \nabla f(x^{k-1}))\)
   end for
until Stop criterions

2) Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) [26]

Algorithm 2. FISTA

Input: \(\rho = 1/L_f, r^1 = x^0, t^1 = 1\)

repeat
   for \(k = 1\) to \(K\) do
      \(x^k = \text{prox}_\rho(g)(r^k - \rho \nabla f(r^k))\)
      \(t^{k+1} = \frac{1 + \sqrt{1+4(t^k)^2}}{2}\)
      \(r^{k+1} = x^k + \frac{t^{k-1}}{t^{k+1}} (x^k - x^{k-1})\)
   end for
until Stop criterions
end for

until Stop criterions

3) Composite Splitting Denoising (CSD) [55, 56]

Algorithm 3. CSD

Input: $\rho = 1/L, \alpha, \beta, \{z_i^0\}_{i=1,\ldots,m} = x_g$

repeat

for $j = 1$ to $J$ do

for $i = 1$ to $m$ do

$x_i = \text{argmin}_{x} \frac{1}{2\rho} \|x - z_i^{j-1}\|^2 + g_i(B_i x)$

end for

$x^j = \frac{1}{m} \sum_{i=1}^{m} x_i$

for $i = 1$ to $m$ do

$z_i^j = z_i^{j-1} + x^j - x_i$

end for

end for

until Stop criterions

4) Composite Splitting Algorithm (CSA) [55, 56]

Algorithm 4. CSA

Input: $\rho = 1/L, x^0$

repeat

for $k = 1$ to $K$ do

for $i = 1$ to $m$ do

$y_i^k = \text{prox}_p(g_i) (B_i (x^{k-1} - \frac{1}{L} \nabla f_i(x^{k-1})))$

end for


\[ x^k = \frac{1}{m} \sum_{i=1}^{m} B_i^{-1} y_i^k \]

end for

until Stop criterions

5) Fast composite splitting algorithms (FCSA) [26]

Algorithm 5. FCSA

**Input:** \( \rho = 1/L, r^1 = x^0, t^1 = 1 \)

repeat

for \( k = 1 \) to \( K \) do

for \( i = 1 \) to \( m \) do

\[ y_i^k = \text{prox}_{\rho} (g_i) (B_i (r^k - \frac{1}{L} \nabla f_i(r^k))) \]

end for

\[ x^k = \frac{1}{m} \sum_{i=1}^{m} B_i^{-1} y_i^k \]

\[ t^{k+1} = \frac{1+ \sqrt{1+4(t^k)^2}}{2} \]

\[ r^{k+1} = x^k + \frac{t^{k-1}}{t^{k+1}} (x^k - x^{k-1}) \]

end for

until Stop criterions

6) Fast composite splitting algorithms used for Compressed Sensing (FCSA-MRI) [20]

Algorithm 6. FCSA-MRI

**Input:** \( \rho = 1/L, \alpha, \beta, r^1 = x^0, t^1 = 1 \)

repeat
for $k = 1$ to $K$ do
\begin{align*}
    x_g &= r^k - \rho \nabla f(r^k) \\
    x_1 &= \text{prox}_\rho(2\alpha||x||_{TV})(x_g) \\
    x_2 &= \text{prox}_\rho(2\beta||\Phi x||_1)(x_g) \\
    x^k &= (x_1 + x_2)/2 \\
    t_{k+1} &= \frac{1 + \sqrt{1+4(t^k)^2}}{2} \\
    r^{k+1} &= x^k + \frac{t_{k-1}}{t_{k+1}}(x^k - x^{k-1})
\end{align*}
end for
until Stop criterions

7) Fast composite splitting algorithms with Low Rank Tensor Completion (FCSA-LRTC) [58]

Algorithm 7. FCSA-LRTC

Input: $\rho = 1/L_f, \alpha, \beta, r^1 = x^0, t^1 = 1$
repeat
    for $k = 1$ to $K$ do
        for $i = 1$ to $m$ do
            \begin{align*}
                y^k_i &= \text{prox}_\rho(\alpha_i||x_i||_*) (\gamma^k_i - \rho A^*(A(r^k_i - b)))
            \end{align*}
        end for
        \begin{align*}
            x^k &= \frac{1}{m} \sum_{i=1}^m B_i^T y^k_i \\
            t_{k+1} &= \frac{1 + \sqrt{1+4(t^k)^2}}{2} \\
            r^{k+1} &= x^k + \frac{t_{k-1}}{t_{k+1}}(x^k - x^{k-1})
        \end{align*}
    end for
until Stop criterions
Algorithm 8. Biorthogonalization SVD

**Input:** \(m, n, A\) where \(A\) is \(m \times n\).

**Output:** \(\Sigma, U, V\) so that \(\Sigma\) is diagonal, \(U\) and \(V\) have orthonormal columns, \(U\) is \(m \times n\), \(V\) is \(n \times n\), and \(A = U \Sigma V^T\).

1. \(U \leftarrow A\). (This step can be omitted if \(A\) is to be overwritten with \(U\))
2. \(V = I_{n \times n}\)
3. Set \(N^2 = (\sum_{i=1}^{m} \sum_{j=1}^{n} u_{i,j}^2)\), \(s = 0\), and \(\text{first} = \text{true}\)
4. Repeat until \(s^{1/2} \leq \epsilon^2 N^2\) and \(\text{first} = \text{false}\).
   a. Set \(s = 0\) and \(\text{first} = \text{false}\).
   b. \text{for} \(i = 1, \ldots, n - 1\)
      i. \text{for} \(j = i + 1, \ldots, n\)
         - \(s \leftarrow s + (\sum_{k=1}^{m} u_{k,i} u_{k,j})^2\)
         - Determine \(d_1, d_2, c = \cos(\theta)\), and \(s = \sin(\varphi)\) such that:
           \[
           \begin{bmatrix}
           c & -s \\
           s & c
           \end{bmatrix}
           \begin{bmatrix}
           \sum_{k=1}^{m} u_{k,i}^2 & \sum_{k=1}^{m} u_{k,i} u_{k,j} \\
           \sum_{k=1}^{m} u_{k,i} u_{k,j} & \sum_{k=1}^{m} u_{k,j}^2
           \end{bmatrix}
           \begin{bmatrix}
           c & s \\
           -s & c
           \end{bmatrix}
           =
           \begin{bmatrix}
           d_1 & 0 \\
           0 & d_2
           \end{bmatrix}
           
         - \(U \leftarrow UR_{t,j}(c, s)\) where \(R_{t,j}(c, s)\) is the Givens rotation matrix that acts on columns \(i\) and \(j\) during right multiplication.
         - \(V \leftarrow VR_{t,j}(c, s)\)
5. \text{for} \(i = 1, \ldots, n\)
   a. \(\sigma_i = \sqrt{\sum_{k=1}^{m} u_{k,i}^2}\)
   b. \(U \leftarrow U \Sigma^{-1}\)
9) Principal Component Analysis (PCA) [39, 83]

**Algorithm 9. PCA**

Generate an initial solution $Old\_Config$

for $n = 0$ to # of iterations

   Generate a stochastic perturbation of the solution
   
   $if \ Fitness(\ New\_Config) > Fitness(\ Old\_Config)$
   
   $Old\_Config: =\ New\_Config$
   
   Exploration()

   $else$
   
   Scattering()

   $endif$

end for

Exploration()

for $n = 0$ to # of iterations

   Generate a small stochastic perturbation of the solution
   
   $if \ Fitness(\ New\_Config) > Fitness(\ Old\_Config)$
   
   $Old\_Config: =\ New\_Config$

   $endif$

end for

return

Scattering()

$$p_{scattering} = 1 - \frac{Fitness(\ New\_Config)}{Best \ Fitness}$$

$if \ p_{scattering} > \ random(0, 1)$

   $Old\_Config: = \ random \ solution$
else
    Exploration()
end if
return