A Qualitative Study of the Use of Content-Related Comics to Promote Student Participation in Mathematical Discourse in a Math I Support Class

Jeni M. Halimun
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A QUALITATIVE STUDY OF THE USE OF CONTENT-RELATED COMICS
TO PROMOTE STUDENT PARTICIPATION
IN MATHEMATICAL DISCOURSE
IN A MATH I SUPPORT CLASS

by

Jeni M. Halimun

A Dissertation

Presented in Partial Fulfillment of Requirements for the
Degree of
Doctor of Education

In
Leadership for Learning
Teacher Leadership

In the
Bagwell College of Education
Kennesaw State University
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2011
Dissertation Signature Page

The dissertation of

Jeni M. Halimun 000393647  Adolescent Education Mathematics Concentration

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In many silent moments
I prayed for Your compassion
Finally, God hastened an end
My heart is flung open
Filled with joy and chant
You grant me love and existence

To Dr. Mary Garner, my in-depth gratitude
Patience and trust you poured out
Steward of Math, you are undisputed
A milestone carved on the wood
To be refined by your fortitude
Distinguished and the best simply put

To Dr. Amy Hillen, thank you
Enlightened me with the spirit of a guru
Our inspirations entwine and continue
As a journey to pursue treasure
What makes mathematics true

To Dr. Deborah Wallace, keeper of fire
Your light house beams advice and desire
Tirelessly toils to anchor waver
Bountiful blessings, to render me wiser

To all my teachers,
Dedicated and clever
With minds of philosophers
Never cease to ignite wonder
Talent and interests you foster
Your courage makes life richer
Resilient love flows like a river
Lifted me up on your shoulder
Intellect and humanity you offer
Linking between the hemispheres
Always serving to bridge the barrier
May your field blossom with lavender
To you my debt is beyond infinite number
May your deeds return as spring showers
and my happiness brings you sunflowers

To my beloved family and friends,
Your prayers were my heavenly oasis
You refreshed my strength in this quest
You are the village who raised
And laced comics into thesis
Cured my pain with your embrace
Kept me posted to send sweet bliss
A bond always to cherish
We had crisis and success
But with faith, we grew to harvest
Golden moments of our wishes

My husband, my field of serenity
Savvy and witty you kept my sanity
To endure and win my endless study
You sustained me with tasty culinary
For this Ed.D is no fortune cookie
At last, liberty is a sweet melody
Savor the coffee and sail to the Indies
Everything in between your love is worthy
DEDICATION

I dedicate this dissertation to all Math teachers who give their love of teaching and learning to all students.
ABSTRACT

A QUALITATIVE STUDY OF THE USE OF CONTENT-RELATED COMICS TO PROMOTE STUDENT PARTICIPATION IN MATHEMATICAL DISCOURSE IN A MATH I SUPPORT CLASS

by

Jeni M. Halimun

The purpose of this study is to contribute to existing research on classroom discourse by investigating whether a content-related comic that is closely linked to the learning task can stimulate mathematical discourse in a real high school classroom. To build a math-talk learning community and to analyze the effectiveness of content-related comics in eliciting student participation, I employed the combined theoretical frameworks consisting of a Hufferd-Ackles’ et al. (2004) math-talk learning community and Nathan and Knuth’s (2003) social and analytical scaffolding. The combined frameworks provided a multilevel analytical tool for studying classroom interactions. The results suggested that the content-related comics helped students to become more comfortable and independent in expressing their thinking during class discussion. Recommendation for practicing mathematics educators include: classroom teachers should be encouraged to partner with college level researchers to study mathematical discourse in the classroom, and pre-service and in-service teachers should learn and practice the skills and methods for successfully conducting a whole-classroom discussion. Further research is needed to
investigate the impact of content-related comic activity in a long term interaction with a larger population. Focusing on a relatively simple method of incorporating comics in a mathematical task, this study can serve as a practical example of implementing a feasible and concrete tool to encourage discourse in the mathematics classroom.

*Keywords*: analytical scaffolding; classroom discourse; content-related comics; math-talk learning community; participation; social norms; social scaffolding.
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CHAPTER I

STATEMENT OF THE PROBLEM

This study addresses the problem of low student participation in classroom discourse. One primary cause of students’ low participation is the traditional way of teaching and learning in which students are accustomed to solving routine mathematical problems using prescribed procedures (Herbel-Eisenmann & Cirillo, 2009; Silver & Smith, 1996; Silver, Kilpatrick, & Schlesinger, 1990), rather than actively participating. The motivation to improve student participation in mathematical discourse emerged from a real classroom situation that I faced daily as a teacher: The same few high achievers consistently dominated all discussions in my Math I Support class while the rest of my students rarely contributed to class discussions. This was troubling to me because, as noted by the mathematics education community, students’ active participation is vitally important to their learning (Hatano & Inagaki, 1991; Hiebert & Wearne, 1993; Hufferd-Ackles, Fuson, & Sherin, 2004; Silver, Smith, & Nelson, 1995; Silver & Stein, 1996; Wood & Sellers, 1996). As Selley (2005) observes, “when students have the opportunity to figure out an approach to a problem; discuss, argue, and justify their ideas; and wrestle with challenging mathematics, they are truly engaged in their learning” (p. 2). Thus, my students who rarely participated were missing important opportunities to learn.

Mathematical Discourse: An Important Feature of Classrooms

*Discourse*, as defined by Sfard (2002), includes any act of communicating that influences communicative effectiveness, including body movements, situational clues, and interlocutors’
histories. In the practice of teaching and learning, Lampert (1990) asserts “mathematical discourse is about figuring out what is true, once the members of the discourse community agree on their definitions and assumptions” (p. 160). The shift toward student-centered classrooms in which students become actively involved in making sense of mathematical situations is central to reformers’ vision of desirable school mathematics (Silver, 2009; William & Baxter, 1996). In the mathematics classroom students are learning to think critically in a mathematical way with an understanding that there are many different ways to a solution and sometimes more than one right answer to be compared in a class discussion (Baldree, 2004; Lampert, 1990). Thus, the class discussion is the site where students have the opportunities to focus, elaborate, and reflect on the strategies used for figuring out a problem. Similarly, the National Council of Teachers of Mathematics ([NCTM], 2000) stated “interacting with others offers opportunities for exchanging and reflecting on ideas; hence, communication is a fundamental element of mathematics learning” (p. 348).

The reform vision is portrayed in the Principles and Standards for School Mathematics (NCTM, 2000) hereafter called “the standards” and another document that focuses on teaching and assessment, the Professional Standards for Teaching Mathematics (NCTM, 1991) hereafter called “the teaching standards.” These two documents catalyzed a national movement for teaching reform (Schoenfeld, 2002; Silver, 2009; Silver & Stein, 1996). Similar to the national movement, the Georgia Performance Standards ([GPS], 2008) places emphasis on communication and encourages students to explore multiple strategies, to reason, and communicate using mathematical ideas. In order to meet the demands of the GPS (2008) curriculum mandated at the time of this study, in particular Math I, teachers were called to change the traditional routines for producing and sharing knowledge.
The mathematical discourse literature has documented previous studies that have implemented mathematical discourse to change traditional ways of teaching and learning. Lampert (1990) explores how classroom discourse changes the meaning of knowing and learning in school by posing problems but not providing answers and supporting social interactions to generate mathematical arguments between the teacher and students and among the students themselves. O’Connor and Michaels (1993, 1996) describe classroom conversation as providing opportunities for aligning students with one another and with the content of the academic work while socializing them to take the role of thinkers, hypothesizers, predictors, analysts, and defenders. Schoenfeld (1989) describes the creation of communities of students doing mathematics in a culture of sense-making. Furthermore, Schoenfeld (1989, 2002) emphasizes that, while creating a community of learning, focusing on the structure of the classroom mathematics discussion is the most important effort to insure all students learn mathematics.

Two important aspects were derived from the research studies mentioned above. First, coming to know mathematics is a process that needs social interactions within a classroom community. Second, the interactions provide an opportunity for students to learn through thinking, talking, agreeing, and disagreeing about mathematics. These common views provide new ideas for mathematics classroom instruction that requires teacher and students to interact. Discourse should help the success of all members of a classroom community (Ball, 1993; Bauersfield, 1995; Cobb, Wood, & Yackel, 1993; Hufferd-Ackles, 1999; Hufferd-Ackles et al., 2004; Lampert 1990; Nathan & Knuth, 2003; NCTM, 1991, 2000).

Theoretical frameworks have been developed to study discourse in the mathematics classroom. These theoretical frameworks have been cited frequently in articles about mathematical discourse, including O’Connor and Michaels’ (1993) participant framework
through revoicing, Nathan and Knuth’s (2003) analytical and social scaffolding, Hufferd-Ackles’ et al. (2004) math-talk learning community, and Forman’s (2001) sociocultural framework. Hufferd-Ackles et al. (2004) presents research creating a mathematics discourse community and a theoretical framework to initiate a mathematics classroom community in which the teacher participates in the discussion and students find solutions to problems, articulate, and defend them. Similarly, Nathan and Knuth (2003) formulate the analytical and social scaffolding framework to explain and examine the development of a whole-classroom discussion through researching a teacher’s effort to change her classroom practice.

Although the above research studies have been undertaken and reported in the literature since the early 1990s, Franke, Kazemi, and Battey (2007) and other proponents of discourse practices have indicated that the initiate, respond, and evaluate (IRE) pattern still persists in mathematics classroom practices (Hiebert & Stigler, 2000; Hufferd-Ackles, 1999; Spillane & Zeuli, 1999). Research studies indicate that the implementation of mathematical discourse requires the teachers and students to change thinking about the nature of mathematical knowledge (Hufferd-Ackles, 1999; Hufferd-Ackles et al., 2004; Lampert, 1990; Romagnano, 1994). Lampert (1990) asserts that to understand how to transform mathematics instruction to content and methods derived from the teaching standards (NCTM, 1991) and the mathematics school standards (NCTM, 2000), research and theory need to be applied to the conditions of the real classroom context. Similarly, Franke et al. (2007) state, “There is only little known about what teachers need to do to support classroom discourse in a way that opens participation and supports the development of students’ knowledge and identities” (p. 230). This study is a response to the call for research on how a teacher can implement classroom discourse in her classroom and to examine practical action in a concrete situation (Lampert 1990; Silver, 2009).
Mathematical Discourse: Challenging to Facilitate

Although teachers embrace standards-based instruction, turning a teacher-centered classroom into one in which mathematically eager students participate is no trivial task (Herbel-Eisenmann & Cirillo, 2009; Silver & Smith 1996). An atmosphere of trust and mutual respect is requisite for building an effective classroom learning community where students willingly engage in investigation and discourse (Silver & Smith, 1996). Often, however, students are slow to join the discussion because they are used to watching their teachers explain mathematics concepts, and they accept their roles as passive learners (Hatten, 2009; Herbel-Eisenmann & Cirillo, 2009; Pimm, 2009). In light of the literature about the existing research in classroom discourse, the current study focused on finding ways that encourage students to participate in discourse to share their thinking with the teacher and other students.

A large-scale implementation of a mathematics reform project documented findings about the general ways teachers can support mathematical discussion. The Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project aims to help students develop a meaningful understanding of mathematical ideas through engagement with challenging mathematical tasks (Silver & Smith, 1996; Silver & Stein, 1996). Through the QUASAR project, teachers have the opportunity to initiate and establish mathematics discourse communities in their classrooms. The researchers of QUASAR project find that creating a classroom community in which students explain and actively construct their mathematical ideas through interactions with others in QUASAR middle school classrooms is critical for the development of mathematical discourse communities (Silver et al., 1995).

The first step in orchestrating mathematical discourse is to create an atmosphere in which each student can feel safe to ask questions and express his or her thinking (Chapin, O’Connor, &
Anderson, 2009; Hufferd-Ackles et al., 2004). The challenge, then, is to make mathematics discourse accessible for beginners. As Silver and Smith (1996) suggest, “It seems reasonable to begin in a safe, possibly non-mathematical space, in which students may initially be more comfortable, and then move gradually to settings in which the mathematical ideas are salient in the discussion” (p. 24). One way to do this is by relating the unfamiliar mathematical experience to familiar experiences (Artzt & Newman, 1997; Clark, 1998; Curcio & Artzt, 1998). The familiarity and the relevance of the context of a problem seem to give rise to student responses and subsequent success (Clark, 1998). Using familiar and relevant contexts for mathematical activities, the teacher can support the students by building their confidence, which eventually will encourage them to contribute to mathematical discourse (Silver & Smith 1996).

One possible solution: Utilizing Humor to Provide Access

Research indicates that most students enjoy humor and are familiar with drawings and story-telling, which renders comic strips an engaging and appealing classroom tool (Hutchinson, 1949; Reeves, 2007; Wright, 1976). The mathematics classroom is an especially promising site for comics, because prior studies suggest that creating a sense of familiarity can foster students’ interest in challenging tasks (Clark, 1996; Dienstbier, 1995; Garner, 2006; Ziv, 1988). Consistent with the hypothesis that content familiarity fosters active participation, studies show that students appear to respond well to puzzles and comics (Hutchinson, 1949; Gorham & Christophel, 1990; White, 2001; Ziv, 1988). Thus, including comics in mathematical learning is a way to employ familiar items in order to pique students’ interest in a subject that is otherwise considered difficult and unfamiliar (Dientsbier, 1995; Garner, 2006; Mitchell, 2005; Torok, McMorris, & Lin, 2004; Weaver & Cottrell, 1987; White, 2001). Similarly, Clark (1998) and MacGregor’s (1998) research studies indicate both the familiarity and the relevance of the
context of a problem seem to have an effect on whether the problem is accessible to the students. As found in a very old research study of the use of comics as instructional material, students are more likely to participate in class when exposed to recognizable material (Hutchinson, 1949). They are more comfortable contributing their responses when they hear uninhibited incorrect answers and laughter from their peers (Garner, 2006; Mitchell, 2005; Weaver & Cottrell, 1987; White, 2001). The use of comics can potentially create this sense of familiarity, a non-mathematical and safe space, which eventually may stimulate students’ participation in mathematical discourse.

Statement of Purpose and Research Question

This research addresses the problem of low student participation in mathematics discourse by introducing content-related comics—comic strips and puzzles with embedded mathematics questions or problems—into the classroom. In particular, the following research question will be explored: To what extent does teaching with content-related comics support student participation in mathematical discourse? Seeking a way to elicit student participation in our whole-classroom discussion, I hypothesized that students’ enjoyment of and familiarity with comics would improve student participation in mathematical discourse in my Math I Support class. To explore the research question, an action research methodological tradition, which includes participant observational method, was selected (Bartolini Bussi, 1998; Cestari, 1998; Garfinkel, 1967; Lampert, 1990). Student engagement in mathematical discourse was examined without comics and with comics through action research.

Significance of the Study

This study contributes two innovative actions from the high school classroom context. First, a math talk learning community was implemented and analyzed in my Math I Support
class using a combination of theoretical frameworks from the literature. The combined theoretical frameworks consist of a math-talk learning community framework (Hufferd-Ackles et al., 2004) and analytical and social scaffolding framework (Nathan and Knuth, 2003). Second, a method for increasing student participation was tested to find whether a content-related comic strategy can support mathematical discourse in a real high school classroom.

This proposed study combined theoretical frameworks to equip teachers to understand classroom interactions in a complete manner. This is because our attention cannot be focused on one facet only, such as students’ mathematical ideas, but rather to the whole process of growth, be it interaction around mathematics content or classroom social norms (Erickson, 1996; Nathan & Knuth, 2003; Wood, 1998). The combined theoretical frameworks enabled me to coordinate a multilayer analysis that explains different facets of classroom discourse where each is complementary to another. In particular, the analytical and social scaffolding framework brings classroom social norms to the forefront of classroom discourse development. Using the combined theoretical frameworks, the analysis includes the description of the interactions between the students and myself, and among the students themselves from different perspectives.

Of further significance, teaching in an ordinary high school classroom provided me the opportunity to implement these theoretical frameworks in a real classroom and in a concrete way. As a teacher I have the opportunity to continually assess and observe my students. This access advantage, coupled with systematic research inquiry, provided concrete and theory-informed results directly applicable to the classroom environment.

A large number of studies have proposed ways to engage students in mathematical discourse (Sfard, 2002; Silver, 2009). Clearly, there is a strong interest in mathematical
discourse. However, the trend to promote mathematical discourse is dominated by college level education researchers, and the actual implementation is mostly by college level instructors (Silver, 2009). Research of mathematical discourse in high school is needed, as this has not become a common practice among classroom teachers (Lloyd, Wilson, Wilkin, & Behm, 2005; Spillane & Zeuli, 1999; Stigler & Hiebert, 1999).

This research sought implementation closure between researchers at the college level and educational practitioners at other levels. As discussed before, there is a barrier between college level researchers who delve into conceptualized theories and practice of mathematical discourse and classroom teachers who are too overwhelmed to make change possible in their classroom (Black, Harrison, Lee, Marshall, & Wiliam, 2003; Erickson, 1982; Silver, 2009). As a teacher-researcher, I responded to this academic dichotomy by designing a systematic inquiry into the use of content-related comics as a strategy for stimulating discourse. Thus, in this study the methodological action research project was used to connect educational theory and research studies to my classroom practice and examine practical action in a concrete situation so that theory and practice develop interactively (Hubbard & Power, 1999; Johnson, 2002; Lampert, 1990; Romagnano, 1994).

Creating or increasing the level of mathematical discourse as part of regular classroom practice is a common and widespread challenge in high school education. This fuels the current systematic inquiry of whether comics can serve as an effective tool for stimulating discourse in classroom community. The method used is an instructional plan in which the teacher-researcher poses a mathematical problem using an ordinary object which, in turn, can become a tool to initiate mathematical discourse. To investigate the effectiveness of content-related comics in eliciting student participation, I examined whether there is an increased level of questioning,
responding, explaining, and arguing mathematical ideas in our whole-classroom discussion. Eventually, the results of this study can inform high-school educational practitioners, especially mathematics teachers, of the usefulness of content-related comics in fostering active classroom participation.

Conceptual Framework

Two theoretical frameworks were used in this study: 1) Hufferd-Ackles’ et al. (2004) math-talk learning community framework and 2) Nathan and Knuth’s (2003) analytical and social scaffolding framework. The combined frameworks from Nathan and Knuth (2003) and Hufferd-Ackles’ et al. (2004) are grounded in social constructivism theory that proposes the two levels of discourse, “at one level, the topics of discourse were mathematical, [and] at the other level they were social norms that regulate the activity of doing and talking about mathematics” (Cobb et al., 1993, p. 105). Classroom social norms represent a set of rules about the expectations and obligations for students’ learning behaviors that regulate the activity of doing and talking about mathematics (Cobb et al., 1993; Cobb, 1994; Cobb & Bauersfeld, 1995). As part of the development of mathematical discourse, classroom social norms become a new way of knowing mathematics and guiding student participation (Gutierrez, 1993; Wood 1998).

While implementing the GPS Math I curriculum, Hufferd-Ackles’ et al. (2004) math-talk framework was applied as the initial step to understand and build a math-talk learning community in which teacher and students use discourse to support the mathematical learning of all participants. The math-talk framework contains key components of a math-talk learning community, describing how to evaluate and monitor the degree of students’ participation when taking part in a math-talk learning activity.
In a math-talk learning community, a whole-classroom discussion was facilitated for students to question, explain math thinking, propose mathematical ideas, and lead discourse for learning. Each part of this activity corresponds to levels 0 to 3, indicating the progress of the students as they become more engaged in a math-talk learning community. The combination of the four components and the level of growth in math-talk provide a basis from which I developed a theoretical framework influenced by other frameworks, as discussed below, to examine mathematical discourse in the current study.

In building a math-talk learning community, the importance of social participation in the classroom must be taken into account (Lemke, 1990; O’Connors and Michaels, 1996). Since the comics provide familiar topics related to daily experience (Garner, 2006; White 2001), mathematical activities were designed using content-related comics to encourage student contribution to a math-talk learning community. Therefore, the use of content-related comics provided an avenue for all students to participate. Principally, I anticipated the content-related comics would provoke students to advance from level 0 to 1 and higher in questioning and explaining mathematical thinking, which is a critical step in stimulating initial participation and development of classroom social norms.

Upon establishment of the first level of math-talk, the teacher and students created classroom social norms as the background in which students felt more comfortable sharing their thinking, ideas, arguments, questions, and revisions (Cobb, 1994; Hufferd-Ackles et al., 2004; Rogoff, 1990). In this atmosphere, more participants engaged in mathematical and social practices by negotiating ideas, constructing plans, and achieving goals through collaborative discourse, enabling them to move to levels 2 and 3 of the math-talk learning community (Cobb et al, 1993; Hufferd-Ackles et al., 2004; Lampert, 1990).
In addition, the examination of math-talk components and level of math-talk was combined with the analytical and social scaffolding framework described in Nathan and Knuth’s (2003) research (“A Study of Whole Classroom Mathematical Discourse and Teacher Change”). This provided another analytical tool that examined the social interactions that occur as the background during classroom discourse (Bartolini Bussi, 1998; Nathan & Knuth, 2003). The analytical and social scaffolding framework includes (a) the concept of horizontal and vertical interactions, and (b) the form and content of discourse as a means of describing how participants communicate with one another using social and analytical scaffolding (Nathan & Knuth, 2003; William & Baxter, 1996; Wood, 1998). Combining a math-talk theoretical framework with concepts of horizontal and vertical interaction—and social and analytical scaffolding from Nathan and Knuth’s (2003) theoretical framework—the teacher-researcher examined different patterns of interaction that emerge within the classroom.

Definitions of Terms

This study used the following definitions of terms:

*Activity structure* is a pattern of organization where events of specific kinds tend to follow one another in a more or less definite order (moment-to-moment basis). A classroom lesson has this type of activity structure (Lemke, 1990; O’Connor & Michaels, 1993).

*Classroom social norms* refer to a set of rules about the expectations and obligations of an individual member’s behavior that influences the regularities of classroom community. In other words, a student knows that he is expected to explain his thinking (Franke et al., 2007; Cobb & Yackel, 1996; Wood, 1998).

*Comics* are form of pictorial art that includes features such as (1) continuing characters, (2) frames that show action, and (3) dialogue in balloons with a humor theme (Berger, 1989).
Content-related comics prompts consist of comic strips, puzzles, and questions focusing on the students’ interests and their relation to mathematical tasks.

Cognitive behavior is mental activity demonstrated by learning, thinking, remembering, and the performance of one’s knowledge (Ormrod, 2008).

Discourse is defined as acts of communicating and any aspects of communication that influence its effectiveness: for example, body movements, situational clues, and interlocutors’ histories (Sfard, 2002).

Episode is a whole-classroom discussion consisting of a warm-up or closing activity.

Interlocutor is the one who takes part in dialogue or conversation (Merriam-Webster, 1993).

Math-talker refers to individuals who respond/talk during our whole-classroom discussion.

Math-talk learning community refers to a classroom community in which the teacher and students use discourse to support the mathematical learning of all participants (Hufferd-Ackles et al., 2004).

Prompt is a tool to elicit the students’ responses whereby students explain and understand a given problem. In this study, the teacher uses a content-related comics prompt to invoke the students’ mathematical thinking (Lesh, 1985; Resnick, 1989).

Scaffolding refers to the support for a student’s cognitive activity provided by an adult when students and teacher perform a task together; or the support provided by another student in a joint problem solving activity (Resnick, 1989).

Analytical Scaffolding is the scaffolding of mathematical ideas for students (Williams & Baxter, 1996).
Social Scaffolding is the scaffolding of norms for social behavior and expectations regarding discourse (Williams & Baxter, 1996).

Socialization refers to the long-term process by which personal habits and traits are shaped through participation in social interactions that involve rights and responsibilities (Resnick, 1989; O’Connor & Michaels, 1996).

Summary

The purpose of this study is to contribute to existing research on classroom discourse by investigating whether a content-related comic that is closely linked to the learning task can stimulate mathematical discourse in a real high school classroom. The method used is an instructional plan in which the teacher-researcher poses a mathematical problem using an ordinary object which, in turn, can become a tool to initiate mathematical discourse. Focusing on a relatively simple method of incorporating comics in a mathematical task, this study can serve as a practical example of implementing a feasible and concrete tool to encourage discourse in the mathematics classroom.

Organization

Chapter 2 is a review of the literature relevant to this study: theoretical perspectives about communication in the mathematics classroom, theory of humor and mathematics, and mathematical activities for classroom discourse implementation. Chapter 3 outlines the research rationale and procedure, which includes learning task sequences and data collection method. Chapter 4 reports the findings of the current study. Finally, chapter 5 presents discussion of results, conclusion, and implication of the study.
CHAPTER II

REVIEW OF THE LITERATURE

Introduction

This study investigated how content-related comics in mathematical teaching can support participation in a math-talk learning community. A math-talk learning community is a community in which teacher and students use discourse to support the mathematical learning of all participants (Hufferd-Ackles et al., 2004). The following section reviews the literature on classroom discourse. Most relevant to this study are three categories of literature: theoretical perspectives about classroom discourse, theory of humor and mathematics, and mathematical activities for classroom discourse implementation.

My review of literature on classroom discourse revealed that the development of classroom interactions in which individual students reorganize their beliefs about their own role, others’ roles, and the general nature of mathematical activity requires greater attention (Cobb et al., 1993; Cobb & Yackel, 1996, Lampert, 1990, 2001; Franke et al., 2007). Cobb and his colleagues propose that “students’ mathematical learning is influenced by both the mathematical practices and the social norms negotiated and institutionalized by the classroom community” (Cobb et al., 1993, p. 114). Classroom social norms are defined as the set of rules about the expectations and obligations of an individual member’s learning behavior (Cobb & Yackel, 1996; Wood, 1998). Consistent with Cobb and colleagues’ view about classroom social norms influencing the inquiry approach to mathematics, Silver and Smith (1996) and project
researchers in Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) emphasize that developing the social norms should precede the development of mathematical discussion, especially at the initial stage of forming a mathematical discourse community (Silver et al., 1995). The barrier to implementing classroom social norms in discourse practice, as researchers have indicated, is that students are not accustomed to sharing their thinking publicly (Pimm, 2009; Silver et al., 1990; Silver et al., 1995; Silver & Smith, 1996). Thus, practicing classroom social norms as mathematical tools and standards of knowing mathematics can create resistance especially for high school students who have already experienced a great deal of traditional school (Silver et al., 1990; Silver & Smith, 1996). Proponents of classroom discourse emphasize that further study needs to elaborate the development of classroom social norms in a way that opens participation and supports the development of students’ knowledge and identities (Cobb et al. 1993; Franke et al., 2007; Lampert, 2001).

Considering that classroom social norms underlie the success of initiating and maintaining mathematical discourse that enable students to support each other’s learning, in this study the researcher developed mathematical activities to cultivate classroom social norms. The design of mathematical activities that include content-related comics aligns with the goal of fostering discourse where teachers invent teaching and learning strategies for students to collaborate and construct mathematical concepts (Cobb, 1994; Cobb et al., 1993; Cobb & Yackel, 1996; Franke et al., 2007; Hufferd-Ackles et al., 2004; Lampert, 1990). Thus, this study took the form of a qualitative study in which the teacher-researcher examined her own classroom as she taught in an ordinary setting (Bartolini Bussi, 1998; Lampert, 1990). The results of this study contribute to the field of classroom discourse practices, since at present only a few studies have
used the tools and concepts of discourse analysis in secondary level classrooms (Herbel-Eisenmann-2009; Silver, 2009).

Previous studies have suggested valuable methods for establishing and maintaining the norms that govern classroom interactions and mathematical work (Cobb et al., 1993; Cobb & Yackel, 1996; Lampert, 1990). For example, Cobb and Yackel (1996) argue that the development of classroom social norms and individual students’ beliefs are related. Under this view, individual students are seen as actively contributing to the development of classroom mathematical practices. The underlying classroom social norms afford and constrain what is learned, how it is learned, and which students learn it (Cobb, 1994; Cobb & Yackel, 1996; Franke et al., 2007; Wood, 1998). Following up on Cobb and his colleagues’ work, I narrowed the research topic to the study of creating opportunities for participation. In the current study, the teacher used students’ contributions to ask more questions and to support the learning of both the individual and the group. To achieve this goal, the researcher needed to coordinate several strategies, including: getting students to participate together in ways that challenge one another’s thinking and justify their ideas, designing content-related comics as a context of learning, and developing a math-talk learning community.

My first strategy for creating opportunities for participation, following Cobb et al. (1993), was to adopt a perspective that focuses on the culture of the classroom community so as to understand the role of individual participants in a discourse community. The theoretical perspective for this study is rooted primarily in the social constructivist’s approach that includes an interactionist lens for analyzing the evolution of social norms (Cobb & Yackel, 1996). Since the social constructivist perspective is a combination of two prominent theories—the
sociocultural and the constructivist theories—my literature review includes a discussion of both of these perspectives.

For the purpose of launching an innovative strategy, the literature review exposes theory and practices to integrate comics as a teaching tool and align this tool with other mathematical activities (Glenn, 2002; Hutchinson, 1949; Jensen, 1999; Ormrod, 2008; Jonas, 2004). Since the main focus of interest was to examine participation in a discourse community, this study used the components and levels of a math-talk learning community framed by Hufferd-Ackles et al. (2004), which provide an elaborate path for analyzing student’s discursive actions while building a discourse community. As an analytical tool in this study, the theoretical perspective of a math-talk learning community relies on social constructivist and sociocultural theories (Cobb, 1994; Hufferd-Ackles, 1999; Hufferd-Ackles et al., 2004)

Overview of Chapter 2

Three categories of literature are relevant to this study: theoretical perspectives about communication in a mathematics classroom, theory of humor and mathematics, and mathematical activities for classroom discourse implementation. To sufficiently understand communication in the mathematics classroom, the first part of the review covers major theoretical approaches. Many scholars use sociocultural theory (Vygotsky, 1978, 1986, 1987) to provide a viewpoint for studies of language and learning processes (Sierpinska, 1998; Forman, Minick, & Stone, 1993; Hicks, 1996; Resnick 1991). Several other theoretical perspectives that provide lenses to understand communication are constructivist (Piagetian), interactionist (Brunerian), and social constructivist (Bauersfeld, Krummheuer, & Voigt, 1988; Cobb, 1994; Cobb & Yackel, 1996; Sierpinska, 1998). These theoretical perspectives influence certain aspects of sociocultural theory and vice versa (Cobb & Yackel, 1996; Sierspinska, 1998). As Cobb and
Yackel (1996) and other theorists indicate, there exists an apparent consensus that these perspectives are at least partially complementary (Cobb, 1994; Confrey, 1995; Hatano, 1993; Resnick, 1991; Smith, 1996).

The second part reviews theory of humor and mathematics and the potential usage of comics as instructional materials. The existing theory about humor and mathematics, coupled with the classroom discussion theories, provided a foundation for the research question guiding this study.

The third part of the literature review evaluates empirical studies focused on the development of mathematical activities to promote classroom discourse. This review draws from a broad range of classroom discourse studies, and eventually aims to construct a theoretical framework and a qualitative methodology implementable in the classroom.

**Review of Theoretical Perspectives**

*Sociocultural Theory*

Research in the field of mathematical discourse classifies communication patterns in the traditional mathematics classroom as either a teacher-dominated or a transmission-oriented model of instruction (Cazden, 2001; Doyle, 1985; Mehan, 1985). The teacher-dominated pattern, also known as the IRE sequence, consists of a teacher-initiated question, student response, and teacher evaluation. Instructional practices employing the traditional approach tend to foster a passive learning in which students seldom participate in their own learning of mathematics (Stigler & Hiebert, 2000). In contrast, in the reform-oriented classroom, the instructional strategy focuses on developing a community of discourse in which students take a more active role in building on one another’s thinking (Cobb et al., 1993; Lampert, 1990; Hufferd-Ackles et al., 2004; Michaels, O’Connor, Resnick, 2007; Silver & Smith, 1996; Silver, 2009).
The literature broadly uses the term discourse as language-in-action which is language used in a variety of contexts as part of social practice (Hicks, 1996; Sierpinska, 1998). For the scope of teaching and learning mathematics, Sfard (2002) defines discourse as acts of communicating and anything that goes into communication that influences its effectiveness, such as body movements, situational clues, and interlocutors’ histories. The broad meaning of the term discourse implies observable instances for analysis of classroom discourse (Sfard, 2002). Hypothesizing the link between learning mathematics and mathematical discourse, Sfard (2002) further proposes that “thinking may be conceptualized as a case of communication” (p. 26). This hypothesis is related to Vygotsky’s (1987) idea that “thinking arises as a modified private version of interpersonal communication” (Sfard, 2002, p. 26). Notably, the idea that thought is an internalization of initially social processes (Resnick, 1989, 1991) is compatible with the claim that children learn through participation in social contexts (Hicks, 1996).

The above authors’ works represent only a few of an increasing number of articles about theoretical perspectives of learning that focus on discourse to alternate IRE discourse structures. The major trend in the study of discourse during the past decades draws from Vygotsky’s (1986, 1987) work on the relation between natural language, spontaneous thought, and scientific concepts (Sierpinska, 1998). This trend derives its name—sociocultural theory—from its emphasis on the socially and culturally situated nature of mathematical activity (Cobb, 1994; Hicks, 1996). The proponents of sociocultural theory view human cognition as varied and sensitive to cultural context, and urge that we must seek to understand the mechanisms by which people actively shape each other’s knowledge and reasoning processes (Forman et al., 1993; Resnick, 1991). On the contrary, constructivists assume everything an individual knows is personally constructed. For example, a constructivist teacher will guide the student to form his
own ideas and understanding of mathematical relations and properties (Bartolini Bussi, 1998). Resnick (1991) propounds the constructivist view that directly experienced events are only part of the basis for knowledge construction because people also build their knowledge from a variety of instances (asking questions, discussing problems, and explaining ideas) and sources (in the forms of writing, pictures, gestures, sounds etc). Principally, theorists who lean toward social phenomena hold a view that people are, by nature, social creatures, and most of their learning depends on the people around them (Lave, 1985; Ormrod, 2008; Resnick, 1991; Sierpinska, 1998; Wertsch, 1991).

Using the viewpoint of sociocultural theory, a significant number of theorists have developed an understanding of the role discourse plays in children’s learning (Hicks, 1996; Palinscar, Brown, & Campione, 1993). This observation regarding the role of discourse supported by sociocultural theory opens a channel of inquiry in educational context and the teaching and learning processes (Hicks, 1996; Forman et al., 1993). Building on Vygotsky (1978) and Mead’s (1934) ideas, Resnick (1989, 1991) proposes that social experience can shape habits and skills of interpretation and meaning construction available to individuals, including students in the classroom setting. Similarly, Ormrod (2008) explains Vygotsky’s (1986) proposal that complex mental processes begin as social activities where children gradually internalize ways of thinking that they first use in social settings. Developing from the age of two, the child discovers “the symbolic function of words” (Vygotsky, 1962, p. 43) and starts learning to name things. Put another way, “thought becomes more verbal; speech becomes more intellectual” (Sierpinska, 1998, p. 42). These statements nicely summarize Vygotsky’s social, cultural, and historical theories of learning which postulate that language and other culturally significant symbolic systems are the means for achieving thinking or a specific goal that emerges during the
event of active individual participation in the social practice (Cobb, 1994; Hicks, 1996; Minick, 1989; Moll, 1990).

Vygotsky’s theory regarding the internalization of dialogues initially experienced in social context gave rise to research studies on children’s social construction of knowledge through participation in activity mediated by language (Hicks, 1996; Palinscar et al., 1993; Resnick, 1991; Wertsch, 1991). The investigation of the role that discourse plays in children’s learning has become a focus of study across disciplinary and subdisciplinary boundaries, including the fields of psychology and education (Hicks, 1996; Resnick, 1991; Sierpinska, 1998). In the mathematics classroom, for instance, there is a mutual relationship between thought and language that follows certain assumptions (Sierpinska, 1998). When teachers set demanding standards on the intellectual activities in which students ask questions, express their thinking, and justify their claims publicly to better understand different forms of mathematical explanations, the teacher can then facilitate discourse to align students’ thinking with one another and with the content (Franke et al., 2007; Hatano and Inagaki, 1991; Lampert, 1990; O’Connor and Michaels, 1996). This makes it possible to advance students’ logical thinking to a higher level (Cobb et al., 1993; Resnick, 1991; Sierpinska, 1998).

The above theory is congruent with the hypothesis from studies of classroom processes showing that organized and consistent intellectual practices in which students take on various roles to socialize their thinking and talking are associated with complex thinking and problem solving (Chapin et al., 2009; Lampert, 1990; O’Connor & Michaels, 1993, 1996; Palinscar et al., 1993; Resnick, 1989). The findings from these socialized learning studies encourage the shift from instruction to socialization where together the teacher and students build a community of validators (Cobb et al., 1993; Resnick, 1989). The teacher’s role is to engage students in a form
of language socialization by encouraging students to participate in various roles and stances in a
dialogue between the students (Cobb et al., 1993; O’Connor & Michaels, 1993, 1996). O’Connor and Michaels (1996) assume that students who are frequently and successfully engaged in these intellectual activities “are learning interactional routines and practices that will continue to work for them in other settings” (p. 64). However, it is equally important to understand what entails the socialization of learning as Resnick (1989) warns:

> When we describe the process by which children are socialized into these cultural patterns of thought, affect, and action, we describe long-term patterns of interaction and engagement in a social environment, not a series of lessons in how to behave or what to say on particular occasions. If we want students to treat mathematics as an ill-structured discipline —making sense of it, arguing about it, and creating it, rather than merely doing it according to prescribed rules—we will have to socialize them as much as to instruct them. (p. 58)

Researchers are increasingly recognizing the importance of Vygotsky’s concept of learning in social interaction (O’Connor & Michaels, 1996). They examine social interactions to develop effective teaching and learning conditions in which joint problem solving occurs, guided by the teacher who is skilled in conducting intellectual activities through discourse (Erickson, 1996; Palinscar et al., 1993). The intellectual activities in teaching and learning are beneficial when the engagement of expert (teacher) and novice (student) occurs in the zone of proximal development (ZPD) (Bartolini Bussi, 1996; Erickson, 1996; Hufferd-Ackles et al., 2004; Palinscar et al., 1993; Seeger, 1998). Palinscar et al. (1993) incorporate Vygotsky’s (1978) idea on the ZPD. She explains, “Vygotsky proposed that this region could be best understood by considering both the actual developmental level of the individual and the potential
developmental level” (p. 44). Researchers suggest that the teacher supports the viable interests and participation of the learners in ZPD by providing assistance to complete and extend the actions and insights of the student (Erickson, 1996; Hufferd-Ackles et al., 2004; Palinscar et al., 1993).

My study grew from the sociocultural tradition of teaching and learning. Vygotsky’s (1978) zone of proximal development concept provides a logical framework to analyze the teacher and the students’ movement through their own learning zones of proximal development to build a math-talk community in which participants use discourse to support the mathematical learning of all (Hufferd-Ackles et al., 2004). Consistent with Vygotsky’s (1978) emphasis on the importance of social interaction between the novice and more knowledgeable others in the ZPD and the role of language as psychological tools for thinking (Cobb, 1994), in the current study, the participants in a math-talk community assisted one another in a recursive process as they moved through several levels of development. Supporting classroom discourse through the ZPD for long term implementation, I anticipated that students would participate in mathematical and social practices by negotiating ideas, constructing plans, and achieving goals as a classroom community. The predicted outcome of frequent and consistent engagement in this collaborative and intellectual activity would be the improvement of students’ math-talk level from assisted to independent (Hufferd-Ackles et al., 2004).

Cobb (1994) and Rogoff (1990) summarize the social interaction approach through ZPD as the interplay between individual mathematical activity and participation in social engagement that increasingly enables the student to take the role of the expert. The authors propose that this social participation constitutes the foreground of a child’s mathematical development with active individual learning as the background. Similarly, my study aimed to increase individual
participation in discourse through social practices. Rooted in sociocultural theory, the observation and analysis of individual participation moving from one level to the next level of math-talk were described in a framework of math-talk learning community (Hufferd-Ackles et al., 2004).

Constructivist and Social Constructivist Theories

In contrast to the sociocultural view about the influence of individual participation in cultural practice, Cobb (1994) and his colleagues take the constructivist approach to “analyze thought in terms of conceptual processes located within the individual” (p. 14). In the constructivist view, knowledge refers to the internal mental constructions of the individual (Bransford, Brown, & Cocking, 2000; Ormrod, 2008; Sierpinska, 1998). Under the constructivist’s view, when the teacher gives students a problem, the individual student will construct the concept from her mental structures as an answer to a problem that she considers her own (Ormrod, 2008; Sierpinska, 1998). Essentially, constructivists view students’ mathematical activity as psychological and individualistic by nature (Cobb, 1994; Cobb & Yackel, 1996). However, the constructivists’ stance that learning is about self-organization displays kinship to Vygotsky’s idea that knowledge cannot be learned as recipes or by rote memorization of rules and formulas (Sierpinska, 1998).

In constructivism, learning is a process of self-organization in which the student reorganizes his or her construction of knowledge to eliminate conflicts (Cobb, 1994; Cobb & Yackel, 1996; von Glasersfeld, 1992). Conflict arises as part of a child’s self-organization process (von Glasersfeld, 1989). Speaking from the constructivist viewpoint, Sierspinska (1998) explains that there is a continuous conflict between the student’s spontaneous thinking and the student’s non-spontaneous learning of concepts (for example, formulas and definitions learned at
school are different from those used at home). In this model, conflicts in the individual student’s mathematical interpretations might become a driving force to mathematical development (Cobb & Yackel, 1996; Sierpinska, 1998; von Glasersfeld, 1992).

Although the individual construction process is central to the constructivist camp, Cobb (1994) and von Glasersfeld (1989) account for the implicit role of social interaction and assert that the major source of conflicts for the individual’s cognitive development is interaction with others. This observation that the constructive activity of self-organization occurs as the individual interacts with others leads to the path of blending the sociocultural and constructivist perspectives (Cobb, 1994; Cobb & Yackel, 1996). Consequently, Cobb and Yackel (1996) revised their early approach that “social interaction was viewed as a catalyst for otherwise autonomous psychological development because it influenced the process of mathematical development but not its products, increasingly sophisticated mathematical ways of knowing” (Cobb & Yackel, 1996, p. 212). The authors found in their classroom-based research (Cobb, Yackel, & Wood, 1989) that students and teacher jointly establish the classroom participation structure that constitutes the social norms, sociomathematical norms, and mathematical practices in the course of their classroom interactions (Bauersfeld, 1980; Cobb, 1994; Cobb & Yackel, 1996). This finding led to the new conjecture that while students contribute to the evolution of social norms, they reorganize their individual beliefs about their own role, others’ roles, and the general nature of mathematical activity (Cobb et al., 1989). Cobb’s et al. (1989) study on the development of social norms, sociomathematical norms, and mathematical practices will be addressed after the discussion of theoretical perspectives.

The above account correlates psychological and sociological terms in analyzing individual mathematical development in the classroom. Cobb and Yackel (1996) redefined their
psychological concept of autonomy as a characteristic of an individual mathematical activity “to be social through and through because it does not develop apart from their participation in communities of practice” (p. 214). The authors coordinate the two major theoretical principles, sociocultural and constructivist theories, into the social constructivist perspective by formulating their complementary aspects that mathematical learning involves both active individual construction (Glasersfeld, 1992, 1995) and enculturation (Rogoff, 1990) processes. Departing from the radical constructivist position, the social constructivist perspective views learning as social practices in which the social practices of an individual form the background against which his self-organization comes to the foreground (Sierpinska, 1998; von Glasersfeld, 1992, 1995; Wood, 1998). Conversely, active individual construction processes constitute the background against which guided participation in social practices comes to the foreground (Cobb, 1994; Rogoff, 1990; Sfard, 2002).

Cobb and Yackel (1996) indicate the advantage of two ways of analyzing classroom activity, psychological and social analyses, afforded by the social constructivist perspective. In classroom mathematical practice, “analysis from the psychological constructivist perspective brings out the heterogeneity in the activities of members of the classroom” (Cobb & Yackel, 1996, p. 214). Complementing the psychological organization of beliefs, social constructivist perspective adopts the interactionist approach that gives another lens to analyze the sociological aspects of classroom microculture, namely classroom social norms, sociomathematical norms, and classroom mathematical practices.

As the researcher planned to explore the zone of proximal development concept (Vygotsky, 1978) where students engage in guided participation, she applied the social constructivist perspective to analyze individual participants in social practices, including
teachers, constructing their own knowledge and reflecting upon and discussing this knowledge (Cobb & Yackel, 1996; Hufferd-Ackles et al., 2004). The social constructivist perspective is relevant to my study because it combines the social and psychological lenses, enabling researchers to view learning in two ways. This is possible by examining student’s beliefs and understandings, and locating these beliefs within the context of classroom norms and practices (Bowers, 2004). Grounded in social constructivist perspective, when conducting psychological constructivist analysis, the researcher focused on individual students’ activity as they participated in a math-talk learning community and documented their re-organization of their beliefs (Cobb & Yackel, 1996). Based on this perspective, the analysis described the interpretations and mathematical thinking of individuals as they participated in a math-talk learning community.

The Interactionist Perspective

The interactionist viewpoint is useful to impartially observe the evolution of social interaction processes in the zone of proximal development (ZPD) through which students participate and improve their habits and skills to acquire a conceptual understanding (Cobb & Yackel, 1996; Erickson, 1996). The interactionists’ focus is to study classroom communication as an impartial observer (Cobb & Yackel, 1996; Sierpinska, 1998; Steinbring, 1998) and accept that “ordinary language is all right” (Wittgenstein, 1969, p. 28). Juxtaposed to the interactionist view, Piaget and Vygotsky’s theories tend to analyze speech activity in terms of how it can be corrected. Furthermore, the constructivism and sociocultural theories associate psychological research with the study of the development and the process of the individual mind, while the interactionist’s theory incorporates psychological research for which the object of study is the interactions, not the psychological subject (Sierpinska, 1998). Although their psychological focuses are different, the interactionist’s view complements the constructivist’s view in that both
see communication as a process of mutual adaptation wherein individuals negotiate meanings by continually modifying their interpretations (Cobb, 1994; Bauersfeld, 1980; Bauersfeld et al., 1988).

Furthermore, the interactionist approach is integrated within the social constructivists’ perspective. In social constructivist analysis, the interactionist perspective gives another lens to put forth social interaction that stimulates and shapes the processes of individual construction and brings shared meaning to the foreground of our attention (Erickson, 1996; Sierspinska, 1998). Principally, the interactionist perspective examines sociological aspects of classroom microculture that constitutes classroom social norms, sociomathematical norms, and classroom mathematical practices.

Interactionist proponents have developed the idea of shared activity in which negotiation of meanings occurs and becomes part of a discourse (Bruner, 1985; Kanes, 1998; Sierspinska, 1998). Since the particular context of language use influences meaning, members of a community of discourse find the meanings of mathematical explanations when they share these tools (language as psychological tools for thinking [Vygotsky, 1978]) with others (Gergen, 1995; Kanes, 1998; Sierpinska, 1998). The shared activity is one of the facets of classroom discourse where teacher and students shape and contribute to the evolution of social norms and mathematical practices (Bauersfeld, 1988; Cobb, 1994). Consequently, analysis of classroom discourse based on an interactionist perspective proposes that individual student’s mathematical activity (for examples, mathematical beliefs, values, and conceptual reorganizations) and the classroom microculture are reflexively related (Cobb et al., 1989; Cobb, 1994; Voigt, 1992). Likewise, the interactionist approach regarding discourse practices claims that the type of
knowledge depends on the types of communication and interactions in which the individual participates in the process of learning (Bruner, 1985; Sierpinska, 1998).

In line with the interactionist’s view, the teacher’s responsibility is to engage students “in a form of language socialization: socialization that is directed to bringing children into school-based intellectual practices manifested in ways of talking” (O’Connor & Michaels, 1996, p. 15). The interactionist approach through social construction of knowledge is supported by Vygotsky’s perspective that proposes participation in a wider culture influences an individual’s learning (Cobb, 1994). For example, students construct the meanings of words, formulas, and diagrams through a process of interacting in a community, namely discourse; these meanings become cultural representations and norms for interacting (Cobb & Bauersfeld, 1995; Hufferd-Ackles et al., 2004; Sierspinska, 1998).

Furthermore, when students and teacher are engaging in discourse, certain aspects of inquiry mathematics activity such as explaining, justifying, and collaborating become taken-to-be-shared objects of reflection (Cobb et al., 1993). Some researchers observe that the movement from engaging in a process to being able to treat the process as an object of reflection provides students an opportunity for improvement of their specific experience in mathematical and social practice (Herbel-Eisenmann, Cirillo, & Otten, 2009; Lampert, 2001; Schoenfeld, 1989). This account is consistent with research findings that describe two levels of discourse in a mathematics classroom. “At one level, the topics of discourse were mathematical, at the other level they were social norms that regulate the activity of doing and talking about mathematics” (Cobb et al., 1993, p. 105). Similarly, Lampert (2001) develops classroom social norms using activities related to mathematical problem solving strategy. These activities require students to (a) articulate conditions in a problem, (b) make conjectures, and (c) revise ideas based on
mathematical evidence. Lampert (2001) posits that these practices in mathematics lead students to learn Polya’s (1954) intellectual virtues, those of intellectual courage, intellectual honesty, and wise restraint. In other words, by becoming a participant who enacts his obligation and expectation, and interacting with others to negotiate meaning, a student reorganizes individual beliefs about his own role, others’ roles, and the general nature of mathematical activity (Cobb & Yackel, 1996; Wood, 1998).

Consistent with the interactionist perspective, which is useful to examine the evolution of social norms, the teacher put forth effort in the current study for the creation of classroom norms to support and sustain classroom discourse through innovative action. The interactionist’s view, in particular Bauersfeld’s (1988) contribution, was vital to my study because he endorses “the local classroom microculture rather than the mathematical practices institutionalized by wider society as his primary point of reference when he speaks of negotiation” (Cobb, 1994, p. 15). A math-talk framework proposed by Hufferd-Ackles et al. (2004) is compatible with Bauersfield’s (1988) model of construction of knowledge by which teacher and students create social norms and mathematical practices in the course of their classroom interaction. Grounded in social constructivist perspective, a math-talk theoretical framework outlines the development of discourse community in which students gradually internalize new roles by (a) questioning, (b) explaining mathematical thinking, (c) becoming sources of mathematical ideas, and (d) taking responsibility for learning (Hufferd-Ackles et al., 2004). Describing the key components as well as the level of growth in a math-talk learning community, the structure of this framework provides teachers with steps to develop classroom social norms.

To understand the complexity of discourse activity where classroom social norms are intertwined with individual student learning, Cobb and Yackel (1996) propose that, when
conducting a social analysis from the interactionist perspective, the observer’s analytical position shifts to that of an outsider whose objective is to document the evaluation of social norms. In this case, the interactionist perspective brings classroom social norms to the foreground for analysis while individual student learning becomes the background. Conversely, in conducting psychological constructivist analysis, the observer focuses on individuals as they participate in a community of learning.

Classroom Microculture

Bauersfeld’s (1988) view on interactionism leans towards constructivism in that he emphasizes that “learning is characterized by the subjective reconstruction of societal means and models through negotiation of meaning in social interaction” (p. 39). Referring to Bauersfeld’s frame of thought, Cobb and Yackel (1996) explain subjective reconstruction processes that involve the teacher and students who shape norms for creating a classroom microculture that includes classroom social norms, sociomathematical norms, and classroom mathematical practices. Each of these constructs gives rise to individual construction processes that include a student’s beliefs about her own role, others’ roles, and the general nature of mathematical activity; mathematical beliefs and values; and mathematical conceptions and activity. The researcher analyzed a classroom microculture under the social constructivist perspective by using the interactionist lens. In applying an interactionist lens, a classroom microculture came to the foreground for analysis while individual construction of mathematical meaning became part of the background (Cobb, 1994; Cobb & Yackel, 1996).

Classroom social norms. The discussion of classroom social norms centers on the words “obligation and expectation” (Lampert 2001, Wood, 1998; Cobb & Yackel, 1996). Classroom social norms are the set of rules about the expectations and obligations of an individual
member’s behavior that influences the regularities in communal or collective classroom activity and are considered to be jointly established by the teacher and students (Cobb & Yackel, 1996; Wood, 1998). For instance, the teacher is an authority agent who initiates, guides, and organizes the inquiry and renegotiation processes. Cobb et al., (1993) describes renegotiation processes as parts of classroom discourse development needed to shape the classroom participation structure (Cobb & Yackel, 1996; Erickson, 1986; Lampert, 1990; Lemke, 1990). To analyze students’ mathematical discourse, it is important to consider classroom social norms that a teacher cultivates to promote discursive practice, because an individual’s participation in a particular form of social interaction influences his or her psychological development (Cobb et al., 1993; Cobb & Yackel, 1996; Wood, 1998). This proposal is consistent with Vygotsky’s (1987) view that psychological development depends on the social situation in which the individual acts.

Cobb and several other discourse proponents confirm the above mentioned line of theory based on a year-long teaching experiment in a second-grade classroom (Cobb et al., 1993; Cobb et al., 1989). In this project, researchers developed instructional settings in second-grade mathematics classrooms compatible with the constructivist approach. The second-grade classroom teacher engaged her students in collaborative small group and whole-class discussion. Implementing new classroom norms, the teacher attempted to involve the students in the process of negotiating mathematical meanings when she and the children performed and talked about mathematics.

Cobb et al. (1993) further explain that the teacher initially asked questions about how students interpreted and attempted to solve mathematics problems; thus at this level, the discourse is centered on mathematical content, talking about mathematics, or in Wood’s (1998) words, “knowing what to say” (p. 170). However, students have their own expectation about
classroom interactions when asked to participate in sharing their thinking and interpretation. The teacher coped with this conflict between her own and the children’s expectations by the process that Cobb et al. (1993) call the renegotiation of classroom social norms. Cobb et al. (1993) illustrate the process of renegotiation in which the teacher framed the situations; for example, when a student gave an incorrect answer, the teacher would make the incorrect answer an explicit topic of conversation. “Is it okay to make a mistake, Jack?” (p. 98). Similarly in terms of developing classroom social norms, Lampert (1990) coached her students to use the remarks, “I want to revise my thinking” (p. 159) and “I want to question so-and-so’s hypothesis” (p. 187). The researchers describe that the teacher used her authority in guiding students to exercise know how to talk in which the students could say what they really thought mathematically (Cobb et al., 1993; Lampert, 1990; Wood, 1998). At this level, the conversation builds classroom social norms that are distinct from the content of mathematics. Although the researchers distinguish between the action of supporting students in providing explanation and the action of constructing their mathematical understanding, they see that both interactions are interdependent (Cobb et al., 1993; Wood, 1998).

Likewise, the two levels of conversation in mathematical discourse found by William and Baxter (1996) identified two kinds of instructional scaffolding. First, a teacher’s action is related to analytic scaffolding which is the scaffolding of mathematical ideas for students to support students’ learning of mathematical content during classroom interaction. Second, social scaffolding refers to the teacher’s initiating and organizing of the scaffolding of norms for social behavior and expectation regarding discourse to facilitate students’ participation in classroom interactions (Williams & Baxter, 1996). Social scaffolding is compatible with the teacher’s efforts to renegotiate classroom social norms since both have the same goal of eliciting
contributions from all students to whole-class conversation (Cobb & Yackel, 1996; Nathan & Knuth, 2003). Although William and Baxter (1996) distinguish analytic scaffolding from social scaffolding, they emphasize the interplay of social and analytic scaffolding as instrumentally intertwined to the flow of discourse, which has a significant effect on knowledge construction.

In the classroom where an inquiry approach is part of the routine, students are contributing to the evolution of social norms, which is the background against which they reorganize their individual construction of knowledge as part of classroom community (Cobb et al., 1993; Cobb, 1994; Cobb & Yackel, 1996). Results from quantitative and qualitative research support the above finding that students learn mathematics with greater understanding in a classroom in which they explore, investigate, reason, and communicate their ideas (Hatano & Inagaki, 1991; Hiebert & Wearne, 1993; Wood & Sellers, 1996). To provide concrete evidence, Hiebert and Wearne (1993) and Wood and Sellers (1996) investigated the relationship between teaching and learning mathematics in the classrooms using an alternative approach.

Hiebert and Wearne (1993) analyzed the classroom discourse of six classrooms involved in this study, classrooms A, B, C, D, E, and F, with classrooms D and F receiving alternative instruction. The researchers analyzed classroom discourse in two ways. First, the researchers calculated who talked during the lesson, the teacher or the students, and how much they talked. For example, each exchange was coded into one of the following categories of spoken words: 1-2, 3-5, 6-10, 11-25, 25-50, and 50-100. Second, to indicate the nature of the conversation, the researchers classified and coded the kinds of questions the teacher asked. In this study, Hiebert and Wearne (1993) categorized four different groups of questions about the mathematics lesson.

Teachers’ questions that went beyond asking for the recall of facts or procedures ranged from 1.6 per lesson in classroom B and C to 20.1 in classroom D and 23.3 in classroom F.
Classrooms A and E were located within this spectrum with 5.3 and 11.4 of those kind of questions, respectively. Classroom discourse in terms of student responses showed that in classrooms A, B, and C, about 10% of students’ responses were six words or longer, in classroom E, nearly 25% of students’ responses were six words or longer, and in classrooms D and F, about 35% of students’ responses were six words or longer.

Based on the measures of number of problems per lesson and amount of time spent per problem, the six classrooms emerged in three general profiles. Classrooms A, B, and C worked more problems per lesson and spent less time on each problem. Classrooms D and F, which received alternative instruction, worked fewer problems and spent more time on each problem. Classroom E fell between these two distinct profiles. In term of the contextual nature of the problems, there appear to be two profiles: Classrooms A, B, and C emphasized written symbols in place value tasks and solved nearly all computation problems using only written symbols (rarely using hands-on materials or pictures). Meanwhile, classrooms D, E, and F included more alternative representations (pictures and hands-on material) in place value tasks, and students wrote computation problems in story contexts.

The qualitative data analysis was coupled with the statistics tests to report the changes in student performances. At the beginning of the year, the performance of students on each of the problems suggested that classrooms A, B, C, and D were at comparable levels and they scored about one standard deviation below classrooms E and F. The results of statistical tests at the end of the year showed that the largest gain in percentage points was found in classroom D for many types of items. Classroom F, which started the year with higher performance, still showed gains on most types of problems and ended the year as the highest achieving classroom.
Wood and Sellers (1996) conducted a quantitative research study to examine the result of different instructional methods. The research study included five schools and nineteen third-grade classrooms. Six classes received problem-centered mathematics instruction for two years in second and third grade classes. Another six classes received problem-centered mathematics instruction for one year in third grade, and the rest of the third-grade classes received textbook-based instruction.

Problem-centered mathematics instruction generally reflects a social constructivist theory of knowing, compatible with the reform vision in mathematics education. The activity generally begins with children working in pairs for 20-25 minutes, followed by whole-class discussion for another 15-20 minutes (Wood & Sellers, 1996). The instructional approach in a problem-centered classroom appears to be similar to Hiebert and Wearne’s (1993) study in which students who receive reform-based instruction spent more time with each problem and engage in inquiry-oriented mathematics. Consistent with the reform document recommended by the NCTM (1989, 1991), the problem-centered project focuses on children developing their understanding of arithmetic concepts through collaborative activity allowing students to discuss a variety of ideas, questions, and strategies to find different ways to solve problems (Wood & Sellers, 1996).

At the beginning of the project, the researchers established baseline data for a sample involving 265 students placed in problem-centered mathematics classes, and 152 students assigned to textbook instruction (Wood & Sellers, 1996). The results of an independent t-test on the first grade Indiana Sequential Test of Educational Progress (ISTEP) indicated that no initial differences existed between students scheduled to enter second-grade problem-centered classes and students scheduled to be placed in textbook-based classes in the five elementary schools (Wood & Sellers, 1996). Similar baseline data were obtained from this group of students prior to
their entering the third grade. The students in the problem-centered classes, and those in the
textbook classrooms, were at comparable levels of achievement when they began third-grade.

Following the two years of intervention on mathematics instruction, 1990 to 1992, data
analyses were conducted in three levels. Wood and Sellers (1996) described three different
analyses used to obtain information from all students in the sample who received different
instruction, by using three instruments: the ISTEP, an Arithmetic Test containing an Instrumental
scale and Relational scale (Wood & Cobb, 1992), and a Personal Goals and Beliefs
Questionnaire (Nicholls, Cobb, Yackle, Wood, and Wheatley, 1990). The results of analyses
pointed out significant differences in arithmetic learning for students in two-year problem-
centered classes on the standardized achievement test and arithmetic test. These groups of
students scored higher on standardized measures of computational proficiency as well as
conceptual understanding. The findings also indicated that these students after experiencing two–
years of problem–centered mathematics activity held stronger beliefs about the importance of
investigating different ways to solve problems (Wood & Sellers, 1996).

As my review of classroom discourse literature pointed out, creating a classroom culture
that supports the development of understanding through ongoing mathematical discourse is a
challenging process (Pimm, 2009; Silver et al., 1990; Silver & Smith, 1996). Generally, students
experience the traditional classroom during their previous school years and are accustomed to
inferring the responses the teacher had in mind rather than articulating their own thinking (Cobb
The observation about students’ resistance against the NCTM vision of teaching and learning
(NCTM, 1989, 1991) is apparent in the observation and analysis of classroom discourse in the
QUASAR project. “QUASAR is a national educational project aimed at fostering and studying
the development and implementation of enhanced mathematics instructional programs for students attending middle school in an economically disadvantaged community” (Silver & Stein, 1996, p. 476). Silver and researchers of the QUASAR project conducted multiple five-year studies that included six schools beginning in the 1990-1991 school year (Silver & Stein, 1996).

Silver and Smith (1996) described one of the studies through the QUASAR project similar to Hiebert and Wearne (1993) and Wood and Sellers’ (1996) research studies, which centered on how to help students develop a meaningful understanding of mathematical ideas through engagement with challenging mathematical tasks. Over a period of five years, the mathematics teachers in the project collaborated with the researcher from a local university to implement mathematics instructional programs through an emphasis on mathematical thinking, reasoning, and problem solving (Silver & Smith, 1996; Silver & Stein, 1996). The instructional practices in the QUASAR classroom yield to the Curriculum and Evaluation Standards (NCTM, 1989) and the teaching standards (NCTM, 1991) that suggest teachers engage students with challenging mathematical tasks, enhancing students’ level of discourse about mathematical ideas, and involving students in collaborative mathematical activity (Silver et al., 1995).

The researchers of QUASAR project described three different analyses used to assess the effectiveness of instruction in QUASAR schools. They administered the QUASAR Cognitive Assessment Instrument (QCAI) as one of the methods to examine changes in student performance over time (Lane, 1993; Silver & Lane, 1993). Silver and Stein adopted Lane’s instrument developed to assess students’ mathematical problem solving, reasoning, and communication for this project: “The QCAI consists of a set of open-ended tasks designed to assess students’ knowledge of a broad range of mathematical content, their understanding of mathematical concepts and their interrelationships, and their capacity to use high-level thinking
and reasoning processes to solve complex mathematical tasks” (as cited in Silver & Stein, 1996, p. 505).

The results of comprehensively evaluating student performance on QCAI tasks administered in all three grade levels from the first three years (over the period of Fall 1990 and Spring 1993) of the project showed evidence that “students developed an increased capacity for mathematical reasoning, problem solving, and communication during that time period” (Silver & Stein, 1996, p. 505). A second method of evaluation used a variety of tasks from the 1992 National Assessment of Educational Progress (NAEP) for the eight-grade students at five of the QUASAR schools. The results were compared to those of NAEP’s national sample and disadvantaged urban sample. The findings from the analysis of student performance on the NAEP were that QUASAR students performed at least as well as the national sample on seven of the nine tasks. Silver and Lane (1995) posit that this is an important outcome, considering the fact that the national sample had significantly outperformed the disadvantaged urban sample on all nine tasks.

A third method of evaluation examined whether QUASAR instruction was linked to the increased number of QUASAR students’ enrollment and success in Algebra coursework (Silver & Stein, 1996; Tate & Rousseau, 2007). Silver and Stein (1996) reported that students from QUASAR schools both qualified for and passed algebra in ninth grade at significantly higher rates than students prior to the implementation of QUASAR.

Through the QUASAR project, teachers have the opportunity to initiate and establish mathematical discourse communities in their classrooms. Silver et al. (1995) found that creating an atmosphere of trust and mutual respect in QUASAR middle school classrooms is critical for the development of mathematical discourse communities. They pointed to the importance of
classroom social norms developed around how to use language in mathematical discourse. Silver (1996) suggested that teachers support and attend to the language of the students, develop a shared vocabulary, respect a range of ways of using language to share ideas, and scaffold the sharing of information in a form everyone can understand. Among the findings from the QUASAR project, Silver and Smith (1996) emphasized that the teacher’s efforts to develop the classroom social norms are critical when implementing classroom discourse, in particular, at the initial stage of discourse practice. Silver and Smith (1996) proposed a careful attention and implementation in terms of building student confidence to contribute to mathematical discourse:

While trying to establish a discourse community, a teacher may legitimately decide that pressing students for more discussion of mathematical ideas must wait until a later time. Even teachers who want their students to understand that mathematical ideas are the topics most valued in discussions in their classroom may decide it is prudent to move toward that goal one step at a time. If one sees the development of classroom discourse communities as a journey, then it seems reasonable to begin in a safe, possibly non-mathematical space, in which students may initially be more comfortable, and then move gradually to settings in which the mathematical ideas are salient in the discussion. (p. 24)

Since education is essentially a social process (Dewey, 1938), interaction between teacher and students is crucial in supporting students’ beliefs about their role and mathematical activity in the classroom (Cobb et al., 1993; Cobb & Yackel, 1996). Social interaction strengthens relationships between students and teachers, which in turn fosters an emotional and intellectual climate encouraging thinking, risk-taking, and involvement (Hanson, 1995; Lampert, 2004; Marzano, 1992). The challenge, then, is to invite students into the realm of social interaction that stimulates participation and changes their role to become active contributors in a
math-talk learning community (Glenn, 2002; Hufferd-Ackles et al., 2004; Jonas 2004). One way is by incorporating comics in the teaching style. Incorporating comics in the teaching method is in line with ample research that has demonstrated that learning is more effective when students find the material interesting and engaging (Alexander, 1997; Garner, 2006; Garner, Alexander, Gillingham, Kullikowich, & Brown, 1991; Hutchinson, 1949; Ormrod, 2008; Ziv, 1988). Further arguments based on classroom research support that the familiarity and the relevance of the problem correlate to students’ responses (Clark, 1998).

Clark (1998) examined the success of students solving problems set in different contexts, and their choice of contexts when one was available. The sample research studies involved one large group of students at the secondary school level, and one large group at the first-year university level, in New Zealand. Clark (1998) proposed a hypothesis that “the familiarity and the relevance of the context of the problem both seem to have an effect on whether the problem is attempted and how successful that attempt is” (p. 289). The investigation included the analysis on context of problem and student performance at the secondary level by using the selected questions of the 1981 Second International Mathematics Study (SIMS) results for grade 9. Clark (1998) continued the study by examining the outcomes from the Equity in Mathematics Education group in 1989 that replicated part of the 1981 SIMS study with a sample of 800 students. Analysis from the 1981 SIMS selected four questions: (a) question model boat, (b) question buying cloth, (c) question constructing pipeline, and (d) question ribbon. Results showed that boys arrived at the correct answer 10 and 12 percent more frequently than girls on questions model boat and constructing pipeline. Meanwhile, girls achieved the correct answers more often than the boys on questions buying cloth and ribbon by 3 and 2 percent. The results of this study pointed out that female students performed well when the context of the problem
related to their own domain. Based on this result, Clark (1998) suggested that perhaps in the initial experience, it is wise to give girls more familiar material; examples that have meaning and significance to them. Clark (1998) emphasizes that to change their mathematics performances, female students need to be confident that the material is accessible and important to them.

Clark (1998) found similar phenomena at the university level when analyzing two different first-year statistics courses at Victoria University in New Zealand: STAT 131 Data and Probability, the course recommended for students majoring in mathematics, physics, chemistry, computer science, and engineering, and STAT 193 Statistics for the Natural and Social Sciences, recommended for those majoring in biological sciences, social sciences, commerce, and medicine. In general, female students consistently earned a larger percentage of A grades than their enrollment in the STAT 193 classes. For example, in 1988, almost half of the female students in STAT 193 earned an A while they made up only 36% of the total enrollment. The success of female students in STAT 193 was consistent throughout the years 1988 to 1993.

Comparing the results of STAT 131 and STAT 193 examinations by gender, Clark described that, overall, male students performed better from 1989 to 1990. She further pointed out that female students were not successful on questions involving technical and abstract problems. The greater number of female students receiving an A in STAT 193 than in STAT 131 may be related to the STAT 193 context of problems appealing more to the females in the class. Clark’s assumption was supported by the results of the 1991 scores. The STAT 131 exam was less abstract, and females scored slightly better overall. Further analysis on choice of context was done with the STAT 193 students. The instructor asked the same question, but assigned two different versions. The first version referred to *psychologist*, and the second one to *concrete strengths*. Of the 217 students, 166 chose the first option on the paper, but if they were female,
they chose this problem more often when it concerned the psychologist. But, if the first option was about concrete, female students favored the second option.

Clark (1998) posited that this study reaffirmed the importance of teachers’ efforts to exercise sensitivity in their choice of appropriate language based on the demographics of their class. “When mathematics problems are selected for assessment, the practice of skills, or the learning of concepts, the role of the teacher is vital because the choice of language and context for these problems is central to whether the problem is accessible to the learner” (p. 301).

Clark’s (1998) proposal to make mathematics more accessible for most of the students can be tied to the use of comics in the classroom. Having considered the potential of comics to draw student participation (Hutchinson, 1949; Wright, 1976), the current study used content-related comics that enhance students’ participation and develop students’ positive disposition toward doing mathematics. The content-related comics served as a focal point around which student and teacher negotiated norms of how to raise questions, invoke explanation, share thinking, and build understanding collaboratively. Thus, this study investigated the question: To what extent does teaching with content-related comics help to support student participation in mathematical discourse? A math-talk learning community framework was applied to observe students’ degree of participation in shaping classroom social norms reflecting components of (a) questioning, (b) explaining math thinking, (c) proposing mathematical ideas, and (d) leading discourse for learning.

For the sake of the organization of this literature review, the discussion on theory and existing research studies about humor and comics in the mathematics classroom will follow that of classroom mathematical practices.
**Sociomathematical Norms.** Cobb and Yackel (1996) suggest that a mathematics classroom may adopt social interaction from any subject matter area. This proposal is similar to Resnick (1989) recommending the shift from *instruction* to *socialization* if teachers want to initiate and sustain classroom cultures where students explain, justify, and argue in social interaction (Cobb et al., 1989). For instance, Cambione, Brown, and Connell (1989) reported that there was less discussion and argument in mathematics classes compared to social studies classes, even when the teacher and students are the same participants in each class. Shifting from the traditional approach, Lampert (2004) shared that teaching is about building relationships between student and teacher, and between students themselves and mathematics, all while engaging in constructing mathematical meaning. In short, it is about working together to negotiate meaning.

Delving into the specific nature of students’ mathematical activity, Cobb and Yackel (1996) described mathematics classroom norms that go beyond the general norms for managing discourse. Central to discourse activity is student and teacher collaboration to develop and establish the norms to engage in mathematical explanation, disagreement, or revision of a mathematical explanation. Lampert (1990, 2001) presents an inquiry approach to mathematics that groups class activities into three problem solving phases. First, students find and articulate the conditions in a problem to plan for appropriate strategies. Comparing Lampert’s mathematical activity with Polya’s (1954), the first activity reflects problem-solving behaviors that are known as the phase of understanding and planning (Artzt, 1996). Second, students conjecture, solve the problem, and explain their reasoning. In Polya’s term, the second activity exhibits behavior of carrying out the plan. Finally, students revise their conjectures based on mathematical evidence and clarification of conditions. The last activity corresponds to Polya’s
phase of looking back, verifying, and watching and listening (Artzt, 1996). Based on Polya’s (1954) method of problem solving, Lampert’s (1990, 2001) outlined the routine that the teacher needs to model in inviting and persuading her students to talk about their thinking. In her classroom discourse, Lampert (1990, 2001) described how her students display alternative responses and disagreement. For instance, the fifth grader used the statement, “I want to question so-and-so hypothesis” (p. 159). The questioner followed this statement by providing logical reasoning for her challenge to the proposed solution. The person who gave the answer may or may not respond with a revision. Lampert (1990) suggested that the teacher prompts the class to volunteer their reasoning, “Can anyone explain what they thought so and so was thinking?” (p. 159). The teacher should refrain from ratifying the students’ answer with the purpose of shifting the authority for validating what is reasonably true in mathematics to the individual student and the community in which the revision is offered.

Aligned with Lampert’s (1990) idea, Cobb and Yackel (1996) propose that the growth of intellectual autonomy is related to the level of participation in the community of validators in which students engage in mathematical argumentation to justify an agreed-upon proof that one or more of the solutions must be correct. Furthermore, Cobb and Yackel (1996) contend that the teacher supports the growth of autonomy when students move to the foreground of the math-talk. Speaking of intellectual autonomy as a primary goal of mathematical discourse in the classroom, the authors warn that it is not sufficient for students to demonstrate frequent participation and contribute their mathematical thinking (Cobb & Yackel, 1996). The students need to step up to the next level of their individual ways to judge when to share their thinking and what counts as an acceptable mathematical contribution. In summary, Cobb and Yackel (1996) expect that “the students could themselves judge what counted as a different mathematical solution, an insightful
mathematical solution, an efficient mathematical explanation” (p. 213). The authors give the term sociomathematical norms to mathematical activity that involves students actively negotiating their own criteria to evaluate a mathematical explanation. In this interaction, students and teacher critically pursue what makes something true or reasonable in mathematics (NCTM, 1991). In other words, “Knowing that one is expected to explain one’s thinking is a social norm; knowing what counts as an acceptable mathematical explanation is a sociomathematical norm” (Franke et al., 2007, p. 239).

The above-mentioned sociomathematical norms include the criteria to indicate a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation for a particular classroom discourse. The goal to promote students in becoming intellectually autonomous and demonstrate sociomathematical behaviors is aligned with the goal to build a math-talk learning community (Hufferd-Ackles et al., 2004) in the current study. In setting a plan to use comics to improve participation in mathematical discourse practice, I used the model of math-talk learning community based on Hufferd-Ackles’s (2004) research study, a one-year project implementing the research-based curriculum in a third grade mathematics classroom. To design an in-depth study, I incorporated another model of discourse based in a middle school setting. The framework of a math-talk learning community contains the following key components (a) questioning, (b) explaining math thinking, (c) proposing mathematical ideas, and (d) leading discourse for learning, as well as the level of students’ progress (level 0 to 3) as they become autonomous in a math-talk learning community while the teacher moves to the background of mathematical activity (Hufferd-Ackles et al., 2004). In this case, the growth of students’ participation in a math-talk learning community through demonstrating and internalizing the math-talk components can be further developed into
the sociomathematical norms for a particular classroom (Hufferd-Ackles, 1999; Hufferd-Ackles et al., 2004).

Classroom mathematical practice. The third construct of classroom microculture is classroom mathematical practice. Cobb and Yackel (1996) propound that classroom mathematical practice is associated with the development of a classroom community in building mathematical truth that becomes the individual student’s mathematical learning. Observing the development of classroom mathematical practice enables the researcher to identify the global shift that occurred over a period of several weeks. In light of this observation, Cobb and his colleagues suggest that analyzing classroom mathematical practice is appropriate since, in documenting instructional sequences, the analysis accounts for mathematical learning as it occurs in the social context of the classroom (Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997).

Content-related comics present mathematical tasks that became a part of instructional sequences in this study. However, because of the limited time and space in this study, the content-related comics were analyzed in the scope of classroom social norms. The instructional sequences were reported to present a complete picture of classroom discourse. Cobb and Yackel (1996) predict that there is a relationship between each classroom microculture construct and the activity of the individuals who participate and contribute to it.

Mathematics and Humor

Paulos (1980), in Mathematics and Humor, theorizes a comparable pattern between humor and mathematics. The author purports that mathematics and humor are forms of intellectual play. The emphasis in mathematics being more intellectual, in humor more play. For instance, comic strips invite the readers to make sense and react through logic, pattern, and rules
presented in the form of dialogues or drawings and story-telling by the comic characters. Similarly, logic, pattern, rules, diagrams, and word-problems are essential in mathematics and also call for students to respond with their thinking skills (Schoenfeld, 1989). In humor, the logic is often inverted, patterns are distorted, rules are misunderstood, and structures are confused (Paulos, 1980). Yet these transformations are creative insights and must make sense on some level. Understanding the incongruity in a given story is essential, as one arrives at “getting the joke.” (Paulos, 1980, p. 11)

Paulos (1980) supports his claim that humor and mathematics share the features of cleverness and economy, playfulness, combinatorial ingenuity, and logic by showing the famous young Carl Gauss’ solution to finding the sum of the first hundred integers. The ten-year young Carl presented his answer in a few seconds. The teacher surprisingly sees only one number: 5,050. Carl then had to explain to his teacher that he found the result because he could see that, $1 + 100 = 101, 2 + 99 = 101, 3 + 98 = 101$, so that he could find 50 pairs of numbers that each add up to 101. Thus, 50 times 101 will equal 5,050. This example is one of the combinatorial ingenuity problems that the author offers to illustrate the similar qualities of humor and mathematics. Paulos (1980) and Koestler (1964) recognize that a bridge between humor and mathematics exists in riddles, trick problems, paradoxes, and brain teasers. The key recipe in presenting humor and mathematics is that the combination is more intellectual than most jokes, lighter than most mathematics (Paulos, 1980).

Additionally, Paulos’ example to combine humor and mathematics can be related to problem solving usage in the classroom. Stanic and Kilpatrick (1989) propose that one of the problem solving themes regarding its usage is problem solving as context. In term of context, Stanic and Kilpatrick (1989) describe that recreation is one of the roles of problem solving. The
intention to give recreational problems is for motivation since it includes student interests. “Presumably, such problems fulfill a natural interest human beings have in exploring unusual situations” (Stanic & Kilpatrick, 1989, p. 13).

The purposeful plan needs to consider the freshmen’s development of thinking and social interest founded in learning theory (Bransford et al., 2000). Consulting the literature of human learning theory, I found that the use of humor in mathematics is compatible with Ormrod’s (2008) theory of attention and cognition that emphasize the importance of attention as the first step to engage learning. This theory aligns with a deliberate plan to probe students’ attention as it leads to learning. Ormrod (2008) proposes that attention to stimuli increases with variety, novelty, and incongruity of presentation, as well as with the passion and enthusiasm of the teacher. Jensen (1998) claims that students pay attention to the strongest stimuli within their environment. He also supports the use of contrast and novelty in order to stimulate the state of attention in the cortex area of the brain—something that can be accomplished through riddles, trick problems, paradoxes, brain teasers, and content-related comics (Provine, 2000; White, 2001; Ziv, 1988).

**Comics as Instructional Materials**

The above theories are parallel with research in the field of learning motivation that recognizes the ways to make a boring task more interesting and enjoyable (Dienstbier, 1995; McGhee, 1989; Ormrod, 2008; Provine, 2000). In particular, Dientsbier (1995) launched a research study to link the arousal quality of humor and task performance.

**Humor and Positive Prediction**

Dienstbier (1995) hypothesized that presenting humor before taxing tasks will lead to sympathetic nervous system (SNS) arousal and subsequently to positive predictions that the tasks
will be challenging, with successful outcomes rather than threatening. The study involved college students of forty men and forty-one women. The humor condition group watched a twelve-minute excerpt of a made-for-TV Bill Cosby routine. Meanwhile, the control group watched a non-humorous videotape lecture that described and analyzed all the segments in the Cosby routine. Following the show, participants were informed that they would be engaged in some mundane tasks (proofreading and finding words with the letter “a”). Prior to the task, all participants responded to the survey on how challenging and energizing they actually found the task to be. The result showed the increase of participants’ feeling of energy without increasing their feeling of tension, as compared to participants exposed to the non-humorous video. The results of the significant test supported Dienstbier’s hypothesis. The humor condition produced a greater impact on Energy than on Tension (p< .01). Participants viewing the humorous video also indicated a preference for engaging in a challenging, rather than an easy, activity (p< .04).

Humor and Content Retention

In molding the theory about humor and learning, researchers investigated whether using humor judiciously could enhance learning in a specific subject (Garner, 2006; Ziv, 1988). To test the hypothesis that curriculum-specific humor positively impacts students’ retention and recall, Garner (2006) conducted the experimental study in a research methods and statistics course that involved 94 college students. The instruction for this experiment was designed for three separate hour-long lessons. Students had access to this computer-based lesson in two-week periods. The author chose the topic of research methods and statistics because students had identified this as one of the dreaded courses. At the conclusion of each session, participants were asked to complete a brief survey to provide their assessment of the lesson presented. The survey used the format of Likert-type items, and addressed questions relevant to the evaluation of the material.
For example, one of the questions included asked “How well do you believe it communicated the important information? (Garner, 2006, p. 178). At the conclusion of the three sessions, all participants took an additional exercise that required them to recall content that had been delivered over the three viewings (Garner, 2006).

The results of the analysis variance revealed a significant difference between the two groups: the group with humor treatment had higher ratings for their overall opinion of the lesson and of the instructor, $F (1, 92) = 43.33, p < .001$. Most importantly, students in the humor group significantly recalled and retained more information regarding the topic $F (1, 92) = 73.81, p < .001$ (Garner, 2006). Garner (2006) suggested that judicious humor used in a Research Methods and Statistics course, identified as one of the scary courses in college, could have a positive effect on student enjoyment and content retention.

The research studies relating the use of humor and its positive effect on teaching and learning included another distance education class for which Mitchell (2005) completed a dissertation on “Learning through Laughter: A Study on the Use of Humor in Interactive Classrooms” (p. 1). Similar to Garner (2006), Mitchell explored the use of planned, relevant humor in four interactive video classrooms that included 54 trainees employed by the University Extension Service and 147 Childcare Givers who received training provided by the university. Statistical tests revealed an increase in test scores in those groups receiving the humor treatment.

**Humor and Positive Classroom Atmosphere**

Goldsmith (2001) conducted a qualitative study to find student opinion regarding the quality and efficacy of online learning. The study included 400 responses representing students from 72 courses in 15 colleges and universities who were members of the Connecticut Distance Learning Consortium. The students were asked to respond to three open-ended questions: 1) “Is
there anything about the online classroom that has made it easier for you to learn, achieve your academic goals, or participate in class discussions (as compared to an on-ground classroom)?” 2) “Is there anything that has made it harder?” and 3) “We'd appreciate any other comments about your experience with this online course. Any observations, suggestions or criticisms about the strengths or weaknesses of online learning in particular would be welcome?” (Goldsmith, 2002. p. 4)

Goldsmith (2001) described the analyses of students’ responses that provided (a) an insight into why they chose online courses, (b) their opinions on the asynchronous conferencing (threaded online discussion) used by most faculty, and (c) their attitudes toward the role the faculty plays in online classes. In general, analysis results suggested that students emphasized the importance of flexibility, good communication and interaction. Goldsmith (2001) explains that the results of student evaluation of faculty gathered in this study as appreciative of faculty who are completely present and bring their personality into the course in ways that enhance learning. In particular, the teacher’s ability to demonstrate a sense of humor by making a lesson humorous is considered helpful to fully bringing the students into a virtual classroom.

*Humor and Learning*

Early research conducted by Kaplan and Pascoe (1977) found college students improved retention when instructors used humorous examples relevant to the material taught and relevant to items on the test. Ziv (1988) confirmed Kaplan and Pascoe’s study by conducting the initial study and replicating the same method with different populations and instructors. Ziv (1988) conducted the first study in a one-semester College Statistics course involving 161 students. The second experiment included 132 students in a one-semester Introductory Psychology course. The results indicated that the experimental groups who received content-related humor in their lesson
consistently scored higher than the control groups. Equally important to note, in Ziv’s (1988) experiments, the teachers who participated in the study received training to implement humor for the lesson they taught. Ziv (1988) supported Kaplan and Pascoe’s (1977) proposal that when planning a lesson, the purpose of humor is to clarify the main concepts.

The research on humor-learning relation proposed by the above researchers encounters a lack of evidence from other research studies. Casper (1999) conducted research in two Introductory Psychology classes for a dissertation entitled; “Laughter and Humor in the Classroom: Effect on Test Performance.” The hypothesis that laughter enhanced long-term memory was not supported; statistical tests on the relationship between learning and arousal did not yield any significance results in the experimental group. However, Casper (1999) reported the findings that males performed better on a test over material that was presented with laughter, and females performed better on a test over material that was presented with no laughter. Whisonant’s (1998) research entitled “The Effect of Humor on Cognitive Learning in a Computer-Based Environment” did not provide evidence to support the relationship between humor and cognitive learning. The first hypothesis, “Humorous treatment groups will score significantly higher on content posttests than non-humorous control groups,” was not supported by the result of study (Whisonant, 1998, p. 33). The second hypothesis, “Humorous treatment groups will score significantly higher on enjoyment of the instructional unit than non-humorous control groups,” was not supported by statistical test results of this study (Whisonant, 1998, p. 33). Of some note is the fact that, the humorous comic strips were not necessarily related to the curriculum content of the Advanced Psychology and Foundation of Psychology course selected in this experiment.
Comics for Classroom Humor

Generally, students recognize the social factors of humor because of the playful elements in comic strips (Hutchinson, 1949; Kessler, 2009; Wright, 1976). The students most likely feel inclined to respond to a comic prompt which leads the class to focus on the topic at hand (Glenn, 2002; White, 2001). White (2001) conducted a study entitled “Teachers’ Report of How They Used Humor with Students Perceived Use of Such Humor” (p. 337). One hundred twenty-eight faculties in public and private universities participated in the first round of the survey concerning the use of humor in the classroom. Meanwhile, the second round survey involved 206 students responding to questionnaires on how students perceived humor used by teachers. The researcher compared the responses of the students polled with that of the faculty surveyed, and found that the statement: “Humor could be used to create a healthy learning environment,” received 93% and 91% (agree/strongly agree) response from the faculties and students respectively (White, 2001, p. 339). In this study, the author reported an important finding that many instructors responded to the open-ended question about how they used humor. They volunteered that using cartoons or comics was one important way to insert humor into their classes (White, 2001).

Comics and Student Participation

Research studies on instructional materials recognized the potential of comics dating back over sixty years. Hutchinson (1949) reported a study conducted by Curriculum Laboratory of the University of Pittsburgh and the Comic Workshop of New York University of the use of comics as instructional materials in the classroom. This study included 438 teachers from primary to high school levels. The results indicated that comics appealed to students who usually hesitate to partake in literature, social studies, and science classes. The study also indicated that the use of comics increased individual participation at the completion of the project evaluation of
the study by a survey resulting in a 79% “yes” responses to the statement “Increased individual participation” (Hutchinson, 1949, p. 244).

The above mentioned research studies provide evidence relating to the use of humor and its positive effects on teaching and learning (Dienstbier, 1995; Garner, 2006; Ziv, 1988), as well as creating a positive social environment (Glenn, 2002; White, 2001). Consistently, Glenn (2002), and Jonas (2004) assert that when used appropriately, humor prior to a formal meeting or classroom instruction can ease the tension, break the ice, and infuse collaboration. The potential pedagogical use of humor, as indicated by the above research studies, inspired my study to investigate if comic strips can be used as an engaging and appealing classroom tool based on the idea that all students enjoy humor and are familiar with its drawings and story-telling. (Glenn, 2002; Jonas, 2004; Weaver & Cottrell, 1987; Wright, 1976). Potentially, comics could entice students to participate in discussion (Glenn, 2002; Hutchinson, 1949; Weaver & Cotrell, 1987; White, 2001). Thus, the theory of humor in mathematics (Paulos, 1980) and a series of research studies on humor-learning (Garner, 2006; Hutchinson, 1949; Mitchell, 2005; Ziv, 1988) underlie the question I addressed in this research.

Mathematical Activities for Classroom Discourse Implementation

In this section, the broad range of classroom discourse literature is discussed, including theoretical constructs and empirical studies of discourse that provide discussion and analysis of discourse as a mediating tool in the teaching-learning process (Forman et al., 1993). The review culminates in a description of the theoretical framework that provides a set of principles linked to the outcomes of implementing classroom discourse.

As the review of the classroom discourse literature indicates, in the past two decades researchers have been developing the theoretical frameworks and methodological tools to
describe classroom discourse (Steinbring, Bartolini Bussi, & Sierpinska, 1998). Researchers have contributed to the study of classroom discourse, ranging from general study of discourse as a communication system in social contexts (Cazden, 2001) to mathematical communication and conversation in a mathematics classroom (Silver, 2009; Truxaw & DeFranco, 2007). However, the IRE pattern persists to dominate the mathematics classroom (Franke et al, 2007; Spillane & Zeuli, 1999; Stigler & Hiebert, 1999). Recognizing the core challenge is in the implementation of mathematical discourse, the discussion will give deeper attention to discourse specifically related to classroom instruction including verbal moves and other features of mathematical discourse.

**Discourse Structure**

*Reciprocal Teaching*

Grounded in social cultural theory, which emphasizes the engagement in the zone of proximal development, Palinscar et al. (1993) explained first-grade classroom discourse in which a unit on animal survival was taught through reciprocal teaching. The teachers implemented an instructional program using two key components: teacher scaffolding and task materials employing analogous themes. The authors argued that using analogous themes (for example, a sequential topic of polar bear, Eskimos, penguins, and the hippopotamus) encouraged students to look for core commonality between the subjects. Palinscar et al. (1993) described the strategies of reciprocal teaching in which teachers scaffolded by helping students use prior knowledge, explained, revised, summarized, and clarified their idea in group discussion. Teachers used these strategies to invite students to share their thinking collaboratively as classroom interaction norms that determine the success of classroom discourse. The degree of success was indicated when students increasingly take on the central role to lead the discussion while teacher scaffolding
ceases. Reciprocal teaching introduces a model in which the teacher initiates the structure of classroom interactions as a background, while student elicited thinking in the form of questions, explanations, arguments, and conjectures come to the foreground (Forman, et al., 1993; Franke et al. 2004). Results from reciprocal teaching studies indicated that the overall performances for the experimental groups are 30 to 50 percent higher compared to the control group (Palinscar et al., 1993). From their early work in 1984, Palinscar and her reciprocal teaching team have introduced one of the ways to create opportunities for rich conversation as well as productive student outcomes (Franke et al., 2007).

*Funnel and Focusing Pattern*

Similar to the goal of reciprocal teaching, Cobb, Stephan, McClain, and Gravemeijer (2001) and Wood (1998) propound that teachers create opportunities for students to reflect on their own understanding and reasoning in mathematics by developing ways to encourage students to express and explain their mathematical thinking to others. Teacher’s efforts to build the new classroom social norms based on mathematical inquiry, center on the development of an atmosphere that supports student exploring, investigating, reasoning, and communicating about their ideas (Silver & Smith, 1996; Wood, 1998). The inquiry-oriented classroom that takes a form of social practice distinguishing the alternative mathematics classes from the traditional mathematics classes (Cobb et al., 1993; Pimm, 2009; Silver & Smith, 1996; Wood, 1998). The classroom social norms serve as the background of the interaction patterns and serve to constrain or stimulate student’s opportunities to actively construct mathematical meaning (Cobb & Yackel, 1996; Wood, 1998).

Wood (1998) analyzed the nature of classroom discourse through two conceptual tools: form of discourse (know how to talk) and the content (knowing what to say). In analyzing both
form and content, Wood (1998) described the funnel pattern from Bauersfeld’s (1980) proposal. A funnel pattern occurs when the exchange between the teacher and students conveys that the teacher acts as the sole official, giving validation of knowledge and students need only to respond in accordance with the teacher’s expectation (Cobb et al., 1993; Wood, 1998). The funnel pattern reflects certain beliefs about the nature of mathematics and the relationship between teacher and students (Wood, 1998). In contrast, the focusing pattern flourishes in a class in which the teacher expects the students to think about mathematics, to find strategies for problem-solving, and to discuss their ideas with others. From the learning perspective, students face the risk that their thinking is open for public evaluation. In terms of expectation and obligation, students are responsible for sharing their strategy to solve problems with their classmates, who are expected to ask questions for clarification and justification. Through this interaction, the teacher and students create a community of math-talk in which students are participating in the process of communicating about mathematics (Cobb et al., 2001; Wood, 1998).

Random and Turbulent Flow

Seeger (1998) described the discourse pattern of random and turbulent flow as he compared two classroom discourse activities in a research project conducted in Germany. Seeger (1998) indicated that a certain pattern is recurring to express the underlying functional rule, for example, the mathematics language of the students and teacher, the relationship between teacher and students, and the practice of discourse-oriented teaching. Random flow illustrates students seeking the right answer that satisfies the teacher’s question (Seeger, 1998). The searching movements have much in common with a game to fill a slot that is ruled by chance. In addition, this kind of educational discourse displays other characteristics: the teacher initiates all the
questions to elicit student responses; there is no student-to-student or student-to-teacher
discussion; students respond with a short sentence or just one word. Seeger (1998) observed that
as the surface of the classroom interacts “the picture of perfectly ordered flow emerges” (p. 93).

Complementary to the previous discussion about discourse patterns, random flow echoes
the IRE pattern that traditionally and currently exists in mathematics classrooms (Franke et al,
2007; Wood, 1998). In his attempt to explain the persistence of this pattern, Seeger (1998)
purports the thought from Edward and Furlong (1978). The unequal relationship between most
teachers and students in mathematics meanings, symbols, definitions, and experiences creates a
barrier to bringing students’ ideas in a whole-classroom discussion. Contending with the unequal
stance between teacher and students in the pattern of random flow, Seeger (1998) described the
turbulent flow that occurs in another classroom. In this classroom, the first impression was that
the students expressed their own point of view, spontaneously and sometimes without waiting for
his or her turn. Simultaneous discourse occurred as students work on a task with their peers and
the teacher worked with one student on the assigned task. Although the teacher’s intention was to
lead the class in finding the best strategy, the students only wanted to know who had won or how
many scores had occurred. Seeger (1996) indicates that in the turbulent flow discourse pattern,
students contribute to the classroom social norms as the exchanged of idea occurs frequently
between student-to-student and teacher-to-student. Although the chaos and confusion appears to
be the style of interaction, the author suggests that absence of order is not necessarily detrimental
to learning. On the other hand, orderly classroom interaction does not necessarily enhance
learning. The teacher’s approach to encouraging students to articulate their thinking matters
Horizontal and Vertical Interactions

Hatano and Inagaki (1991) support the construction of knowledge through social interaction and describe that the flow of information can occur vertically, from a more capable person to a learner, or pass horizontally in peer interactions. The authors argue that the more knowledgeable member may offer the solution with a brief explanation because the authority position of the more mature member and his idea is not challenged in interaction (Hatano & Inagaki, 1991; William & Baxter, 1996). Conversely, in horizontal interaction, members participate among peers frequently by giving a variety of ideas, arguments, questions, and revisions, because there is an equal level of expertise. Although the horizontal interaction may have the potential to increase productive and meaningful discussion, the authors suggest the balance of vertical-horizontal interaction to achieve construction of knowledge for all participants.

Hatano and Inagaki’s (1991) concept on vertical flow of information is similar to the funnel pattern of Wood (1998) and the random flow pattern of Seeger (1998) that explain the tendency for the students to figure out the response the teacher wants instead of constructing mathematical knowledge for himself. The findings from several research studies including William and Baxter’s QUASAR (1996) study and reform-based teaching from Nathan and Knuth (2003) and Hufferd-Ackles et al. (2004) are consistent with Hatano and Inagaki’s (1991) observation on flow of information. The next discussion will present Truxaw and DeFranco’s (2008) study, followed by other subsequent research studies.

Univocal and Dialogic Discourse

Research studies describing the role of discourse claim that the function of classroom talk can be either to transfer mathematical knowledge to students or to act as a tool for enabling
students to generate new meanings for themselves (Lotman, 1988; Wood, 1998). The univocal discourse reflects the exchange between teacher and students or the lack of this exchange that aims toward specific answers, rather than toward building conceptual understanding (Lotman, 1988; Wood, 1998). Alternately, when the discourse serves as a means to engage students in dialogue to build their thinking, the discourse is characterized as dialogic (Wertsch & Toma, 1995; Wood, 1998). Dialogic discourse is comparable to focusing pattern, where the teacher employs mathematics instructions to design situations in which students learn as they participate more equally in the dialogue (Truxaw & DeFranco, 2008; Wood, 1998).

Truxaw and DeFranco (2008) conducted a research study involving seven middle grades teachers. The authors used sociocultural theory as the primary framework for analysis and discussion of discourse as a mediating tool in the teaching-learning process. Based on Vygotsky’s theory (1978) that verbal interactions can help to develop recursive processes from thought to word and vice versa, Truxaw and Defranco (2008) constructed a theoretical analysis on verbal moves to examine the flow of classroom discourse and to develop models of teaching that included the sequence of forms of talk from univocal toward dialogic. Truxaw and DeFranco (2008) created a map of the flow of classroom discourse to translate the type of talk and verbal assessment. The sequence map was a tool to track the type of talk and verbal assessment indicating the tendencies of the discourse within a sequence toward univocal or dialogic. Looking deeper into Truxaw and DeFranco’s framework of flow of discourse, the classification of type of talk consists of monologic talk, leading talk, and exploratory talk, and accountable talk. Meanwhile, the verbal assessment was classified into inert assessment (IA) (assessment that does not incorporate students’ understanding into subsequent moves, but rather, guides instruction by keeping the flow and function relatively constant), and generative assessment
(GA) (assessment that mediates discourse to promote students’ active monitoring and regulation of thinking about the mathematics being taught). The description of IA and GA sounds similar to descriptions of social and analytic scaffolding described in William and Baxter’s (1996) study.

In their research study, Truxaw and DeFranco (2008) found that the univocal discourse is associated with the deductive methods of teaching in which the classroom practices employ general rules to specific cases. The observation showed evidence of leading talk and IA to transmit meaning to students univocally. Students rarely generate meaning dialogically. “Once the problem was solved, the teaching episode ended” (Truxaw & DeFranco, 2008, p. 507). In contrast to the deductive model, the inductive model of teaching employs a recursive process in which students revisit and revise ideas, strategies, and conjectures related to the problem. The inductive model of teaching is associated with discourse that leans toward dialogic. Teachers that tend to move toward using dialogic discourse will include exploratory talk, accountable talk, and GA judiciously and timely to support classroom discourse.

Revoicing

O’Connor and Michaels (1993; 1996) draw from Goffman’s (1974, 1981) speech activity called “he-said she-said” to develop revoicing that serves to clarify or emphasize an idea and allows the teacher to substitute mathematical vocabulary for everyday words or redirect the conversation. Having a character of non-evaluative reutterance, revoicing is one of the discourse forms that replaces the traditional IRE model and becomes a form of discourse that teachers can use as a way to orchestrate classroom discussion (Forman & Ansell, 2001; Hicks, 1996; O’Connor & Michael, 1993; 1996). Based on the work of O’Connor and Michaels (1993; 1996), Chapin et al. (2009) refined and expanded this discursive move through Project Challenge to propose productive talk moves. The productive talk moves included revoicing, repeating,
reasoning, adding on, and waiting. However, revoicing takes a more prominent place compared to the other talk moves, since with this tool the teacher can frame the content and encourage students in sharing their mathematical thinking (Forman & Ansell, 2001; Franke et al, 2004). The work of O’Connor and Michaels (1996) and Forman, Larreameandy-Joerns, and Brown (1998) provided the ways of orchestrating discourse through revoicing, wherein the teacher aligns student participation to achieve a learning goal; thus, revoicing is a mediating tool to build a new conceptual understanding.

To analyze speech activity using revoicing in the classroom, O’Connor and Michaels (1996) introduce the participant framework that encompasses (a) the ways that participants are aligned with or against each other and (b) the ways they are positioned relative to topics and specific ideas (for example, a statement in a textbook). The participant framework is the tool to navigate conversational turns in asking students for more elaboration, demonstration, and justification about the topic at hand. Implementing the participant structure, the teacher provides the opportunity for students to shift roles in constructing, co-constructing, and sharing meaning. The authors suggest that a teacher purposefully set the social norms in which participants’ responsibilities and rights support the revoicing activity.

Revoicing practiced in classroom discourse can accomplish learning goals, but can also limit productive discourse (Franke et al., 2004). For example, non-evaluative rephrasing and redirecting creates conflict between the teacher’s academic objectives and a pedagogy that honors the inquiry process, particularly when it means leaving incorrect mathematical ideas on the table (Ball, 1993; Lampert, 1990; Nathan & Knuth, 2003; William & Baxter, 1996). In relation to my study, revoicing appeared in social and analytical scaffolding, in math-talk
components, and other talk moves embedded in student interaction among themselves and between students and the teacher.

*A Study of Whole Classroom Mathematical Discourse*

William and Baxter (1996) define *discourse-oriented teaching* as actions taken by a teacher that support the creation of mathematical knowledge through discourse among students. Principally, William and Baxter (1996) assert that it is a complex undertaking for teachers to embark from the traditional teaching method where scaffolding is a one-way passage to reform-based instruction where the teacher involves students in jointly establishing the social and mathematical agenda. Discussion on William and Baxter’s (1996) research study will be combined with Nathan and Knuth’s (2003) research study since they complement each other’s findings.

Nathan and Knuth (2003) described a comparison of the first two years of a middle school mathematics teacher’s efforts to change her classroom practice to align with the vision of reform-based mathematics instruction. The authors described the theoretical perspective that consists of three major concepts. First, the teacher and the students interactively provided analytic and social scaffolding for one another (William & Baxter, 1996). This observation is compatible with Cobb’s et al. (1993) description about two intertwined levels of mathematical discourse that emerged as the teacher and students talked about and did mathematics in the whole-class setting. As I have described earlier in the *Classroom Social Norms* section, at one level the students and teachers engage in the process of renegotiation of classroom social norms (social scaffolding). Williams and Baxter (1996) note that providing social scaffolding is one method to ease “the tension between discourse-oriented teaching and socializing students into mathematical discourse” (p. 24). They describe that one of the most direct techniques the teacher
used to encourage students to discuss their thinking was to ask them to explain their thoughts to her. For example, when the teacher-researcher checks the group of students working to compare the different methods of factoring, she will ask, “Can you try to tell me that?” She also may add, “Tell me in your words because you will really understand it better” (p. 29). In asking the student to explain his thinking, she justified her request with reasoning that was consistent with discourse-oriented teaching (Williams & Baxter, 1996).

At another level, the students participate and construct their mathematical understanding (analytic scaffolding). The discourse tends to shape the content (know what to say) of math-talk as a means to construct knowledge. Nathan and Knuth (2003) echo William and Baxter’s findings that each type of scaffolding is instrumental in the building of reform-based instructional practices. In light of the goals of reform, a teacher’s attempt to change her mathematics instruction faces the dilemma of teaching the curricular content with mathematical precision while still honoring the student’s ideas and claims that are mathematically incorrect (Ball, 1996; Nathan & Knuth, 2003; Sherin, 2002; Silver & Smith, 1996). McClain and Cobb (2001) gave first account on this dilemma; when, for example, the participant teacher in their research study provided social scaffolding by accepting all student contributions equally.

In promoting a discourse environment, the teacher did not evaluate students’ contributions directly (an example of analytic scaffolding) that were particularly valued mathematically. William and Baxter (1996) similarly observed the tension when the “teacher distances herself from the development of analytic scaffolding, choosing to affect mathematical understanding indirectly by directly affecting task selection and social norms” (p. 37). The result of Nathan and Knuth’s (2003) research study indicated that when student-led discussion increases, the content of discourse lacks mathematical precision. “Ideally, balance between the
social and analytic demands is reached when students’ own social constructions of mathematical ideas are also connected to the ideas and conventions of the mathematical community” (Nathan & Knuth, p. 179, 2003). Thus, the challenge continues, for the teacher needs to be skillful and judicious in providing the scaffolding and structure within which students can efficiently and reasonably produce knowledge (William & Baxter, 1996).

Second, as Hatano and Inagaki (1993) introduced the concept of vertical and horizontal flow of information, Nathan and Knuth (2003) proposed that simultaneous flow of information occurs between the teacher and students, and among the students themselves. Aligned with the social constructivist perspective that maintains the development of knowledge occurs when the individual interacts with other members, educators put greater emphasis on horizontal information flow that elicits more student-to-student dialogues to construct meanings through revision, elaboration, and inquiry processes (Bauersfeld, 1988; Cobb et al., 1993; Vygotsky, 1978). As Hatano and Inagaki (1993) cautioned early, educators should maintain the balance between horizontal and vertical flow of information to advance mathematical construction for all members.

Nathan and Knuth (2003) applied the concept of vertical and horizontal interaction as an analytical tool to examine the moment-to-moment flow of information among the members (middle school students) of the classroom. The authors analyzed the relative amount of information that flows vertically from teacher to student, student to teacher, or flows horizontally from student to student. At the next level, they analyzed the content of discourse to identify the nature of scaffolding. Taking all these observations to the macro level, the researchers evaluated how these interaction patterns related to the teacher’s practice, the changes or lack of initiation,
to promote social and mathematics curriculum envisioned by the reform-based document (NCTM 1991, 2000).

Third, Nathan and Knuth (2003) brought to the foreground the teacher’s curricular goals and beliefs about student learning and development as these perceptions govern teacher’s actions to enact the mathematics curriculum (Eisenhart, Shrum, Harding, & Cuthbert, 1988; Forman & Ansell, 2001; Thompson, 1984, 1992). The focus on teachers’ belief enabled the researchers to analyze how teachers responded to student’s cognition during the interaction. For example, Forman and Ansell (2001) found the other side of teachers embracing the reform-based instruction through their study (“The Multiple Voices of a Mathematics Classroom Community”); a teacher tended to generalize student’s invented strategy as an effective strategy that other students might want to consider. Based on their study, the same teacher was less likely to elaborate the student’s use of standard algorithm; a teacher noted that this kind of strategy was a traditional approach to the problem from the older generation and students needed to use strategies that make sense (Forman & Ansell, 2001).

Additionally, examining the relation between teacher beliefs and actions, Nathan and Knuth (2003) attempted to investigate whether teacher’s beliefs and goals were congruent with their actions in the classroom. In the same path, Hopkins (2002) encouraged individual teachers to begin the process of identifying a performance gap, or the discrepancies “between behavior and intention” (p. 57). Herbel-Eisenmann and Cirillo (2009) used Hopkins’ strategy as a preliminary process in which teacher-researchers collaboratively examined videotapes and reflective journals of their own teaching to learn what needed to improve in their action-research projects.
Nathan and Knuth’s (2003) research study provided analytical tools that I incorporated into a theoretical framework for my own research. The three major components, (a) horizontal and vertical interactions were molded and elaborated with Hufferd-Ackles’ et al. (2004) math-talk components to outline the road map of creating classroom discourse; (b) social and analytical scaffolding was aligned with Cobb’s et al. (2001) classroom social norms and sociomathematical norms analyses. I located content-related comics within the development of classroom social norms in which it served as a pedagogical tool to stimulate questions, response, and copious participation that lead to mathematical discussion; and finally (c) teacher beliefs and goals were examined as teacher’s reflection on one’s own practice (Hopkins, 2002). Considering the scale of this project, teacher beliefs and goals were not included.

Math-talk Learning Community

In this study, I applied multilayer analysis consisting of Nathan and Knuth’s (2003) three major forces mentioned above and the math-talk learning framework formulated by Hufferd-Ackles et al. (2004). I adopted the math-talk components, level of math-talk, and teacher-means-of-assistance as the primary analytic tools used to analyze the episode of classroom interactions during discursive activity. Hufferd-Ackles et al. (2004) conducted a one-year in-depth case study in which teachers employed the research-based mathematics curriculum, *Children’s Math Worlds* ([CMW] Fuson, Ron, Smith, Hudson, Lo Cicero, & Hufferd-Ackles, 1997) to develop a *math-talk learning community*. The authors defined a math-talk learning community as a classroom community in which the teacher and students used discourse to support the mathematical learning of all members. The CMW curriculum contained a model of classroom discourse where teachers used language and representations that helped mathematics to become personally meaningful to students (analytical scaffolding), while providing context as a
background (social scaffolding) through which students can share their ideas with others (Hufferd-Ackles et al., 2004; William & Baxter, 1996). The Georgia Performance Standards ([GPS], 2008) was the curriculum used in this study and was compatible with CMW because the GPS aligned with the NCTM (2000) standards of communication which state that “instructional programs from pre-kindergarten through grade 12 should enable all students to organize and consolidate their mathematical thinking through communication” (p. 348).

Hufferd-Ackles et al. (2004) conducted a one-year study on the development of the teacher’s practice using the CMW curriculum followed by three phases of data analysis. The authors identified four distinct but related components that characterized the growth of the math-talk learning community over time: (a) questioning, (b) explaining math thinking, (c) becoming a source of mathematical ideas, and (d) taking responsibility for learning. Each of these components indicated the level of the math-talk learning community by demonstrating the growth of individual students from the emerging participant of level 0 to becoming the central figure of level 3, as well as the advancement of the group from few participants to productive members of math-talk learning community.

The math-talk learning community framework presents a step-by-step path: (1) level 0 indicates that the teacher is the sole questioner while students give short responses to the teacher only; (2) level 1 indicates that the teacher asks follow up questions and students explain their mathematical thinking as it is probed by the teacher; (3) level 2 indicates that the teacher asks more open-ended questions and also prompts students to ask questions about their classmates’ work and explains mathematical thinking with some volunteering thoughts; (4) level 3 indicates that student-to-student talk is initiated by the students. In level 3, many questions are “why” questions that require justification. Students defend and justify their answers with little assistance
from the teacher. The framework of the math-talk learning community can be found in Appendix A.

Having learned a math-talk framework and the flow of information that both aim to achieve student-to-student talk (Hatano & Inagaki, 1991; Hufferd-Ackles et al., 2004; Nathan and Knuth, 2003), the researcher saw that each math-talk component and its corresponding developmental level were complementary to horizontal and vertical interactions observed in Nathan and Knuth’s (2003) research study, “A Study of Whole Classroom Mathematical Discourse.” Thus, math-talk components substantiate the analysis of horizontal and vertical interactions of a classroom discourse.

The framework proposed by Hufferd-Ackles et al. (2004) specifies (a) components and levels in the creation of such an environment and (b) the teacher’s means of support needed to build such a community. The first specification has been discussed in length throughout this section. The second requirement centers on teacher actions to support transitions from one level to the next in the math-talk learning community. Hufferd-Ackles et al. (2004) labeled this process means of assistance. Teacher-provided means of assistance often were referred to as scaffolding that guides and encourages students to participate actively in the discourse of a collaborative community. Similarly, Herbel-Eisenman and Cirillo (2009) suggest that teacher-researchers should take time to discuss their new expectations about classroom participation explicitly with their students. Hufferd-Ackles et al. (2004) specify means of assistance, or the teacher’s discourse behaviors, as essential in shaping her efforts to guide students to move to the next level. To move from level 0 to level 1, teachers will begin to focus more on the students’ mathematical thinking as students concentrate more on how they arrive at answers and less on the answer themselves. To move from level 1 to level 2, teachers will increasingly expect
students to take on significant roles in the math-talk learning community and assist them in learning these roles. Moving from level 2 to level 3, teachers facilitate the discourse and help students as they take the central roles in the math-talk learning community. The summary of the teacher’s means of assistance for making the transition is shown in Table 1.

Table 1

*Levels of Discourse in a Mathematics Classroom*

<table>
<thead>
<tr>
<th>Levels</th>
<th>Teacher’s Means of Assistance for Making the Transition to a New Level (Analytic Scaffolding)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher's questions.</td>
</tr>
<tr>
<td>1</td>
<td>The teacher asks students questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.</td>
</tr>
<tr>
<td>2</td>
<td>The teacher asks open-ended questions to elicit student thinking and encourages students to comment on one another's work. Students answer the questions posed to them and voluntarily provide explanations about their thinking.</td>
</tr>
<tr>
<td>3</td>
<td>The teacher facilitates the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.</td>
</tr>
</tbody>
</table>

*Note.* Adopted from Hufferd-Ackles et al. (2004) and Stein (2007).

Using the above rubric, the teacher-researcher modeled, as well as reflected on, the means of assistance to navigate the students through the newly-developed stage interactions. Additionally, the teacher increasingly expected students to take central roles in the math-talk learning community. Using Hufferd-Ackles’ et al. (2004) teacher means of assistance, the researcher was able to improve the use of analytic scaffolding to support students in socializing their mathematical thinking. The itemized analytic scaffolding helped the researcher clarify the
difference between the social and analytic scaffolding that many times blends amid the multilayer discourse pattern. The researcher observed social classroom norms (knowing that one is expected to explain one’s thinking) and sociomathematical norms (knowing what counts as an acceptable mathematical explanation) established by members of the classroom as they progressed through each level to practice analytic scaffolding with one another (Franke et al., 2007, p. 239).

In this study, the objective to create context of learning as the background was implemented through the content-related comics activity. The content-related comics serve as a basis for commonality in which students feel confident enough to contribute to mathematical discourse (Glenn, 2001; Jonas, 2004); hence, it may provide social scaffolding. The use of content-related comics is one way to create a supportive environment (Glenn, 2002; Jonas 2004). In the current study, the teacher opened the channel of math-talk beginning with questioning and explaining mathematical thinking through the content-related comics, which served as mathematical activity in which students inclined to share and learn mathematical reasoning. According to Hufferd-Ackles et al (2004), questioning and explaining mathematical thinking becomes the core of emerging participation followed by proposing mathematical ideas and leading for discourse. These math-talk components occur over time and build successively on one another (Hufferd-Ackles et al., 2004). The description of the combined theoretical frameworks will be detailed in its own section at the end of this chapter.

Mathematical Activity for Classroom Discourse

Significant attention to concept of thinking and communication as an aspect of mathematics education is evident in several research studies and published documents, including such policy documents as the framework of state curricula (Silver & Smith, 1996; Silver, 2009).
The NCTM (1989, 1991) recommends standards regarding communication to promote learning in a social context that necessitates mathematical conversations. In the same vein, the new GPS (2008) encourage teachers to focus on learning processes to achieve the standards in problem solving, reasoning, representation, connections, and communication. Thus, the teacher’s role calls for sophisticated knowledge of both mathematics and learning process. As mentioned earlier, learning mathematics can be assumed as an initiation to mathematical discourse, that is, initiation to a special form of communication known as mathematical construction of meaning (Sfard, 2002; Pimm, 1996).

My awareness and concern about the common challenges encountered by teachers in implementing mathematical discourse grew as I became acquainted with the vast theoretical and methodological literature. Proponents of mathematical discourse acknowledge two primary challenges. First, teachers face student resistance when changing the routines in a classroom setting (Pimm, 2009). Meanwhile, the creation of an atmosphere of trust and mutual respect is the precursor for building effective classroom learning communities where students are willing to engage in investigation and discourse (Silver & Smith 1996; Silver, 2009; Pimm, 2009). The NCTM recommends discourse guidelines in the teaching standards (NCTM, 1991) for the successful implementation of mathematics education reform. However, reforming mathematical teaching has proved challenging, specifically requiring that teachers change traditional teaching practices and develop a discourse community in their classroom (Franke et al., 2007; Hiebert & Stigler, 1999, 2000; Hufferd-Ackles et al., 2004; Sfard, 2002; Silver, 2009).

Several studies report that adapting to a classroom discourse community is challenging whenever the traditional method of teaching and learning has been engrained deeply in students’ perceptions about the mathematics classroom (Silver et al., 1990; Silver & Smith, 1996).
Consequently, to implement discourse-oriented teaching can “involve pushing against a strong tide and almost always create turbulence” (Pimm, 2009, p. 137). Although the teaching standards (NCTM, 1991) describes the roles of teacher, student, and technology in a discourse community, the document says little about how to achieve those standards, leaving teachers in the dark about where to begin creating such discourse practices (Herbel-Eisenmann & Cirillo, 2009; Hufferd-Ackles et al., 2004; Silver & Smith, 1996).

Secondly, teachers find in the context of reform that they encounter a decision-making dilemma of how to predict where a lesson will go, making it more difficult to anticipate and prepare for their role in instruction (Adler, 1999; Chazan & Ball, 1999; Smith, 1996). Adler (1999) argues Lave and Wenger’s (1991) idea to be transparent and create an environment in which teachers attend to pupils’ verbal expressions as a public resource for class. The apparent challenge is that the teacher needs to maintain the productive discourse while simultaneously managing and mediating the shift of focus between mathematical language and the mathematical problem (Adler, 1999; O’Connor & Michaels, 1996).

So eminent are the challenges to implementing discourse-oriented teaching for school mathematics teachers, “It is no wonder IRE remain prevalent,” (Franke et al.2007, p.231) even after nearly two decades since the teaching standards document (NCTM, 1991) suggested the departure from the traditional culture of teaching and learning (Cestari, 1998; Silver, 2009). Nevertheless, researchers and teachers share their viable interests to see mathematical discourse implemented successfully in classroom practice (Herbel-Eisenman & Cirillo, 2009; Silver, 2009; Silver & Smith, 1996; Silver & Stein, 1996; William & Baxter, 1996). They seek detailed classroom processes and its culture in order to understand the step-by-step changes needed for teachers to support productive mathematical discourse (Herbel-Eisenman, 2003; Hufferd-Ackles
et al., 2004; Franke et al., 2007; Silver, 2009; William & Baxter, 1996). For instance, how a teacher manages her role is one factor of classroom processes related to a teacher’s control of mathematical content and students’ responses when orchestrating discourse. My study evolved into an exploration to find a means to create a social climate conducive to discourse. This tool potentially can connect students and teacher in a framework of common understanding allowing them to construct knowledge.

Small Group Communication

From the perspective of learning and teaching, Cobb et al (1993) maintain that small group discussion is rooted in social constructivist approaches, since peer dialogues is a prominent social interaction in which development of understanding is stimulated. Cobb and colleagues find that the small group discussion can be used as a strategy to support student engagement to construct meaning through frequent and successful participation in explaining their methods. Essentially, while working collaboratively in small groups, students create a zone of proximal development for themselves in which they can mutually grow more advanced learning mathematics than they could had they worked alone (Yackel, Cobb, & Wood, 1991). In addition to this main goal, through small group communication, teacher and students regulate and establish classroom social norms that positively influence the establishment of a discourse community (Yackel et al., 1991). As Hatano and Inagaki (1991) share, the comparable level of expertise among peers increases the variety of ideas, arguments, questions, and revisions. Thus, small group discussion is important in creating productive student-to-student discussion before joining a bigger circle for whole-class discussion (Cobb et al., 1993). Allotting time for group discussion also gives the teacher an opportunity to adjust her questioning to assist the small-
group discussions that in turn support the whole group discussion to move to the next level (Hufferd-Ackles et al, 2004).

The study of small group communication often is related to instructional strategy to create a more balanced and student-centered communication (Cobb & Bauersfeld, 1995; Forman & Ansell, 2001; Franke et al., 2004; Hufferd-Ackles et al, 2004). For instance, setting the stage for collaborative and dynamic learning, the teacher forms small-groups of students to engage in problem-solving activities (Curcio & Artzt, 1998; Stacey & Gooding, 1998). Researchers have examined the problem solving behaviors within the context of small groups and suggest that the small group setting appears to provide a natural environment in which increased dialogue and communication about mathematics can occur among students (Artz, 1996; Curcio & Artzt, 1998). Communicating with each other enhances positive interdependence among members of small-groups and increases the effectiveness of problem-solving skills (Stacey & Gooding, 1998). Curcio and Artzt (1998) investigated the effects of small-group communication on problem-solving behaviors among the fifth graders. Motivated by Artzt and Armour-Thomas’ (1992) early study on small group communication, Curcio and Artzt (1998) hypothesized that small group communication gives rise to cognitive and metacognitive behaviors occurring among the members that reflect the thoughts and behavior of expert problem solvers in achieving their mathematics task.

Curcio and Artzt (1998) described student strategies associated with possible problem-solving behaviors and their related cognitive levels. These strategies included (a) reading the problem (cognitive), (b) understanding the problem (metacognitive), (c) analyzing the problem (metacognitive), and (c) planning (metacognitive). The results indicated that “as students work in the small group to complete the graph task, the significance of the interplay of cognitive,
metacognitive, and watch-and-listen behaviors was revealed, which supported the work of Artzt and Armour-Thomas (1992)” (Curcio & Artzt, p. 187, 1998). Furthermore, Curcio and Artzt (1998) posited that the analysis of the interaction indicates how students’ metacognitive behaviors contributed to the building of ideas.

Stacey and Gooding (1998) contributed another study of small group communication in which they investigated the patterns of oral communication associated with successful learning in small groups. This study was complementary to the study of Curcio and Artzt (1998) which emphasized cognitive behaviors. In examining the group of students engaged in problem-solving, Stacey and Gooding analyzed components of interaction as a primary investigation to study small group’s pattern of interaction. After the analysis of discourse based on the type of interaction was completed, a second level of analysis examined the content of student’s discourse. Stacey and Gooding (1998) developed a theoretical framework consisting of the unit of analysis (a) focused interaction, to describe how students used language in interaction (for examples, a request for clarification, agreement, or disagreement) and (b) cognitive strategies (explanation with evidence, thinking out loud, and concrete examples). The authors indicated there was a relation between pattern of interaction and performance of members of the group.

Comparing focused interaction and cognitive strategies in Stacey and Gooding’s (1998) study with two other levels of discourse, this analytical tool can be identified with social and analytical scaffolding (Williams and Baxter, 1996) in the previous mentioned studies. Across the broad review of discourse studies, researchers have identified similar theoretical frameworks in analyzing discourse patterns (Cobb et al, 1993; Cobb et al., 2001; Hufferd & Ackles et al., 2004; Nathan & Knuth, 2003; Stacey & Gooding, 1998; Williams & Baxter, 1996). In particular, Stacey and Gooding (1998) provided a detailed account in analyzing focused interaction to
describe patterns of interaction occurring in a small group discussion. Teacher-researchers can use focused interaction and cognitive strategies to improve mathematics learning through small group discussion (Stacey & Gooding, 1998). To optimize mathematics talk in small group, Curcio and Artz (1998) suggested that the group formation should be based on the teacher’s knowledge of the students’ personalities and intellectual capabilities.

Coupled with small group discussion as one of the strategies to support classroom discourse, attention was given to the role of the task in mathematical discourse. The teaching standards (NCTM, 1991, p. 34) state, “The discourse is shaped by the tasks in which students engage and the nature of the learning environment.” Artzt and Newman (1997) and Cohen (1994) confirmed that the assigned task determines the effectiveness of the small-group strategy. The emphasis on task also comes from other researchers who have explored the role of tasks that support productive discourse (Civil, 1998; Silver & Smith, 1996; Silver & Stein, 1996; Franke et al., 2007). They advised that teachers work from problem solving tasks that allow multiple strategies, connect core mathematical ideas, and are of interest to the students. The above statements affirmed the strategy applied in this study, that is, the use of content-related comic prompts to enhance social and participatory behavior of the students.

*Mathematical Inquiries through Reading*

Siegel, Borasi, and Fonzi (1998) advanced one of the strategies to create mathematical tasks that allow students to construct meaning. The idea of making sense of a difficult text can provide an opportunity to engage students in mathematical inquiries, that is, inviting them to experience and appreciate firsthand the ambiguity, deductive and inductive processes associated with the mathematical thinking of expert problem solvers (Lampert, 1990; Siegel et al, 1998).
Siegel et al. (1998) suggested that teachers design an environment of inquiry and establish the conditions that support a community of learners. This view resembled Lemke’s (1990) description of *activity structure* that suggests teachers are responsible for creating the conditions for successful socialization of all students, specifically in coordinating academic tasks and social participation structures aimed at the development of mathematical discourse (Erickson, 1982; O’Connor and Michaels, 1993). Although Siegel et al. (1998) recognized teachers’ efforts to socialize students into practicing inquiry habits are aligned with the implementation of discourse community; they realized there is limited literature that can help a teacher create an inquiry-based environment.

Exploring the study of reading to learn mathematics, Siegel and colleagues offered the use of inquiry cycle to create a supportive environment for discourse community. In this study, Siegel et al. (1998) investigated the function of reading, in combination with reading and talking in mathematical inquiries carried out in the context of a secondary mathematics classroom. Central to this activity, teacher and students transformed the reading material into a collective text which generates dialogue and negotiation of meaning (Siegel et al., 1998). Siegel and her team developed the study involving three classrooms of 10\(^{th}\) through 12\(^{th}\) graders who engaged in an inquiry cycle with a topic: exploring the alternative geometry known as taxi geometry. Interestingly, the components of mathematical inquiries through reading are compatible with the problem-solving cognitive and metacognitive strategy (Curcio & Artzt, 1998). In this inquiry cycle students read to generate questions and conjecture, to analyze data, to verify, to present the results, and to reflect on what they had done during the inquiry cycle. Siegel et al. (1998) emphasized that when teachers implemented reading in mathematics as an integral curriculum, they provided opportunity for students to find a problem rich enough to pursue.
Comics in the Classroom

One strategy for planning a mathematics activity accessible for beginners is by relating it to their interests and familiar experiences (Clark, 1998; Curcio & Artzt, 1998; Franke et al. 2007, MacGregor, 1998). Based on the idea that there should be harmony between students’ daily activities and their experiences in the classroom, students’ new learning can be extended from their prior knowledge, for instance, their leisure reading (Hutchinson, 1949; Wright, 1976). Due to student interest in comics, an activity that applies comics to classroom material might make a unique contribution to mathematical discourse in the classroom (Kessler, 2009, Reeves, 2007; Wright, 1976).

Educators and researchers continue to organize efforts to incorporate comics into instructional materials. For example, *Mathematics Teaching in the Middle School* (MTMS), a monthly NCTM’s publication offers activities, ideas, strategies for middle school teaching, and since April 1994 has included “Cartoon Corner,” a comic section with embedded mathematical problems. The first editor, Barbara Cain (1994), used cartoons in her own classroom and contributed to the growth of the “Cartoon Corner” department. Her editorship was followed by the subsequent editors who prepared and tested the material to be used in the classroom (Reeves, 2007). Growing interest in integrating comics into the curriculum has also prompted researchers to design and test comic books that teach mathematics (Kessler, 2009; Reeves, 2007). For example, following the request of NCTM Educational Materials Committee, Reeves (2007) collected comics and cartoons to be used in the classroom. Reeves presents the collection of field-tested problems associated with comics and cartoons in the book, *Cartoon-Corner: Humor-Based Mathematics Activities.*
Kessler (2009) asks, “What if you could present mathematics in such a way that even people who did not care much about it would still be interested in the way it was presented, and take away the salient points?” (p. 1). This idea led Kessler (2009) to develop stories embedded with mathematical materials appropriate for elementary grade levels and aligned with the NCTM (2000) standards. Given the potential for comics to bring novelty to the classroom (Jensen, 1998), the current study investigated how content-related comics incorporated into mathematical activities can support student participation in the math-talk learning community.

Content-Related Comic Prompts

The content-related comic prompts (hereafter called the prompt) consist of comic strips, puzzles, and questions focusing on the students’ interests and their relation to a mathematical task. As a tool to generate discussion, the prompts give students something to talk about that leads to mathematical discussion. Referring to researchers’ suggestions about the role of task, the most important criterion in picking a problem is that it has the capacity to engage students in high-level cognitive thinking and reasoning (Civil, 1998; Lampert, 1990; Romagnano, 1994; Stein & Lane, 1996). Lampert (1990) suggests that the goal of setting the problem in mathematical discourse is to engage students in making and testing mathematical hypotheses. Selecting and setting up a high-level task, though, does not guarantee that students’ engagement achieves a high-level thinking strategy (Stein & Lane, 1996; Schoenfeld, 1989). However, in mathematical discourse, a teacher should start with a good task that provides an opportunity to engage students in high level thinking (Bartolini Bussi, 1998; Wood, 1998).

Traditionally a high-level task is associated with a problem-solving task. To understand a problem-solving task in mathematical discourse, discussion turns to problem-solving usage. Schoenfeld (1992) describes the first theme of problem-solving usage which is called problem
solving as context. According to Stanic and Kilpatrick (1989), the five roles of problem solving in terms of context are:

1. *As a justification for teaching mathematics.* “Historically, problem solving has been included in the mathematics curriculum in part because the problems provide justification for teaching mathematics. Presumably, at least some problems related in some way to real-world experiences were included in the curriculum to convince students and teachers of the value of mathematics.”

2. *To provide specific motivation for subject topics.* Problems are often used to introduce topics with the implicit or explicit understanding that “when you have learned the lesson that follows, you will be able to solve problems of this sort.”

3. *As recreation.* Recreational problems are intended to be motivational, in a broader sense than in item 2 above. They show that “math can be fun” and that there are entertaining uses of the skills students have mastered.

4. *As a means of developing new skills.* Carefully sequenced problems can introduce students to new subject matter, and provide a context for discussions for learning new concepts and skills.

5. *As practice.* The vast majority of school mathematics tasks fall into this category. Students are shown a technique, and then given problems on which to practice until they have mastered the technique. (p. 13)

The use of problem solving as context in the recreation role is compatible with Paulos’ (1980) theory about mathematics and humor. Although recreational problem solving as context provides a motivational aspect, the emphasis is to allow students to have fun with the mathematics they have learned (Stanic & Kilpatrick, 1989). Thus, from the perspective of
learning and teaching, in a broader sense, the teacher pays attention to the language and context of problems, activities, and questions used in the classroom to make mathematics more accessible to young students (Clark, 1998; MacGregor, 1998; Siegel et al., 1998). Considering the kind of problem solving appropriate for freshmen students at the high school level, the teacher researcher considered problem solving as recreation and skills to balance students’ interests and connection to higher level mathematical skills. In all content-related comics, the problems presented were tightly linked to the mathematics skills being taught in class. The content-related prompts in this research consisted of the comics “Luann” by Evans (2002) and “Rhymes with Orange” by Price (2009). The content-related prompts for this study are located in Appendix B.

The prompt is also the tool used to achieve the mathematical agenda (Lampert, 1990; Resnick, 1989). The teacher planned and presented the content-related comics as a device to coordinate instructional goals and actions about what the mathematical discourse is going to be about or to answer. For example: What do I want students to learn? Consulting the expert, I observed that Resnick’s (1989) study describes teacher-designed problem solving that aims to study how specific aspects of scaffolded problem-solving practice, together with discussion and argument, may influence the disposition and skills of problem solving. Resnick’s (1989) research study specified scaffolding conditions in three sessions of problem-solving activities:

1. **Planning board with maximum instruction.** The student solved the problem using the planning board. The teacher demonstrated use of the planning board and then participated by providing hints and prompts to further scaffold the problem-solving process and increase use of the board.
2. *Planning board with minimum instruction.* The students solved problems using the planning board. The teacher demonstrated use of the planning board and provided hints and prompts.

3. *Control.* The students solved problems without the Planning Board.

Resnick (1989) explained that the planning board was intended to prompt and support the participants in identifying goals and subgoals and clarifying the relationship of the given information to these goals. Having students recorded their strategies, Resnick (1989) examined the extent to which children in the different groups carried out such a problem-solving task. The planning board provided a roadmap of student’s thinking to achieve the goals. The findings of this study showed that older children use the planning board more efficiently. This result indicates that although the participants from grades four, five, six, and seven received the very brief training and practice; all but the youngest children produced more efficient use of the board. Resnick’s (1989) study provided a model of designing the task to examine a specific teaching strategy and to guide data analysis. Consistent with Lampert’s (1990) line of thought, Resnick (1989) emphasized that the teacher-researcher has to plan the mathematical agenda to align with the specific goal of the study. The content-related comics were designed to support the goal of increasing student participation, and were similar to Resnick’s (1989) task model of using the planning board efficiently in problem-solving tasks.

**Theoretical Framework for Math-Talk**

Having explored numerous research studies on discourse, I synthesized the related theories in this review of literature to formulate a theoretical framework and a qualitative methodology to study mathematical discourse. The combined theories were derived from math-talk and analytical and social scaffolding frameworks (Hufferd-Ackles et al., 2004; Nathan &
Knuth, 2003). Combining these related theories, the researcher was able to describe the nature of classroom discourse using a multilevel analysis to detail the complexities inherent in classroom interactions.

*Figure 1. The Math-Talk Theoretical Framework*

The combined theoretical frameworks of math-talk and analytical and social scaffolding allowed me to observe two phases of discourse in a mathematics classroom. At one phase, the analysis of discourse brings mathematical content to the foreground, and at the other phase the framework examines the background or social norms that regulate the activity of doing and talking about mathematics (Bartolini Bussi, 1998; Nathan & Knuth, 2003).

The first theoretical framework is the basic theory that provides an elaborated path to build a math-talk learning community. This theoretical framework consists of math–talk components, levels, and teacher means of assistance (Hufferd-Ackles’ et al., 2004) as shown in Figure 1. The math-talk components, level of math-talk, and teacher-means-of-assistance became
the primary analytic tools used to analyze the mathematical content which is the foreground of classroom discourse (Bartolini Bussi, 1998; Hufferd-Ackles et al., 2004). In this phase of analysis, the teacher applied the structure and descriptions of the math-talk theoretical framework in which members of a discourse community interactively negotiate meaning to construct knowledge associated with the components and levels of math-talk (mathematical content) or analytical scaffolding (Hufferd-Ackles et al., 2004; William & Baxter, 1996; Wood, 1998). The analysis focused on students engaging in (a) questioning, (b) explaining math thinking, (c) proposing mathematical ideas, and (d) leading discourse for learning. Each part of this activity corresponds to level 0 to 3, indicating the progress of the students as they took part in a math-talk learning activity.

Teacher-means-of-assistance is a part of the math-talk framework associated with teacher’s discourse behaviors. Hufferd-Ackles et al. (2004) describe teacher-means-of-assistance for each level of math-talk (see Table 1) detailing the analytical scaffolding, apart from social scaffolding, provided by the teacher. William and Baxter (1996) propose that analytical scaffolding is intended to support students’ participation to share mathematical reasoning during classroom interactions. Building math-talk in a Math I Support classroom, the teacher-researcher modeled and reflected on the teacher-means-of assistance to perform her discourse behaviors and to embrace the role of a facilitator or co-learner in her own classroom (Herbel-Eisenman & Cirillo, 2009; Hufferd-Ackles et al., 2004). The teacher-researcher used the teacher-means-of assistance to examine her own discourse behaviors as a regular entry in a reflective journal. This approach aligns with Schon’s (1983) term *reflective practice*, which is a “dialogue of thinking and doing through which I become more skillful” (p. 31).
At the other phase of discourse analysis, the students and teacher engaged in the process of renegotiation of classroom social norms (social scaffolding). In classroom discourse practices, the participation of members of a discourse community was important to interactively shape the form of discourse. To initiate and maintain students’ participation, the teacher provided social scaffolding by applying the analytical and social scaffolding theoretical framework. This second theoretical framework provided analytical tools used to examine the social interactions, or stream of information, occurring as the background during classroom discourse (Bartolini Bussi, 1998; Nathan & Knuth, 2003). The analytical tools include (a) the horizontal and vertical interactions and (b) the form and content of discourse as a means of describing how participants communicate with one another using social and analytical scaffolding (Nathan & Knuth, 2003; Wood, 1998). Using analytical and social scaffolding framework, the teacher identified and categorized each sentence or group of sentences in terms of their content. In addition, the whole-classroom framework provided the concept of vertical and horizontal information that classified the pathways of interactions in terms of who talked and to whom the information was addressed. Figure 2 presents the theoretical framework of analytical and social scaffolding.

Figure 2. The Analytical and Social Scaffolding Theoretical Framework
The combined theoretical frameworks are the theories adopted to explain, predict and identify phenomena (for example, relationships, events, or the behaviors), or to construct models of reality (Khan, 2006). Having formulated the combined theoretical frameworks, I was able to conduct and analyze the study within the structures described in each theoretical framework. In the first phase of analysis, I investigated the evidence of actions signifying the math-talk components and levels (foreground of mathematical discourse). In the second phase of analysis, I used the whole-classroom discussion framework to identify the sentences associated with analytical or social scaffolding along with the directionality of each sentence (background of mathematical discourse). The combined theoretical frameworks were used as the basis of conceptual framework to investigate the research question: To what extent does teaching with content-related comics support student participation in mathematical discourse? The combined theoretical frameworks that include the math-talk learning community and analytical and social scaffolding are illustrated in Figure 3.

![Diagram of The Combined Theoretical Frameworks]

**Figure 3.** The Combined Theoretical Frameworks
Figure 3 presents the combined theoretical frameworks. At the first phase, the math-talk framework describes the development of a classroom discourse from the initial stage through the changes in teacher and students interactions as they moved together to a new and higher level of the math-talk learning community (Hufferd-Ackles et al., 2004). The description of changes in student participation from active to passive as well as the teacher’s role from sole questioner to a co-learner needs additional explanation about what happens in the background of a math-talk learning community (Cobb et al., 1993).

The background or the second layer of discourse was examined through the shifting of flow of information and the frequency of social and analytical scaffolding occurring among the members of a classroom. In examining the flow of information and the frequency of scaffolding, the researcher interpreted the renegotiation process of classroom social norms by elaborating on the conversations in terms of who talked, to whom the information was addressed, and what kind of scaffolding was provided. For this purpose, the analytical and social scaffolding framework is appropriate to describe the detailed analysis of spoken sentences (Nathan & Knuth, 2003). The analytical and social scaffolding framework that provided detailed information about classroom social norms and mathematical content needed to be coupled with a math-talk learning community framework. In performing the multilayer analysis, I attempted to document the participants’ progress in a complete manner to identify most forms of interactions in our math-talk learning community.

The combination of theoretical frameworks is the tool that has a capacity to examine classroom social norms as the background that supports the development of a math-talk learning community. I used the information obtained from applying the combined frameworks to explain the development of a math-talk learning community influenced by classroom social norms in my
Math I Support classroom. The description reflected the teacher’s practices and students’ engagement in whole-classroom discussions. Comparing the description of classroom discourse before and after the use of content-related comics allowed the researcher to interpret the changes in student participation. Based on the combined theoretical frameworks, I proposed the conceptual framework that becomes the operational tool to investigate students’ interactions in this study. Figure 4 presents the conceptual framework for this study.

Conceptual Framework

Grounded in social constructivist theory, student’s responses and levels of engagement during discourse activity were examined primarily using the math-talk learning community theoretical framework: “Action Trajectories for Teacher and Student” by Hufferd-Ackles, et al. (2004). The math-talk components, levels of math-talk, and teacher-means-of-assistance became

Figure 4. Conceptual Framework of Math-Talk and Analytical and Social Scaffolding.
the analytic tools used to analyze each sentence (or group of sentences related to the same thought) in a classroom episode as the unit of analysis of discourse activity (Bartolini Bussi, 1998; Hufferd-Ackles et al., 2004). An episode is a whole classroom-discussion consisting of a warm-up or closing activity. The analyses of math-talk components with the level of math-talk were applied as the first phase of analysis as described in Hufferd-Ackles’ et al. (2004) framework. At the second phase, a classroom episode was classified into speech sentences using the analysis of Nathan and Knuth’s (2003) analytical and social scaffolding. This framework provided concepts of vertical and horizontal information with the type of scaffolding that appeared in a math-talk episode. How the combination of the two frameworks worked together is explained below.

**Component (a)—Questioning.** Level 0 questions reflect the traditional teacher-centered classroom where the teacher asks questions that require only a brief answer, and rarely follows up the students’ responses or seeks more explanations. Initiating student participation, the teacher asks level 1 questions to pursue student thinking. For example, “How did you figure that out?” “What is another way to solve this problem?” “Can you show your work on the board?” and other probing questions to develop more understanding of strategies to solve a problem rather than getting an answer. Level 2 questioning indicates the shifting from the teacher as the sole questioner to students beginning to question. “How did you get the answer 5?” “Can you explain it one more time?” “Can I show my work that is different than hers?” Why did you subtract the numbers?” and other questions focusing more on strategies to find the answer. In level 2, students begin to feel comfortable asking their peers for further explanation, modeling their teacher’s habits of inquiry. Level 3 questioning indicates that the students demonstrate
student-initiated questioning in order to understand one another’s thinking and to understand the mathematics content with little assistance from a teacher.

The growth in exercising questioning from level 0 to 3 is influenced by the teacher providing analytical and social scaffolding throughout the discourse. The researcher needs another framework that provides an analytical tool to clarify how the teacher and the students use analytical and social scaffolding. Using Nathan and Knuth’s (2003) analytical and social framework, the sentences of questioning in math-talk episodes were classified as analytical or social scaffolding. To examine the control in a conversation of who talks to whom, the directionality of each speech act was assigned with teacher-to-student, student-to-teacher, and student-to-student talk (Nathan & Knuth, 2003). Essentially, the analytical and social scaffolding framework allows the teacher to examine and describe the evolution of social norms (William & Baxter, 1996).

*Component (b)—Explaining.* The level 0 explanation is the counterpart to Level 0 questioning because both the questioner and explainer are focused on answers only. The teacher does not look for the explanation of student’s thinking; therefore the students give the answer as the end result of the assignment. The level 1 explanation of mathematical thinking indicates that the teacher assists students in their brief initial attempts. The efforts to encourage the student to take the role of explainer require the renegotiation of classroom social norms in which the teacher uses scaffoldings to extend and complete the actions and insights of the students who are taking a risk to share their thinking publicly (Cobb & Yackel, 1996; Palinscar et al. 1993; Williams & Baxter, 1996). The account of a teacher using social and analytical scaffolding to encourage students to participate in explanation needs Nathan and Knuth’s concept of
scaffolding that allows the researcher to examine how the students and the teacher shape classroom social norms to make all participants feel safe in explaining their thinking.

Hufferd-Ackles et al. (2004) explain that level 2 explanation of mathematical thinking begins after students become more comfortable with the process of communicating their thinking for public evaluation. At this level, the teacher still prompts and assists the students in clarifying their thoughts. The teacher’s actions to clarify student’s explanation can be accomplished by the act of revoicing, which allows the teacher to credit a contributor, to clarify the contribution, and to align students to one another for deeper discussion of mathematical ideas (O’Connor & Michael, 1996). The level 3 explanation indicates that students engage in full, confident explaining without teacher assistance. Using the concept of vertical and horizontal information, the researcher examined the explanation at this level whether student-to-student talk was evident throughout the exchanges.

Component (c)—Proposing mathematical ideas. At Level 0, the teacher presents mathematics content while the students work individually to perform the mathematics procedures as instructed by the teacher. Shifting to level 1, the teaching focuses on involving students’ ideas or making connections between their prior knowledge and the lesson presented. The level 2 proposing mathematical ideas indicates that students’ explanations build much of the content. In level 2, proposing mathematical ideas appears as a counterpart of level 2 questioning in which a student comes up with his or her own strategy to respond to other student’s questions. The teacher uses students’ strategies, even those that contain errors, for opportunities to learn. The level 3 proposing mathematical ideas indicates that the students gain confidence that their ideas about mathematics are valid and important for a class discussion. The teacher gives the discourse space that allows students’ strategies to become a topic of discussion.
The analysis using vertical and horizontal flow of information verified evidence of students-to-student talk and the students’ use of analytical scaffolding in proposing mathematical ideas throughout the exchanges. At level 3, the teacher’s role is assisting the flow of the discussion to achieve an instructional goal.

Component (d)—Leading discourse for learning. The last component of math-talk is leading discourse for learning. In the level 0 classroom, students direct their attention and address their responses to the teacher only in order to successfully learn mathematics. The entire communication is teacher-centered and students are passive listeners. The classroom moves to level 1 in their leading discourse for learning as the teacher begins to hold students accountable for listening to one another and as she begins to focus on thinking and not just on answers. To build accountability and scaffold students in leading discourse for their learning, the teacher prompts her students to evaluate other students’ thinking or compare the work of others with their own thoughts. At level 2, leading discourse for learning indicates that the students begin to evaluate other’s mathematical ideas and require justification for their answers. The level 3 of leading discourse for learning occurs as students take the initiative to clarify other students’ work and ideas for themselves and for others during whole-class discussion and small-group interactions. The exchanges among the participants demonstrate that the students are actively engaged in the discourse to understand the topic at hand.

The speech sentences in a math-talk episode associated with leading discourse for learning were examined with the concept of vertical and horizontal information with the frequency of analytical and social scaffolding used in discourse. The information obtained from the second phase of analysis verified that members of the classroom engaged in constructing their understanding and helping one another in their learning.
Summary Chapter 2

A study by Cobb et al. (1993) that was grounded in social constructivism theory provided both the interactionist and the constructivist lenses so that the teacher-researcher could focus on the culture of the classroom community to understand the nature of discourse in the learning of mathematics. In describing the interdependence of classroom social norms and individual student’s construction of knowledge, Cobb et al. (1993) proposed two levels of discourse in the mathematics classroom. “At one level, the topics of discourse were mathematical, [and] at the other level they were social norms that regulate the activity of doing and talking about mathematics” (p. 105). Similarly, William and Baxter (1996) proposed analytical scaffolding, which is “the scaffolding of mathematical ideas for students,” to support students’ learning of mathematical content during classroom interaction, social scaffolding, which refers to the teacher’s initiating and organizing of “the scaffolding of norms for social behavior and expectation regarding discourse” (William & Baxter, 1996, p. 24). The review of the literature also included theory of humor and mathematics to describe content-related comics within the goal of creating and establishing classroom social norms that support the development of mathematical discourse.

In this study, I employed Hufferd-Ackles’ et al. (2004) math-talk framework grounded in social constructivist theory to understand and build a math-talk learning community. In addition, the examination of math-talk components and level of math-talk was combined with the analytical and social scaffolding framework described in Nathan and Knuth’s (2003) research (“A Study of Whole Classroom Mathematical Discourse and Teacher Change”). This provided another analytical tools that examined the social interactions, or stream of information, that
occurs as the background during classroom discourse (Bartolini Bussi, 1998; Nathan & Knuth, 2003).

Table 2 shows the summary of theoretical frameworks mentioned in the above discussion and adopted to make up a conceptual framework for this study. The components and levels of math-talk in this conceptual framework combined with the coding scheme of vertical and horizontal flow of information for each type of scaffolding can be found in Appendix A. The summary version of a math-learning framework is shown in Table 3.

Table 2.

*Theoretical Frameworks Synthesis*

<table>
<thead>
<tr>
<th>Framework</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math-Talk Learning Community</td>
<td>1. Level 0 – 3 correspond to components:</td>
</tr>
<tr>
<td></td>
<td>a. Questioning</td>
</tr>
<tr>
<td></td>
<td>b. Explaining mathematical thinking</td>
</tr>
<tr>
<td></td>
<td>c. Proposing idea</td>
</tr>
<tr>
<td></td>
<td>d. Leading discourse for learning</td>
</tr>
<tr>
<td></td>
<td>2. Teacher means of assistance</td>
</tr>
<tr>
<td>Analytical and Social Scaffolding</td>
<td>1. Vertical and horizontal flow of information</td>
</tr>
<tr>
<td></td>
<td>2. Analytic and social scaffolding</td>
</tr>
</tbody>
</table>
Table 3

*Components and Levels of the Math-Talk Learning Community* (summary version)

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Questioning</td>
<td>Shift from teacher as questioner to students and teacher as questioners.</td>
</tr>
<tr>
<td>B. Explaining mathematical thinking</td>
<td>Students increasingly explain and articulate their mathematical ideas.</td>
</tr>
<tr>
<td>C. Proposing mathematical ideas</td>
<td>Shift from teacher as the source of all math ideas to students' ideas also influencing direction of lesson.</td>
</tr>
<tr>
<td>D. Leading discourse for learning</td>
<td>Students increasingly take responsibility for learning and evaluation of others and self.</td>
</tr>
</tbody>
</table>

Level 0  Traditional teacher-directed classroom with brief answer responses from students.

Level 1  Teacher beginning to pursue student mathematical thinking. Teacher plays central role in the math-talk community.

Level 2  Teacher modeling and helping students build new roles. Some co-teaching and co-learning begins as student-to-student talk increases. Teacher physically begins to move to side or back of the room.

Level 3  Teacher is a facilitator. Teacher monitors all that occurs, still fully engaged. Teacher is ready to assist, but now in more peripheral and monitoring role (coach and co-learner).

*Note.* The complete theoretical framework adopted from Hufferd-Ackles et al. (2004) can be found in Appendix A.
CHAPTER III
METHODOLOGY

The goal of this research was to investigate the impact of content-related comics on students’ engagement in mathematics discourse in a high school Math I Support classroom. At the core, I sought to explore the research question: To what extent does teaching with content related comics support student participation in mathematical discourse? To describe the methodology designed for this study, this chapter is organized as follows: (1) research methods, (2) setting and context, (3) participants, (4) data collection procedure, (5) data analysis, (6) validity and reliability, (7) limitations of the methodology.

Research Methods

This study used two methodological traditions: action research and participant observation. Action research is a methodology through which teachers use systematic inquiry to apply educational theory and research to analyze and improve their classroom practice (Johnson, 2002). Theory and research were applied in a real classroom, in this case to build a math-talk learning community without or with the use of content-related comics in my third period Math I Support class. Action research was employed to examine my own teaching and learning because I strove to become more reflective and focused on learner outcomes by continually assessing and observing my students (Hubbard & Power, 1999; Johnson, 2002; Lampert, 1990). Additionally, the proponents of classroom discourse assert that more details from a real-life classroom context are needed from secondary school mathematics teachers (Franke et al., 2007; Silver, 2009). By
conducted action research, this project will expand the resources and curriculum ideas available to secondary teachers.

Participant observation is an approach to qualitative inquiry that requires the researcher to be a member of the group under study as well as to observe the scene as an impartial observer (Eisenhart, 1988; Marshall & Rossman, 2006). The participant observation method aligns with the social constructivist lens that focuses to understand classroom interactions. Creswell (2009) describes the social constructivist lens:

In terms of practice, the questions become broad and general so that the participants can construct the meaning of a situation, a meaning typically forged in discussions or interactions with other persons. The more open-ended the questioning, the better, as the researcher listens carefully to what people say or do in their life setting. Thus, constructivist researchers often address the “processes” of interactions among individuals...Researchers recognize that their own background shapes their interpretation, and they “position themselves” in the research to acknowledge how their interpretation flows from their own personal, cultural, and historical experiences. (p. 21)

As an approach to a data-gathering method, participant observation includes detailed data collection of classroom observations, audiovisual material, documents, and reports. I employed participant observation to gain insight into the social interactions through which students participate and improve their habits and skills as members of a math-talk learning community (Cobb & Yackel, 1996; Hufferd-Ackles et al., 2004).
Setting and Context

The setting for the study was my third period Mathematics I Support class in River Park High School,¹ which serves a middle-class, suburban community of a large metropolitan city in Georgia. The study was conducted during the first nine-week period in the spring semester of the 2010-2011 school year. The beginning of the spring semester was selected as the time of the study because freshmen students were still adjusting to a new high school environment during the fall semester. Based on Cushman’s (2006) work interviewing students who had advanced from middle school to high school, freshmen students’ new experience included having new teachers in all their classes and perceiving high school subjects as being difficult. Considering new learning conditions typically create apprehension as students encounter challenging problems in the mathematics classroom, Lampert (2004) suggests that the teaching approach should include building relationships—between students and the teacher and among students themselves around mathematics. The fall semester was used to get to know my students within the social environment of the classroom and school.

The Georgia Performance Standards (GPS, 2008) curriculum was introduced in the 2008-2009 school year as the mandated high school instructional program. To support the implementation of the GPS curriculum, Georgia teachers received professional development provided by the school district. This professional development was intended to help teachers employ standards-based instruction in a more active and student-centered way, to develop students’ higher order skills, and to encourage students to explore mathematical ideas. The Math I Support class began to learn the GPS Mathematics I in the previous fall semester. Having received the training and taught Mathematics I for two years with various degrees of success, I

¹ All names (including names of students) are pseudonyms.
held a belief that I was implementing the standards-based (GPS, 2008) teaching. I continued my teaching style in my third period Math I Support class. Teaching Math I standards, I had to spend a great deal of time asking students to participate in sharing their work and explaining their solutions. The low rate of participation in my third period class compelled me to improve my own teaching, so they would be motivated to join our class discussion and share their thinking. Listening and understanding other students’ strategies would help more students participate and be successful in their learning experience (Hufferd-Ackles, 1999).

Tasks and Prompts

The tasks assigned to all students in Math I Support are the learning tasks from the GPS (2008). The GPS places emphasis on communication, and suggests students should communicate mathematically. In particular the GPS (2008) notes that students should:

a. Organize and consolidate their mathematical thinking through communication.

b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

c. Analyze and evaluate the mathematical thinking and strategies of others.

d. Use the language of mathematics to express mathematical ideas. (p. 6)

The above standards align with the NCTM (2000) standards of communication which recommended that students engage in mathematical discourse from pre-kindergarten through grade 12. The goal to achieve discourse standards recommended by the GPS (2008) and NCTM (2000) is consistent with the goal of this study to create and increase the level of participation in a math-talk learning community.

Adopting the above criteria, the learning tasks for the current study were selected. They were the Painted Cubes, Triangles, Constructing Diagonals and Quadrilaterals (hereafter called
Diagonals), and Mean Absolute Deviation (MAD). Similarly, the content-related comic prompts possess the features that satisfy the worthwhile task which is described in the Data Collection Procedures section (Phase 2). In this study, two content-related comic prompts were chosen to stimulate student responses. They were “A Square by Any Other Name” (Evans, 2002) incorporated with the Diagonals task and “A Stone’s Throw Away” (Price 2009) incorporated with the MAD task. The complete learning tasks and content-related comics prompts can be found in Appendix B. The content of the prompts related to problems from the Math I GPS curriculum. In all content-related comics activities, the topic of discourse presented was tightly linked to the mathematics skills being taught in class.

The content-related comics consisted of comic strips with associated puzzles, and questions presented to generate student responses to math problems in everyday situations. I adapted the problem solving prompts from Resnick (1989), who developed a series of writing prompts in the mathematics classroom and explored how scaffolding problem solving shaped the students’ disposition and problem solving skills. The description of Resnick’s (1989) problem solving prompts can be found in the Content-Related Comics Prompts section in chapter 2.

Math I Support in Georgia Performance Standards Curriculum

As described by the GPS (2008), Mathematics I is the first in the sequence of high school mathematics courses designed to prepare students to enter college at the calculus level. The course includes radical, polynomial and rational expressions, basic functions and their graphs, simple equations, fundamentals of proof, introductory level geometry, data analysis and probability. Under the GPS curriculum, Math I Support is a course designed to address the needs of students who have difficulties in mathematics by providing the additional time and attention that is needed for the students to be successful in their current Math I class. The course is taught
daily in a 50 minute period concurrently with Math I, and the teachers collaborate to ensure student success in learning the GPS Math I curriculum. Students receive one math elective credit upon completion of this class. Students continue to GPS Mathematics II, which is the second in the sequence of high school mathematics courses, upon their successful completion of Math I.

At the time of this study, I taught Math I Support, Math II Support, and Math II. The Math I Support class was my third period class. At River Park, students could opt to join college preparatory classes or Advanced Placement courses. The students in my class were all enrolled in the college preparatory diploma track. Students enrolled in Math I Support were recommended by their eighth grade Math teacher or by the request of individual students’ parents. The midterm test performance of this class at the end of first semester was 63. Behavior problems such as talking, being off task, and using inappropriate language occurred frequently, creating a challenge to implementing participation in small group and whole-classroom discussion.

As a teacher with 12 years experience in the high school mathematics department, I participated in the leadership team for Math I and Math II subject areas. Coordinating lesson plans, activities, and assessments to avoid overlap in Math I and Math I Support was accomplished during subject area meetings. I shared the learning tasks and the time line of my study in our subject area meeting before spring semester of 2011. Collaboration with other ninth grade mathematics teachers to initiate classroom discourse is essential. The sharing of ideas and experiences about the progress of classroom discourse can provide the necessary support for the courage in overcoming the initial uncertainties and generating the strategic plan (Black et al., 2004).

Since student interests outside the Math I Support class were important factors of social context to consider when conducting research in a natural classroom setting, I worked to
establish relationships and trust with students outside the classroom in order to support the community I was trying to create inside classroom. For example, I was involved in extracurricular activities such as the Mathematics Club and attended school sporting events and performing arts productions.

Participants in the Study

Participants in this study were the 28 freshmen\(^2\) high school students (fourteen to fifteen-year-old) in my Math I Support class. The class was composed of nineteen females and nine males with a racial composition of fifteen Caucasian, three African-American, and ten Hispanic students. The racial make-up of this class can be compared to the racial composition of the 1,650 students in River Park High School which was 85% Caucasian, 10% African American, and 5% Hispanic (and others) at the time of this study. The racial representation in this high school was similar to the racial make-up of the county’s population. Thirty-five percent of students in this school reported to have free or reduced price lunch.

The list of students’ pseudonyms for third period Math I Support is given in Table 4. The participants in my study included seven students with special needs. Students with special needs are students who have Individual Educational Programs (IEP) provided by the school through a student’s case manager certified as a Special Education teacher. These students were placed in such an inclusion setting as my classroom because they had specific learning disabilities or other identified needs requiring some additional assistance to be successful in the classroom. A second teacher, certified in Special Education and usually in the subject area, may co-teach the class, or provide additional assistance alongside the regular educator during seat work to any student who needs help, including those with special needs. This universal accommodation of help is

\(^{2}\) Two students moved to another school, changing the total on the class roster from 30 freshmen students to 28.
Table 4.

*Math I Support Class Roster*

<table>
<thead>
<tr>
<th>Female Seat Number</th>
<th>Name</th>
<th>Male Seat Number</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Dora</td>
<td>3</td>
<td>Fort</td>
</tr>
<tr>
<td>2</td>
<td>Mae</td>
<td>4</td>
<td>Todd</td>
</tr>
<tr>
<td>5</td>
<td>Kay</td>
<td>8</td>
<td>Troy</td>
</tr>
<tr>
<td>6</td>
<td>Tess</td>
<td>10</td>
<td>Ben</td>
</tr>
<tr>
<td>7</td>
<td>Nina</td>
<td>12</td>
<td>Ramos</td>
</tr>
<tr>
<td>9</td>
<td>Lupe</td>
<td>13</td>
<td>Dan</td>
</tr>
<tr>
<td>11</td>
<td>Hope</td>
<td>15</td>
<td>Patel</td>
</tr>
<tr>
<td>14</td>
<td>Vera</td>
<td>19</td>
<td>Park</td>
</tr>
<tr>
<td>16</td>
<td>Sue</td>
<td>24</td>
<td>Wes</td>
</tr>
<tr>
<td>17</td>
<td>Ali</td>
<td>25</td>
<td>Jed</td>
</tr>
<tr>
<td>18</td>
<td>Rosa</td>
<td>28</td>
<td>Grant</td>
</tr>
<tr>
<td>20</td>
<td>Liz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Jen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Tara</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Juana</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Beth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Novi</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* A seat number is assigned to each participant and used to chart the flow of information during whole-classroom discussion as discussed in chapter 4.

offered to anyone who needs assistance so to avoid a breach of privacy rights of students with special needs. Such assistance was there to help level the playing field so that all students may take advantage of the classroom experience. Due to the limited number of certified teachers in the Special Education department, the school provided a trained assistant in my third period.

I strove to maintain high expectations for all students regardless of their disabilities. Students with special needs were among other populations of students in my class facing disadvantage, including those economically disadvantaged. Although the setting for my research
was a rural county with various economic incomes, comparable populations were subjects of the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project (Silver and Stein, 1996) and Hufferd-Ackles et al. (2004), which indicate that students attending schools in a low socio-economic urban community can engage and perform in challenging mathematical tasks as well as their peers in higher socio-economic communities who are at the same grade level. No matter what geographical location, individual students may not be from just one ethnic, special education, or economically disadvantaged subset, but conceivably from a mixture of two or all three. Regardless of disadvantage, each student was provided access to a mathematical learning environment with the necessary accommodations to be successful. Similar to the QUASAR project, my study focused on what students can do and contribute to their learning experience rather than how their disabilities might or might not impact their achievement.

In addition to employing communicative behaviors to encourage all students to become active participants in a math-talk learning community, I complied with the IEP for each of my seven students with special needs by providing their learning accommodations (ranging from seating preference near supportive peers in a work-group to individual help provided by me). The climate of collaborative work among the case manager, parents, and other teachers of each of these students provided a positive learning network to follow up on the student’s progress. A good relationship is key in getting to know the students and their mathematical experiences that in turn can result in a better understanding of how an individual student participates in a classroom setting (Franke et al., 2007, Lampert, 2004).
Data Collection Procedures

At the initial stage of the current study, I obtained the Institutional Review Board’s (IRB) approval from the local school district and university to study student interactions in my Math I Support class (see Appendix C). Following IRB approval, I sought parental permission (see Appendix D). All parents consented with the request, resulting in a 100% student participation rate.

The study took place throughout a nine-week period, as shown in Figure 5. The nine-week period was broken into three phases: 1) one week to introduce the math-talk learning community; 2) four weeks to collect baseline data; and 3) four weeks to collect content-related comics data. Each phase is described following Figure 5.
Figure 5. Timeline for Activities

Note. Columns denote activities that occurred during each class period. The shapes indicate: warm-up =  , working time =  , and closing =  . Warm-up and closing activities were videotaped to document students interacted in whole-classroom discussions. Working time or small group discussion was documented in field notes. The color-coding indicates Math I standards (see Table 6) built in the learning tasks as shown below:

- **White:** Q&A to identify patterns in table of values and write the related functions. Make sense of functional relationship, write functions, and graph functions.
- **Green:** Explain patterns, functions, and graph of function.
- **Red:** Discover SSS, SAS, ASA, AAS, and HL congruence theorems and prove congruent triangles
- **Pink:** Present how to find the distance of the new bridge
- **Purple:** Q&A with comic to define, compare, and construct quadrilaterals
- **Grey:** Compare properties of diagonals in the quadrilateral family; explain and justify the minimum conditions necessary for a quadrilateral to be true
- **Yellow:** Q&A with comic to explore, collect, and organize data; analyze data by finding 5 number summary statistics, outlier, MAD value, and graph box-and-whisker plot; and compare box-and-whisker plot and explain variability concept
Phase 1: Establishing Math-Talk Norms

Proponents of classroom discourse indicate that the creation of classroom norms that expect students to share their mathematical thinking, to question, and to make sense of other’s mathematical ideas should be the first goal when a teacher begins to implement mathematical discourse (Cobb et al., 1993; Franke et al., 2007; Silver & Smith, 1996). Adapting to a classroom discourse community is particularly challenging for high school students who have experienced traditional school (Silver et al., 1990; Silver & Smith, 1996). Chapin et al. (2009) and Krusi (2009) stress the importance of creating norms and suggest the first step of building a math-talk learning community is asking students to share their thoughts on what makes a good discussion. Furthermore, creating conditions for a productive classroom is an approach to make students “feel that their classroom is a safe place to express their thoughts” (Chapin et al., 2009, p. 11).

Following the experts’ suggestion, the first phase of this study was devoted to establishing classroom norms as an integral plan to build a math-talk learning community. The first phase began on the second day back from winter break. To organize the process so as to allow students to practice sharing ideas, I asked students to list the features of a good class discussion in small groups. Below are most of the responses I received from the students:

1. Listen and take turns.
2. Do your work.
3. Be on task.
4. Ask for help in math work.
5. Participate by sharing answers.
6. Understand math problems.
I also shared my own list of a good class discussion adopted from Chapin et al. (2009) and Krusi (2009) as follows:

- Everyone participates by giving ideas and asking questions.
- Everyone listens while someone else is talking. Take turns.
- Everyone focuses on Math work. All ideas and opinions are respected.
- Wrong or right answers can be used to develop understanding.

I then displayed the groups’ lists, as well as mine on the document camera. I asked the class to compare and contrast the lists. Generally, students’ responses reflected that students understood what was expected in a class discussion. Their first response referred to the practice of listening, with everyone speaking in turn. Based on our class rule that students should “be on task,” the second and third responses also meant students needed to pay attention and regularly complete work on mathematics problems. The fourth and fifth responses acknowledged a class requirement expecting students to share their thinking, ask questions, and correct the explanation during warm-up, group work, and closing activity. The last response prompted discussion revealing that students believed that they needed to understand the mathematics problem at hand before they could participate.

Overall students’ responses and my suggestions were similar. In addition, I stressed the importance to respect each other’s ideas as we were learning to understand the mathematical thinking of other students. At this point, I suggested an additional rule in our class discussion that sharing right or wrong answers can be used to develop understanding. Students agreed to accept this rule, allowing more opportunities for their contribution in mathematical discourse. Finally, the class collaboratively produced the set of rules for math-talk norms, as shown in Figure 6.
Upon returning to the class on the next day, these math-talk norms were made into a wall poster that could easily be seen by the entire class. I could then remind students about our math-talk obligations and expectations by pointing to this poster. While discussion about new math-talk norms was still fresh, I continued to guide student participation in building a math-talk learning community. At the second day of establishing math-talk norms, I briefly showed power point slides of math-talk components and levels and gave examples of these actions (Math-Talk Menu) as listed in Table 5. The goal of this discussion was to clarify expectations and review examples of questioning, explaining, proposing mathematical ideas, and leading discourse for learning; I tried not to coach students on how they would express their ideas. In showing each of these components, I encouraged students to be math-talkers willing to practice each of these components. Math-talkers are individuals who respond/talk during our whole-classroom discussion.
Table 5

**Math-Talk Menu**

<table>
<thead>
<tr>
<th>What kind of questions will students ask?</th>
<th>How do you propose mathematical ideas?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How do you figure out this problem?</td>
<td>1. Think of different strategies to solve a problem.</td>
</tr>
<tr>
<td>2. Is there a different way to solve this problem?</td>
<td>2. Decide if the strategy works in different problems.</td>
</tr>
<tr>
<td>3. Can you explain this problem again?</td>
<td>3. Write a new problem for my group or class.</td>
</tr>
<tr>
<td>4. Can you let someone else explain this problem?</td>
<td>4. Look for examples from notes, textbook, or internet to find if similar strategies have a real world application.</td>
</tr>
<tr>
<td>5. How do you know we have the correct answer?</td>
<td></td>
</tr>
<tr>
<td>6. What if we changed some condition of the problem?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>How do you participate in explaining the problem?</th>
<th>How do you lead discourse for learning?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Show my work on the board and explain my strategies to solve the problem.</td>
<td>1. Help members of my group to solve problems.</td>
</tr>
<tr>
<td>2. Check my work with others’ in my group.</td>
<td>2. Explain my strategies to my partner.</td>
</tr>
<tr>
<td>3. Explain my work that is different than the work of others’.</td>
<td>3. Ask my partner to explain his strategies.</td>
</tr>
<tr>
<td>4. Draw a picture to explain how I solved the problem.</td>
<td>4. Find different strategies in the notes/textbook and share them with my group.</td>
</tr>
<tr>
<td>5. Look in the notes or textbook on how to solve the problems and share it with others.</td>
<td>5. Compare my strategies to others’ and ask why they used that strategy.</td>
</tr>
</tbody>
</table>

I asked students which actions in the math-talk menu were easy and which were difficult to apply in the classroom. Did they have other ideas in building a math-talk learning community?

I noted that students were mostly quiet during this presentation. To find out how they thought about solving problems and discussing their strategy, I asked, “Why do you think that you need to explain your answer?” After few seconds, Novi made a remark, “When I know the answer is correct.” I saw Ramos gave a shrug and other students maintained their passiveness. Having little responses from
my students, I became concerned that students still focused on the answer as the goal of learning mathematics and this presentation was not meaningful at all to them. I tried to lighten up their mood by refreshing their memory on the math-talk norms we discussed yesterday. I asked, “How about wrong answer? Yesterday, we agreed to share right or wrong answer so other can revise it, right?” No response was given upon this probing. I continued to assure them that we are really going to practice what we agreed in our Math-Talk Norms. I asked students “Did you participate today?” Most of them responded, “Yes.” I looked around the class and noticed several hands were up. “I did,” Dora responded, followed by her group members, “We always participate.”

After the presentation, I felt uncertain how to make students share their answers knowing they are afraid that their answer would be wrong. As the literature reported, in the mathematics class, most students perceive they have to work quietly and quickly (Krusi, 2009; Silver et al., 1990; Silver & Smith 1996). In my reflection, I asked myself (as I also needed to change from my old habits) why did I focus on correct answers? What might be some of the benefits of not focusing on the answer, but on the solution process? I came up with these thoughts (I planned to use them to encourage my students throughout this study): The right answer is a quick way to end the problem. We need to share our thinking about why the solution works. We want to think more deeply about strategies to solve a problem. We need to be patient to try different ways; it is interesting to see two or three different ways to solve a problem. I also made a note to myself, be patient, students need encouragement to interact frequently and discuss their mathematics for deeper understanding (Lampert, 1990).

*Phases 2 and 3: Structure of Daily Classroom*

Throughout phases 2 and 3 students in Math I Support engaged in the same structure of activities that provided opportunities to understand the Math I standards presented in the learning
tasks. The purpose of this section is to describe the structure of my third period class where students worked on learning tasks and contributed their ideas in whole-classroom discussions. Each lesson was a fifty minute class period composed of a warm-up activity, working time (small group discussion), and summary or closing activity. Warm-up and closing activities were the times when students interacted in whole-classroom discussions.

**Warm-up activity.** As the first part of an instructional period, warm-up activity plays an important role in focusing students’ attention leading to the topic. Warm-up activities included question and answer (Q&A) sessions to entice students to respond with their thinking and to enhance more participation in a whole-classroom discussion. Hence, I planned math activities related to students’ interests, such as sports, prom, music, food, fashion, arts, and TV shows. In addition, I made a colorful poster to show geometric figures, and used real objects, such as a Rubik’s cube and geometrical shapes, for their learning tasks. The warm-up activity could also be used to discuss students’ concerns and questions from previous lessons, thus functioning as a check method for ensuring students’ understanding. The warm-up activity typically took five to eight minutes.

**Working time.** Following the suggestion in Mathematics I GPS teacher training, students worked in small groups to make sense of mathematical concepts presented in the learning task. As Hatano and Inagaki (1991) suggest, the comparable level of expertise through peer interaction increases the sharing of ideas, arguments, questions, and revision. During the working time, I monitored or intervened in small groups to encourage discussion as a way of finding strategies to solve the problems, and students with special needs received assistance with their work. I accommodated the inclusion students as specified in each Individual Educational Program (IEP) (e.g., pairing with supportive peers in group-work, providing individual help, and allotting additional time to complete tasks). Working time was about twenty-two minutes.
**Closing activity.** The closing activity followed working time and consisted of whole-classroom discussion. To practice reasoning and communicating mathematical ideas publicly, selected problems were presented by an individual or several students as a group. I encouraged students to listen and to ask questions. The presenter(s) would explain the strategies used to solve the problems and answer any questions from the class. Closing activity took about 20 minutes.

To practice classroom discourse that supported and encouraged reasoning, I tried not to agree with a student’s explanation immediately. The presenter’s explanation needed to be justified by another student. I asked my students to propose different solutions. If no students were willing to present the selected problem, I chose one group member to put her work under the document camera and I asked probing questions to start a discussion. Students were encouraged to participate by encouraging them to connect the problem with their experiences: school activities, recreational activities, current events, environmental knowledge, architectural monuments, graduation plans, and others.

**Phase 2: Baseline Tasks**

The Painted Cubes and Triangles learning tasks were used during the four-week baseline period. As required by the GPS Math I curriculum (2008), the Painted Cubes and Triangles learning tasks are situated in contexts that provide students with opportunities to reason, communicate their ideas, and make connections to previous knowledge. The learning tasks selected in this study satisfy the requirements of worthwhile tasks recommended by proponents of mathematical discourse (GPS, 2008; NCTM, 2000; Romagnano, 1994; Stein & Lane, 1996; Silver & Stein, 1996). Research studies that employed standards-based curricula (Romagnano, 1994; Stein & Lane, 1996; Silver & Stein, 1996) describe worthwhile tasks as those that allow the teacher to:

1. Ask open-ended questions allowing for several strategies to possibly different solutions;
2. Provide students with the context for reasoning about mathematical ideas using tables and graphs, as well as making connection between the two;

3. Engage students in discovering, investigating, and applying properties of triangles and quadrilaterals as well understanding the relationship among members of; and

4. Connect students to their real experience to make sense of mathematics (Clark, 1998; MacGregor, 1998).

*The Painted Cubes Learning Task*

In the Painted Cubes learning task, students worked with functions given via tables, graphs, or algebraic formulas, to learn how to use function notation correctly, and to view a function as an entity to be analyzed and compared to others. The task includes generating a table of values using a variety of numbers from the domain, representing functions, and graphing functions. Table 6, items 1 and 2, shows the Math I GPS related to the Painted Cubes learning task.
Table 6

*The GPS Standards Included in the Learning Tasks*

<table>
<thead>
<tr>
<th>Math I GPS Description</th>
<th>Standards Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior (make sense of functional relationship, write and graph functions are shaded white in Figure 5).</td>
<td>MM1A1.e</td>
</tr>
<tr>
<td>2. Communicate with mathematical reasoning by explaining patterns, functions, and graph of functions in a whole-classroom discussion are shaded green in Figure 5.</td>
<td>MM1A1</td>
</tr>
<tr>
<td>3. Understand and use congruence postulates and theorems for triangles (discover SSS, SAS, ASA, AAS, and HL congruence theorems is shaded red in Figure 5).</td>
<td>MM1G3.c</td>
</tr>
<tr>
<td>4. Communicate with mathematical reasoning on how to find the distance of the new bridge (is shaded dark green in Figure 5).</td>
<td>MM1G3.c</td>
</tr>
<tr>
<td>5. Use conjecture, counterexamples, and indirect proof as appropriate (Q&amp;A with comic to define, compare, and construct quadrilaterals are shaded purple in Figure 5).</td>
<td>MM1G3.d</td>
</tr>
<tr>
<td>6. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite (compare properties of diagonals in the quadrilateral family, explain, and justify the minimum conditions necessary for a quadrilateral to be true are shaded grey).</td>
<td>MM1G2.a, MM1G3.d</td>
</tr>
<tr>
<td>7. Compare summary statistics (mean, median, quartiles, and interquartile range) from one sample data distribution to another sample data distribution in describing variability of the data distributions (Q&amp;A with comic to explore, collect, and analyze data by finding 5 number summary statistics, outlier, MAD value, compare box-and-whisker plot, and explain variability concept are shaded yellow in Figure 5).</td>
<td>MM1D3.a, MM1D3.b</td>
</tr>
</tbody>
</table>
The Triangles Learning Task

In the Triangles learning task, the focus is on the minimum information necessary to conclude that triangles are congruent. Students were expected to make and appropriately justify their conclusions. Their justification may be through paragraph proofs, two-column proofs, and other forms of communicating mathematical ideas. Table 6, items 3 and 4, shows the Math I GPS related to the Triangles learning task.

Phase 3: Content-Related Comic Tasks

The two learning tasks from the GPS in this instructional period were the Diagonals and the Mean Absolute Deviation (MAD) learning tasks. The content-related comic “A Stone Throw Away” and “A Square by any Other Name” were adopted from the Mathematics Teaching in the Middle School (NCTM, 2009) and Cartoon Corner (NCTM, 2003) as ways of launching each task, as described below.

The Diagonals Learning Task

The content-related comic (“A Square by any Other Name”) as introduced by its title addressed the topic about square, rectangle, and other quadrilateral figures. The content-related comic activity preceded the Diagonals task allowing students to explore the quadrilateral properties. Building on prior knowledge of quadrilaterals, students conjecture and prove or disprove properties that allow classification of quadrilaterals. The content-related comic led students to engage in the mathematical content of the learning task. The emphasis of this unit is the mathematics and the communication of it, as students demonstrate understanding the relationship between the properties in the quadrilateral family. Table 6, items 5 and 6, shows the Math I GPS related to the Diagonals learning task.
**The Mean Absolute Deviation (MAD) Learning Task**

The content-related comic (“A Stone Throw Away”) provides an activity that engaged students in collecting data and interpreting a box-and-whisker plot. After experimenting with their own data, students shifted to learn the standards in a different population sample. In the MAD learning task, students engaged in analyzing data to understand measure of spread, evaluate and compare five-number summary statistics, and to a graph box-and-whisker plot. Table 6, items 7, shows the Math I GPS related to the MAD learning task.

There was an interval of seven instructional periods between the Diagonals and MAD learning tasks because the students needed to understand counting principles and probability. The interval between these two learning tasks was unavoidable because Math I Support is designed to provide the additional time and assistance needed for students to succeed in Math I. Consequently, Math Support teachers must all follow a set lesson plan so that most students were paced at the same rate and prepared to take the Math I test on the same days. Other days between the learning tasks (see Figure 5) were used to review problems that students still struggled with. For instance, in the Painted Cube task students needed assistance graphing a cubic function and understanding the functional relationship from the table of values to a graph of function. The review was given on Math I standards to ensure students grasped the concept and completed the assignment related to the learning task because students in Math I Support needed additional time and assistance to learn the GPS Math I.

**Data Sources**

Figure 5 shows the variety of data collected to describe classroom interactions throughout this study. Data sources included the transcript of classroom discussion across the learning tasks and supplementary resources which consisted of field notes and a reflective journal.
Transcript of Classroom Discussion

The camera was positioned in the left front corner of the classroom to obtain the most inclusive view of the students and their interactions. The whole-classroom discussions during warm-up and closing activities were videotaped during four learning tasks. The transcription consists of the verbal portions of the videotapes of whole-classroom discussions. The pentagon and oval shapes in Figure 5 represent the recorded whole-classroom episodes. The transcript of a whole-classroom discussion during a warm-up or closing activity was designated as an episode. Data collection for each set of learning tasks, such as Painted Cubes, resulted in two warm-up episodes and two closing episodes. Transcription from videotaped data described as accurately as possible all spoken words from classroom discussions.

Supplementary Resources

Field notes were the first supplementary resource, which consist of data from working time activities or small group discussions. In Figure 5, the circle shapes represent small group discussion. Field notes were the written observations about detailed events during small group discussions that could not be picked up by the videotape. Therefore, the information documented in the field notes supplemented the transcript of whole-classroom discussion. Lampert (1990) suggests that every day the teacher should record detailed field notes on learning tasks, including descriptions of how tasks and interactions are planned and implemented.

Johnson (2002) suggests strategies to record field notes of teacher observation:

a. Use a seating chart and record participants’ actions on the chart.

b. Monitor group work about 10 minutes and record student interactions on the back of the lesson plans.
c. Replay classroom videotape recordings as an additional source of my field notes, immediately after school hours.

I used a two-column format to organize my field notes as recommended by Marshall and Rossman (2006). The comments highlighted specific information that required my attention about the lesson and helped me to adjust my teaching strategy.

The following questions guided my analysis of field notes:

a. How did students respond to the warm-up activity?

b. How did students transition from warm-up to the learning task?

c. How did teacher and students interact?

d. How did students interact with other students?

The dominant themes derived from the above questions were incorporated into the description of the learning task from the videotape transcript to provide detail and a complete picture of classroom interaction.

The reflective journal was the second supplementary resource. The daily recording and reflecting on classroom happenings allows the teacher to review students’ interactions and make necessary and immediate adjustments to stimulate and respond to students’ thinking about mathematical problems (Lampert, 1990). Herbel-Eisenmann and colleagues articulate a series of questions designed to stimulate teacher-researcher’s reflections about practice (Herbel-Eisenman & Cirillo, 2009). Likewise, Lampert (1990) emphasizes that teacher reflection is part of the plan and process to develop classroom social norms through particular social interactions that occur daily.

Reflective journal entries consisted of reactions and interpretations of classroom events added to baseline data and subsequent data. I responded to the following prompts to examine my discourse practice (Herbel-Eisenmann and Cirillo, 2009; Hufferd-Ackles et al., 2004; Gronewold, 2009):
1. Are you dissatisfied with a particular aspect of your practice?

2. Did you learn something in the video segment of your classroom discourse?

3. As you reflect on level 0 to 3 of teacher-means-of-assistance (Table 1) in a math-talk learning community, what is your discourse behavior performance?

4. What did you notice? Was any event unexpected?

5. What would you have done differently?

Similar to field notes, information derived from the reflective journal supported my preparation to facilitate a better math-talk learning community. I incorporated the major themes of the reflective journal as I described the episodes of the classroom videotapes (Miles & Huberman, 1994).

Data Analysis Procedures

Using the protocol for collecting baseline data (Herbel-Eisenman and Cirillo, 2009), that is, comparing typical classroom discourse with discourse that is stimulated by new innovative practices (Nathan & Knuth, 2003), I compared the baseline discourse data with data collected after employing a content-related comics activity. I predicted that the level of participation and number of math-talkers after the implementation of comics activity would improve over the baseline. In particular, I anticipated that the content-related comics would enhance teacher-to-student and student-to-student interactions, as well as encourage students to engage in questioning, explaining, proposing mathematical ideas, and leading discourse for learning (Hufferd-Ackles, 2004). This study focused on episodes with whole-classroom discussions as this type of mathematical discourse receives extensive attention in the standards (NCTM, 2000).

The combined theoretical frameworks described in chapter 2 were applied to perform qualitative data analysis in two phases. The first phase of analysis used the theoretical framework of math-talk and the second phase used the theoretical framework of analytical and social scaffolding. I
performed the procedure for analyzing qualitative data that was described by Creswell (2007) including:

1. Verbal portions of classroom videotapes were transcribed, coded into the component and level described in a math-talk framework.

2. The codes were organized and the frequency of codes was compared in the data through graphs, tables, and charts.

3. The codes were combined to find the emerging themes.

In the second phase, I repeated the above procedure using the coding scheme of analytical and social scaffolding. Figure 7 illustrates the two phases of analysis followed by the description of each phase.

![Diagram of Two Phases of Analysis](image)

**Figure 7.** Diagram of Two Phases of Analysis.

**Note.** TS = teacher to student; TC = teacher to class; ST = student to teacher; SS = student to student; OMN = open invitation non-math; DMS = declaration of math ideas by student; SVM = some volunteer math ideas by student.
**Coding**

The combined theoretical frameworks provided a general classification system that represents pre-existing (*a priori*) codes that guided my coding process (Creswell, 2007). The first phase of data analysis applied the theoretical framework of the math-talk in which the acts of (a) questioning, (b) explaining mathematical thinking, (c) proposing mathematical ideas, and (d) leading discourse for learning were examined as evidence of a math-talk learning community.\(^3\)

For each category, I assigned a scale of 0 to 3 to indicate the degree of student participation (see Figure 8).

![The Math-talk Framework](image)

**Figure 8. The Math-talk Framework**

The math-talk framework provides a description of the components and levels of math-talk. The math-talk components and levels of math-talk became the fine-grained analytic tools used to analyze classroom discourse during a whole-classroom discussion or an episode. The patterns of

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3 This study used the terms proposing mathematical ideas and leading discourse for learning that are originated from the terms “source of mathematical idea” and “responsibility for learning” respectively in the Hufferd-Ackles et al. (2004) framework to express the same actions.
math-talk components and levels that emerged from this analysis were used to explore the research question: To what extent does teaching with content-related comics support student participation in mathematical discourse?

A single sentence (or a group of sentences related to the same line of thought) in an episode was considered to be one unit of analysis. A group of sentences relating to the same idea were usually mine because I repeated myself or re-phrased my words in order to clarify a question or persuade a response from the students. Similarly, students used several sentences to explain their solutions, all of which generally expressed the same thought or methods.

The math-talk analysis was performed by coding every sentence (or a group of sentences related to the same idea) rather than the entire episode to determine the outcome of students demonstrating a certain level of math-talk learning community; hence providing more detail description than coding an entire episode. The detailed explanation of each component and level of math-talk can be found in Appendix A.

Each sentence was coded based on whether the sentence involved: (a) questioning (designated as “AQ”), (b) explaining (“BE”), (c) proposing a mathematical idea (“CP”), or (d) leading in discourse (“DL”) as described in math-talk framework. To eliminate redundancy, these abbreviations are used throughout the text in this chapter and the subsequent chapters. The coded responses were grouped in pink, green, orange, or blue, corresponding to levels 0, 1, 2, and 3 respectively, and indicated the degree of participation in classroom discussion. An example of teacher questioning is as follows:

Teacher: What do you think? Is this situation related to what we discussed yesterday? AQ-1
(Teacher asked a question to probe student thinking; teacher’s questioning is designated as level 1 as the question focused on student methods).
An example of student explaining is as follows:

Cane: You use the Pythagorean Theorem. SBE-1 (Student responded with brief explanation; student’s *explaining* is designated as level 1 explanation as he gave his method as probed by the teacher).

The coding and color-coding of the transcript shows what components emerged and which color appeared most of the time. Following the first coding phase of analysis, I reviewed the coding and color-coding and consulted the theoretical framework of math-talk to see if corrections were needed. I referred to the framework regularly to guide my interpretation about the meaning of every spoken sentence.

To verify interrater reliability of the categories, a teacher-colleague served as another coder and color-coded two sections randomly selected from each learning task. The coder and I reviewed and discussed the math-talk framework to align our perspectives in understanding the components and levels of math-talk (Hufferd-Ackles et al., 2004). The results from the other coder were compared sentence by sentence with my coding scheme to check interrater agreement of the categories (Hufferd-Ackels et al., 2004). Interrater agreement for the first round of color-coding was 93% for all categories (Painted Cubes Task). The differences in ratings were discussed by consulting the framework to reconcile our interpretations. In light of this discussion, interrater agreement reached 100%. The discussion and reconciliation of differences in interpretation of codes increased my confidence in the robustness of math-talk framework (Hufferd-Ackles et al., 2004). Verifying interrater reliability was done also during the content-related comic tasks. The raters agreed 100% for all categories assigned in the transcript sample.

For the next phase of data analysis, I analyzed each of the sentences in an episode using the theoretical framework of analytical and social scaffolding (Nathan & Knuth, 2003), shown in Table
A2 (Appendix A). The application of this framework involved classifying each sentence to examine the flow of information and type of scaffolding. For the flow of information, Nathan and Knuth (2003) coded each sentence with a From/To label, for example, TS indicates a sentence directed from the teacher to a student.

Using this analytical and social coding scheme I classified each sentence into a content category; for example, DM was assigned as a declaration of mathematics concepts or ideas. For each sentence, an additional code labeling A (analytical scaffolding) or S (social scaffolding) was given for the type of scaffolding it represented; for example, DM corresponds to A since it is a form of analytic scaffolding. Some examples showing the application of Nathan and Knuth’s (2003) coding scheme for a content-related comics activity are presented below:

Cane: “You use the Pythagorean Theorem.” ST - DMS - A (Student to teacher, declaration of mathematics, analytic scaffolding)

Teacher: “Okay. How do you find that there is the Pythagorean going on here?” TS - QFM - A (Teacher to student, ask follow-up question, analytic scaffolding)

Teacher: “I like that. Come on. Just repeat it again because you know I want everybody else to hear it.” TS - OMN - S (Teacher to student, open invitation, social scaffolding)

The classification of each sentence (or cluster of sentences that are related to one line of thought) provided information about the content of the speech and the type of scaffolding that allowed the researcher to analyze the development of discourse at two levels: social scaffolding and analytical scaffolding. The combination of Nathan and Knuth’s (2003) two major forces including flow of information, and types of scaffolding obtained from data analyses provided a pattern with which to analyze classroom interaction. The pattern of flow of information informed the researcher about the pathways (for example, teacher-to-student or student-to-student) of interaction among the
members of the classroom. An example of coding using the combined frameworks is shown in Table 7.

Table 7

*Example of Coded Sentences*

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>Cane</th>
<th>You use the Pythagorean Theorem</th>
<th>SBE-1</th>
<th>S - T</th>
<th>DMS</th>
<th>Analytical</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>T</td>
<td>Teacher</td>
<td>I like that. Come on. Just repeat it again because you know I want everybody else to hear it</td>
<td>Non-Math</td>
<td>T - S</td>
<td>OMN</td>
<td>Social</td>
</tr>
</tbody>
</table>

After compiling the data for each learning task using the math-talk framework, the data analysis from this phase provided descriptions to identify the level of math-talk learning community and the emerging patterns of classroom discourse. Similarly, I reviewed the compiled data descriptions through the analytical and social scaffolding framework. This multilevel analysis detailed how the classroom interactions occurred throughout the nine-week period.

Data description included a review of field notes and teacher’s reflective journals. I looked for new math-talkers in small groups and whole-classroom discussions. This information was complementary to the description of our math-talk learning community. Data collection and analysis was monitored frequently. Specifically, the researcher periodically evaluated whether more data needed to be gathered, whether the analysis being done answered the research question directly, or whether data collection or protocols needed modifications (Marshal & Rossman, 2006).

Qualitative data analysis requires an investigator to look for common themes that define “meaningful” information (meaningful to participant or researcher), and to compare units of data to each other (Eisenhart, 1988). Data collected before comics activity took place were compared with data collected after implementing comics activity. I compared the level of math-talk learning
community yielded by each learning task, the flow of information that occurred in each learning task, and the analytical and social scaffolding used in each learning task.

Reliability and Validity

Following the guidelines for qualitative methodologies (Creswell, 2007), the researcher used the subsequent strategies to establish and maintain the reliability and validity of the current study:

1. Description. A clear and complete description of the research process is presented in a research report. This description should guarantee a certain degree of external validity, namely that the research can be replicated by other researchers (Creswell, 2007).

2. Triangulation of data. The researcher used several data sources and methods that allowed her to triangulate data including transcription of classroom discourse, field notes, and the teacher’s reflective journals, thereby establishing credibility or internal validity of this study. The researcher categorized and compared data using multilevel analysis to yield a detailed, in-depth, and complete picture about the current study.

3. Interrater reliability. To verify interrater reliability of coding, another coder coded two segments from each learning tasks chosen randomly from the transcript of classroom discourse. The coding of segments from the first and second episodes in the Painted Cubes learning task resulted in 97% agreement. Using the same method, the results of coding the segments from the first and last episodes in the Triangles, the second and last episodes in the Diagonals, and finally the first and second in the MAD episodes are 100% agreement.

Limitations of Methodology

In this qualitative study, the perspective of the researcher must be considered. One limitation of this study may be researcher bias because I am also a practitioner in the observed classroom. My interpretation possibly affected how to plan and observe classroom interaction and how to code the
transcript. The complex social and behavioral process that occurred during classroom interaction may not be reported completely and accurately, although videotapes of the classroom, field notes, and a reflective journal were employed to record detailed events. Because of my role as teacher-researcher I had familiarity with the students that would have been highly unlikely if I had observed a class in which I was not also the active change-agent. This could have affected student reactions and my ability to distance myself from the situation. Another limitation existed in the learning tasks sequence which followed the Math I common lesson plan as I explained in Data Collection Procedures. Additionally, the limited time frame restricted my research both in terms of how far I could take the class in building a math-talk learning community and in not having a more substantial set of interactions to analyze. To mitigate the limitations of my research methods, I used several data sources and multilevel data analysis.

**Summary Chapter 3**

To analyze the effectiveness of content-related comics in eliciting student participation, a set of co-constructed norms was created, followed by a baseline and content-related comic data collection, in which all whole-classroom discussions were videotaped. Data were analyzed using the combined theoretical frameworks consisting of a Hufferd-Ackles’ et al. (2004) math-talk learning community and Nathan and Knuth’s (2003) social and analytical scaffolding. The combined frameworks provided a multilevel analytical tool for studying classroom interactions.
CHAPTER IV
FINDINGS

Data Description

In the second week of January 2011, my third period Math I Support began to form a math-talk learning community in which the teacher and students used discourse to support the mathematics learning of all participants (Hufferd-Ackles et al., 2004). While building a math-talk learning community, I applied participant observational methods to explore the research question: To what extent does teaching with content-related comics support student participation in mathematical discourse?

Data were collected as described in chapter 3. The data include transcripts of the videotaped classroom discussions together with field notes and reflective journals. To present the findings in detail, this chapter is organized into three parts. Part 1, which is divided into three sections, consists of a discussion of the baseline data focused on initiating a math-talk learning community. Each of the three sections is devoted to examining classroom exchanges using the math-talk learning framework, the flow of information framework, and the analytical and social scaffolding frameworks. Part 2, which is also divided into three sections, consists of a description of the classroom discourse resulting from the use of content-related comics. While the three sections present the results of examining the classroom exchanges using the same frameworks as in Part 1, the math-talk that occurred during the warm-up episodes is included. Part 3 consists of data analysis describing a pattern of math-talk across the learning tasks. The organization of the description of
the data is shown in Figure 9.

Figure 9. Organization of Data Description

Note. * In Part 2, description of math-talk that occurred specifically during the warm-up episodes is included.

Part 1: Baseline Data

The third and fourth weeks of January were used to initiate a math-talk learning community by implementing discourse around the Painted Cubes and Triangles learning tasks. As shown previously in Figure 5 (chapter 3), baseline data collection took place during the Painted Cubes and Triangles learning tasks. Each of these learning tasks used two class periods. Then, the class continued with the review of Georgia Performance Standards (GPS, 2008) in Math I related to the learning task because students in Math I Support needed additional time to learn the GPS Math I.

To describe the math-talk components and levels used in a whole-classroom discussion, every sentence in the Painted Cubes and Triangles episodes was coded based on the Coding Scheme of Math-Talk Components and Levels (see Appendix A, Table A1). The math-talk codes for the Painted Cubes and Triangles episodes are presented in Tables 8 and 9, which are shown next to each other for the purpose of comparison of math-talk between the two learning tasks.
Table 8

*Coding of Math-Talk During Painted Cubes Task*

<table>
<thead>
<tr>
<th></th>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Non Math</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AQ</td>
<td>BE</td>
<td>CP</td>
<td>DL</td>
<td>AQ</td>
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<td>7</td>
<td>102</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
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<td>37</td>
<td>2</td>
<td>2</td>
<td>21</td>
<td>2</td>
</tr>
<tr>
<td>Subtotal 1</td>
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<td>T = 62</td>
<td>S = 44</td>
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<td>Math-talk lines</td>
<td>281</td>
<td></td>
<td></td>
<td></td>
<td>Non-Math-talk lines</td>
</tr>
</tbody>
</table>

*Note.* AQ = Questioning, BE = Explaining, CP = Proposing, DL = Leading Discourse, S = Students, T = Teacher

Table 9

*Coding of Math-Talk During Triangles Task*

<table>
<thead>
<tr>
<th></th>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Non Math</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AQ</td>
<td>BE</td>
<td>CP</td>
<td>DL</td>
<td>AQ</td>
</tr>
<tr>
<td>Students</td>
<td>7</td>
<td>18</td>
<td>0</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>Teacher</td>
<td>4</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>23</td>
</tr>
<tr>
<td>Subtotal 1</td>
<td>S = 25</td>
<td>T = 9</td>
<td>S = 52</td>
<td>T = 58</td>
<td>S = 35</td>
</tr>
<tr>
<td>Math-talk lines</td>
<td>247</td>
<td></td>
<td></td>
<td></td>
<td>Non-Math-talk lines</td>
</tr>
</tbody>
</table>

*Note.* AQ = Questioning, BE = Explaining, CP = Proposing, DL = Leading Discourse, S = Students, T = Teacher
What Math-talk Learning Community Level is Apparent in the Painted Cubes Learning Task?

Questioning and Explaining in the Painted Cubes Episodes

In the Painted Cubes episodes, I used level 0 more than level 1. There are 37 instances in which I asked level 0 questions, requiring only a brief answer, without asking a follow-up question. Although I balanced level 0 questions with 34 level 1 questions, the students mostly centered their 102 responses at level 0. In contrast, students gave 24 level 1 explanations, only one fourth of their level 0 explanations.

The following excerpt reports part of the exchanges in the Painted Cubes’ problem. The task presents a table that asks students to fill in the number of painted faces that they observed from building a large cube with edge lengths of 3, 4, 5, 6, and 7 inches as shown in Figure 10. In the previous problem, students answered the questions: (a) how many faces can you paint in one cube? (b) Two cubes? To answer these questions, students built a large cube out of smaller cubes. In doing this hands-on group activity, students were able to make sense of the term “face” that refers to a painted yellow surface. Then, they counted the painted faces appearing in the large cube. In problem number 2, since there were not enough small cubes to build a large cube, students needed to predict and recognize the pattern of numbers of painted faces all the way to an edge length of 7 inches. Students used the table of values where the edge length of the large cube is used as input and the output is the number of painted faces.

The exchanges took place after students gave up investigating in their small groups to find out if there was a pattern to their input and output values. I was confident, with hints and help, that students could recognize the relation between the input and output values. Thus, I began a whole-classroom discussion and encouraged students to find the formula of 2 faces.
Table 10.2

<table>
<thead>
<tr>
<th>Edge length of large cube</th>
<th>Number of centimeter cubes</th>
<th>Number of small centimeter cubes painted on</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3 faces</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td></td>
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</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 10. Problem Number 2 in the Painted Cubes Learning Task.

Note. The cube in this figure represents the large cube with edge length of 2.

1 Teacher: So, all of this, alright, listen up, we just found. What is twelve?
2 Todd: The greatest common factor in the series of numbers.
3 Troy: I said that, I said the greatest common factor.
4 Teacher: Todd after you said greatest common factor, what else did you say?
5 Todd: On the y.
6 Teacher: On the y. On the y, okay, on the output. The output has the greatest common factor twelve. Alright, so twelve.
7 Troy: Guys, we’re almost there. This is twelve times what? Alright, listen up, so therefore, what is the output when you have twenty? twelve times what? Because this one--^4 Okay guys, listen up. When you have three, you have twelve times what?
8 Ramos: One.
9 Teacher: When you have four?
10 Todd: Every time you subtract two to from the number. So it’s twelve times eighteen for twenty.
11 Teacher: This is twelve times eighteen; please explain how do you get eighteen?
12 Mae: Wouldn’t it be seven?
13 Teacher: Alright, Todd said every time subtract by two.
14 Todd: So you subtract by two from--
15 Teacher: You subtract two from the input. Alright, so please, guys. Twenty minus two. Guys we just need to focus on

^4 ‘--’ in the excerpt indicate the speaker was interrupted by someone else, or, when the speaker sort of ‘interrupted’ herself and changed her train of thought.
this. Focus on the work. Alright, Todd already saw
that we subtract two. Now, if it’s n, what do we do?
Guys, if it’s three you said twelve times what? Guys,
do you see this? Alright, so, let’s do this. Ali, you can
contribute alright. If it’s three, the input, the output is
twelve times one. Four-- I can wait. Let me just call out
the minutes that I have to wait and you have to wait
after the bell rings.

Students: No!!!
Todd: Think outside the box guys.
Teacher: Yeah, think outside the box. I expect Ali, many of you,
Park? Yes, go ahead.
Park: n, twelve n minus two.
Teacher: Alright, if it is n the output is twelve times n minus two.
So that is the formula for two faces, we got it.

Lines 7 and 15 show students responded with one word or a brief description of their
thinking, which were coded BE at level 0. These responses were the counterparts of the teacher’s
AQ of level 0 in which I used “what” in lines 5, 10, 11, and 28 to focus on the answer rather than
on the strategy to find the answer.

Proposing and Leading in the Painted Cubes Episodes

In the Painted Cubes, I elicited students’ responses in 21 instances (see Table 8) by
using another student’s idea (CP = proposing mathematical ideas) as shown in lines 5, 8, and 22.
Meanwhile, students proposed mathematical ideas 6 times in which their ideas were used in
discussion to elicit more responses. Todd proposed his idea in line 17 and Mae interjected her
idea in line 21. Student proposing mathematical ideas in this episode was coded at level 1. There
were 12 instances of leading discourse at level 1, in which students continued the conversation
by repeating what other students said. Students also gave non-mathematical responses to
stimulate more answers from others. In line 35, Todd led others by saying: “Think outside the
box guys.” In the background, students repeated what he just said. This remark most likely
encouraged Park to share the formula that Todd had almost gotten in line 23. Notably, Park
completed Todd’s thinking and came forth with the formula for 2 faces when input is n (line 38). Park said his correct answer the only time he contributed in this whole-classroom discussion. His quiet manner was such a contrast to the noise in this classroom that I barely noticed his response.

*Math-Talk Learning Community Level 0 in the Painted Cubes Learning Task*

According to field notes students could not make progress in their small group discussion. The students could not begin with the strategy to find the pattern of 2 faces. I anticipated in my lesson plan that students would predict the pattern of 2 faces and 3 faces after building 3 by 3 units of a larger cube. The plan did not work; therefore I decided to start a whole-classroom discussion to find the formula of 2 faces and 3 faces of painted cubes. The entire four episodes ended in 335 lines of exchanges that reflected the traditional teacher-centered classroom where the initiate, respond, evaluate (IRE) pattern dominated. Classroom discourse overall reached a math-talk learning community level 0. As Table 8 shows, non-math-talk occurred 49 times. Non math-talk was analyzed using the analytical and social scaffolding coding scheme (Nathan & Knuth, 2003).

*What Math-talk Learning Community Level is Apparent in the Triangles Learning Task?*

*Questioning and Explaining in the Triangles Episodes*

The coding of the Triangles episodes is shown in Table 9. The composition of math-talk components and levels indicate a change in the number of responses from level 0 to levels 1 and 2. This change is likely due to changes in my practice after examining the videotape of whole-classroom discussions, as a recommended process in action research, in the Painted Cubes episodes. I identified a mismatch between my standards-based teaching intention and the actual practice. Considering this initial finding is an important “performance gap” (Herbel-Eisenmann & Cirillo, 2009, p. 20), I made changes to my practice. In the Triangles episodes there was less
AQ at level 0 (4 instances) than in the Painted Cubes (37 instances). I shifted to pressing more questions focused on student thinking (level 1) and multiple strategies from different students (level 2), which resulted in students’ BE moving to levels 1 and 2 as well (see Table 9). The shift from level 0 is shown by the decrease of student’s BE at level 0 from 102 instances in the Painted Cubes episodes to 18 instances in the Triangles episodes. The increase of student responses in levels 1 and 2 of BE indicate that students began to explain with minimal volunteering thoughts, which is a brief description of their thinking with 29 instances (level 1), and to explain by providing fuller descriptions of their mathematical thinking with 29 instances (level 2). The increase in student participation was indicated by the number of questions they posed in two learning tasks. The comparison showed the increase from 2 to 12 questions (level 1) and from 0 to 2 questions (level 2).

The excerpt of the Triangles episode below is taken from a whole-classroom discussion on problem number 3:

3. Next the class decided to use only two sides and one angle. They chose sides of 5 inches and 7 inches with an angle of 38°. Using these measures, construct a triangle and compare it to other students’ triangles.

a. Are any of the triangles congruent? Explain.

b. Does it matter what order the two known sides and known angle are in?

The above problem has the built-in probing questions that required students to respond with explanation. The whole-classroom discussion explored several cases:

- Case 1: Angle between the two given sides
- Case 2: Angle opposite the 5 in. side
- Case 3: Angle opposite the 7 in. side
While exploring the above cases, students began to participate and support each other’s ideas.

The excerpt below is an example that shows Tess extended Todd and Dan’s strategy in discussing Case 2.

Teacher: Letter B. Does it matter what order the two known sides and the known angle are in?
Todd: Yes.
Teacher: Why does it matter Todd?
Todd: It would be wrong.
Teacher: It would be wrong. Okay. In what order, let’s make a plan here. We are going to prove if it is not in that order, and you said it matters. Can you explain, can you predict, why does it matter?
Dan: The triangle would be different.
Teacher: The triangle would be different. And how different?
Can anybody make comment?
Tess: Because it’s not in order.
Teacher: It’s not in order. Please explain?
Tess: Like if it’s side-side-angle, the angle would be moved--like over (she moved her left hand up into the air).
Teacher: Over?
Teacher: Now, the angle is not in between. Guys, the angle now is not in between 5 and 7. Construct your triangle in this order.
Mae: Ms. H did we do this right?
Teacher: Alright, can someone tell me why we did this kind of plan?
Jed: Cause I want a grade.
Teacher: You want a grade. Okay. Listen, class, class. Why did we do this thing?
Jed: To do it backwards.
Teacher: Yeah, we do it backwards. Why do we do this plan? Don’t we want to answer this?
Troy: No. (He smiled and pretended he did not say this remark.)
Teacher: When you change the order the two known sides we want to find out that the order matters. Can you
compare? Guys compare your triangle with others.

Teacher: So--guys, it’s different. Guys listen up. That’s why we do this. You have five inch, still side, side and this is not the order that we did on case number one. This is why we do this, and you said that your triangles are different. That’s why when the sides are not in order like side, angle, side, you do not have what kind of triangle?

Ramos: Congruent.

I had to probe with several questions to get students to explain their mathematical thinking. For example, lines 22, 26, and 29 were my probing to elicit student thinking about the goal to construct a triangle with two sides and an angle when the angle is not formed by the two given sides. My direction was critical to guide students to prove that side-side-angle or angle-side-side is not one of the congruent triangles rules. The Triangles learning task involved construction work and congruent triangles skills. These concepts in themselves are relatively new in the freshmen mathematical experience. Students’ responses using explanations at levels 1 (lines 10, 13, and 42) and 2 (lines 15 and 19) indicate that students demonstrated some degree of participation in sharing their new knowledge of congruent triangles.

Proposing and Leading in the Triangles Episodes

Students continued to support one another by repeating, completing, and responding to each others’ comments. I noted (from the field notes) that students who sat in a group of math-talkers often echoed and added to what their peers said. Nina, Todd, Ben, and Troy would respond in sequence or almost at the same time when Troy or Nina gave the answer. On many occasions Ramos responded after Todd explained his thought. Although Ramos was separated from Todd for the purpose of classroom management, Ramos seemed to tune his ears to what Todd said in this class and would add his own response. Similarly Troy would support Jed’s remarks to keep the discussion alive. The math-talk coding identified 9 instances where students
became more engaged by repeating what other students said or completing another student’s idea. These responses were coded as leading in discourse level 1.

On the other hand, I created the opportunity to facilitate student listening to and helping other students beginning at level 1 by setting up small group discussions, and at level 2 by encouraging students’ sharing their thinking in a whole-classroom discussion. The math-talk framework states that student leading in math-talk level 2 is signified by students listening to one another and making comments on other students’ responses. Instances of students demonstrating DL in discourse at level 2 occurred 3 times. Below is an example of the exchanges in which Troy’s response clarified order of the sides in congruent triangles matters, which was the goal of exploring each case in problem number 3.

Teacher: Your construction results are not the same as your friends’. Alright, you can have five inch. Listen up please. You can have five inch and you still have seven inch and forty degree, and these two are not congruent triangles. That’s why we do this. Alright?

Tara: It’s going to be side, side, angle regardless.

Troy: No, I got angle-side-side.

Teacher: That’s why the order of the sides matter. Alright?

Because side-side-angle, and angle-side-side are two different triangles.

Tara: You said you can’t use angle-side-side.

Teacher: That’s right, we just proved it. Your triangle and Dora’s triangle are not the same. Alright? That’s why this is not the rule of congruent triangles, and we just proved it.

Troy (line 60) argued Tara’s result based on his own construction. Students proposed mathematical ideas (CP) 2 times; these ideas were coded as math-talk level 1 because student ideas were used in discussion, but not explored. For instance, in lines 59 and 64 Tara interjected
the idea regarding the result of our investigation. I used her idea to underline our findings about one of the rules of congruent triangles.

Math-Talk Learning Community Level 1 in the Triangles Learning Task

In terms of a math-talk learning community, the Triangles episodes reflected math-talk learning at level 1 because most of the sentences were coded as level 1. Most of the time, the teacher was the main source of ideas in conducting the whole-classroom discussion. Students’ participation was characterized by the teacher pursuing student thinking; thus the teacher played the role of questioner. Although students asked questions showing their engagement in our discussion, those questions were directed to me and focused on the work assigned to them and the work of other groups. In the Triangles task, Dan, Tess, and Grant were new math-talkers. Fort, who was skillful in constructing triangles, helped Ramos in two class periods. The small group work took place during this learning task. The small group session allowed students to interact with each other and seek individual help from me. A quiet student like Sue would ask for help on how to label the congruent triangles. Meanwhile engaged students would declare, “We are done,” when they successfully constructed a triangle congruent to another student’s triangle in their group.

The length of math-talk decreased from 334 to 302 lines of exchanges in the Triangles episodes. The lower number of exchanges indicates more success in small groups, in which students constructed and compared the results of congruent triangles with less teacher-guided direction than occurred in the Painted Cubes episodes. Color-coding identified 55 instances of non-math-talk to be analyzed in the next discussion.
Flow of Information

To describe the flow of vertical and horizontal information in a whole-classroom discussion, every sentence in the Painted Cubes and Triangles episodes was coded to indicate the pathways of interactions from teacher to student (vertically), teacher to class (vertically) student to teacher (vertically), or student to student (horizontally) as described in the Coding Scheme of Analytical and Social Scaffolding (see Appendix A, Table A2). Figures 11 and 12 present the flow of vertical and horizontal information (Hatano & Inagaki, 1991; Nathan & Knuth, 2003) that occurred during the Painted Cubes and Triangles episodes. Separate nodes represent each student, myself, and the whole class as a unit participant. Nathan and Knuth (2003) introduced this diagram to represent the pattern of interactions at the global level. The colored lines between the nodes identify discourse exchanges. The frequency of interactions for each math-talker was measured by the tallied number of sentences spoken by and received by that node. The line thickness represents relative frequency of the responses. The number of sentences that represents vertical and horizontal flow of information illustrated in each of these figures is summarized in Tables 10 and 11.

The flow of information in Figure 11 indicates that there were three groups with at least one math-talker who contributed their responses consistently. In particular, Nina (7), Troy (8), Mae (2), Todd (4), and Ramos (12) became the prime math-talkers of their groups. Jed (25) participated equally with the above students in the Painted Cubes episodes. His participation marked a new side of Jed, who usually never wrote or solved his mathematics problems, and failed his freshmen Math I. The entire class was supportive of Jed’s participation.
Figure 11. The Painted Cubes Episodes Flow of Information

Note. The number in the circle indicates student’s seating chart in the class roster (Table 4).

The line thickness represents relative frequency of the responses.
### Table 10

**Vertical and Horizontal Information in the Painted Cubes Episodes**

<table>
<thead>
<tr>
<th>Vertical (Teacher-to-Student)</th>
<th>Vertical (Teacher-to-class)</th>
<th>Vertical (Student-to-teacher)</th>
<th>Horizontal (Student-to-student)</th>
<th>Total</th>
</tr>
</thead>
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<tr>
<td>%</td>
<td>Counts</td>
<td>%</td>
<td>Counts</td>
<td>%</td>
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<td>65</td>
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<td>Social</td>
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<tr>
<td>Total</td>
<td>12</td>
<td>73</td>
<td>31</td>
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</tr>
</tbody>
</table>

### Table 11

**Vertical and Horizontal Information in the Triangles Episodes**

<table>
<thead>
<tr>
<th>Vertical (Teacher-to-Student)</th>
<th>Vertical (Teacher-to-class)</th>
<th>Vertical (Student-to-teacher)</th>
<th>Horizontal (Student-to-student)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
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<tr>
<td>Social</td>
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<tr>
<td>Total</td>
<td>25</td>
<td>75</td>
<td>33</td>
<td>100</td>
</tr>
</tbody>
</table>
New math-talker

- Dashed black is analytical scaffolding from teacher.
- Dashed red is social scaffolding from teacher
- Black is analytical scaffolding from math-talker.
- Red is social scaffolding from student.

Figure 12. The Triangles Episodes Flow of Information

Note. The number in the circle indicates student’s seating chart in the class roster (Table 4).

The line thickness represents relative frequency of the responses.
The participation of members of other groups may have influenced the group including Novi (27) and Grant (28). After the Painted Cubes episodes, the students willingly expressed their thinking more often by imitating their peers. For example, Novi and Grant contributed more in the Triangles episodes. Similarly Tara (22), who sat in Jed’s group, gave more responses in the Triangles episodes. The vertical flow of information includes social scaffolding instances (dashed red line, see Figure 11) in which I invited Ali, Beth, Fort, Juana, Tess, and Sue, all of whom had not contributed before, to join our class discussion. The coded sentences representing social scaffolding from teacher to student were recorded 3% out of 335 total coded sentences (see Table 10).

As shown in Table 10, the vast majority of information flowed vertically either as analytical scaffolding from the teacher to the class (19%) or to an individual student (19%), or as students’ responses and questions (46%) directed to me. In the Painted Cubes episodes, as charted by the flow of information, I was the main source of the information or the hub of this whole-classroom discussion. The description of our whole-classroom discussion diagrammed in Figure 11 is consistent with the result of coding categories of math-talk components and levels. I maintained the practice of a traditional teacher-centered classroom which revealed a performance gap (Herbel-Eisenmann & Cirillo, 2009) between my assumption about standard-based teaching and my actual discourse practice.

In the Triangles episodes, Figure 12 shows that the participants began to demonstrate student-to-student talk or horizontal flow of information as exchanges took place between Dan (13) and Tess (6), Grant (28) and Todd (4), and Tara (22) and Troy (8). The vertical flow of information includes social scaffolding instances (dashed red line, see Figure 12) in which I invited Vera, Liz, and Rosa to join our class discussion during the warm-up activity. Vera and
her group usually engaged in the task but they did not contribute in explaining their work. The coded sentences representing social scaffolding from teacher-to-student were recorded 3% out of 302 total coded sentences (see Table 11).

The nodes in Figure 12 include new math-talkers Tess, Dan, and Grant (indicated by stars). Although students began to address each other directly, only 3 sentences were coded as student-to-student talk (1%), in contrast to 109 sentences (36%) of student-to-teacher talk (see Table 11). The overall pattern of the flow of information still indicates my role as the center of our discussion. The above result is consistent with the analysis using the math-talk framework as described in the Triangles episodes (see Table 9).

Analytical and Social Scaffolding in the Baseline Episodes

To describe the analytical and social scaffolding used in a whole-classroom discussion, every sentence in the Painted Cubes and Triangles episodes was coded based on the Coding Scheme of Analytical and Social Scaffolding (see Appendix A, Table A2). The results of coding work for the Painted Cubes and Triangles episodes are presented in Tables 12 and 13. The codes for analytical and social scaffolding are classified into four groups of spoken sentences including teacher-to-student, teacher-to-class, student-to-teacher, and student-to-student. Each group of spoken sentences is divided into several content categories listing the variety of the speech based on the Coding Scheme of Analytical and Social Scaffolding (Nathan & Knuth, 2003).

Analytical Scaffolding in the Painted Cubes Episodes

The students and I provided a variety of social and analytical scaffolding for each other as shown in Table 12. A total of 335 sentences were coded in the Painted Cubes episodes. Student responses totaled 153 sentences with all analytic speech directed to me, except for one occurrence of social scaffolding that invited the whole class to think outside the box.
Table 12

**Coding of Analytical and Social Scaffolding During Painted Cubes Task**

<table>
<thead>
<tr>
<th>Scaffolding</th>
<th>Teacher to Student</th>
<th>Teacher to Class</th>
<th>Student to teacher</th>
<th>Student to student</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QM</td>
<td>TM</td>
<td>QFM</td>
<td>TVA</td>
</tr>
<tr>
<td>Analytical</td>
<td>1</td>
<td>1</td>
<td>41</td>
<td>14</td>
</tr>
<tr>
<td>Social</td>
<td></td>
<td></td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>Analytical = 63</td>
<td>Social = 10</td>
<td>Analytical = 65</td>
<td>Social = 39</td>
</tr>
<tr>
<td></td>
<td>Analytical = 153</td>
<td>Social = 4</td>
<td>Analytical = 0</td>
<td>Social = 1</td>
</tr>
</tbody>
</table>

Table 13

**Coding of Analytical and Social Scaffolding During Triangles Task**

<table>
<thead>
<tr>
<th>Scaffolding</th>
<th>Teacher to Student</th>
<th>Teacher to Class</th>
<th>Student to teacher</th>
<th>Student to student</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QM</td>
<td>TM</td>
<td>QFM</td>
<td>TVA</td>
</tr>
<tr>
<td>Analytical</td>
<td>7</td>
<td>40</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Social</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>Analytical = 65</td>
<td>Social = 10</td>
<td>Analytical = 70</td>
<td>Social = 30</td>
</tr>
<tr>
<td></td>
<td>Analytical = 109</td>
<td>Social = 15</td>
<td>Analytical = 3</td>
<td>Social = 0</td>
</tr>
</tbody>
</table>
My analytical scaffolding to individual students reached a total of 63 sentences. The highest frequency of analytical scaffolding occurred 41 times in “asking follow-up question” (QFM). Analytical scaffolding from teacher-to-class occurred as I invited the whole class to participate at the beginning of a discussion, or in the middle of class exchanges, to involve students in our ongoing discussion as well as to focus on the mathematics problem at hand. I recorded 14 instances of open invitation with “math question” (OM). The transcript of classroom exchanges also included mathematical explanation to scaffold student’s thinking, coded as “explaining and eliciting student idea” (TRE), which occurred 29 times. “Declaring of Math principles, facts, rules” (DM) was reflected in Table 12 in 22 instances. Taken all together, there are 65 instances of vertical flow of information from teacher to class.

When students responded and asked me questions it was coded under the student-to-teacher category. Table 12 shows the variety of students’ analytical scaffolding included 99 instances of students responding with declaring of “math principles, facts, rules” (DMS). The students gave responses with “minimal volunteering thoughts” (MVM) 21 times. Low frequency was indicated by 1 instance of respond with some volunteering thoughts (SVM). Students addressed “mathematics questions” (QMS) 7 times.

Comparing student analytical scaffolding and student math-talk in the Painted Cubes episodes (see Tables 8 and 12), the result of 99 instances of DMS is compatible with the 102 times students gave math-talk explanations at level 0. Both coding procedures revealed students’ tendency to respond using facts, rules, or short answers. The traditional teacher-centered classroom in which students give short responses to the teacher only is verified by both the math-talk and analytical scaffolding coding schemes.
Analytical Scaffolding in the Triangles Episodes

The total of 302 coded sentences in Table 13 shows a variety of analytical and social scaffolding in the Triangles episodes. The total responses comprised 109 sentences in student analytical scaffolding directed to me and 3 responses directed to other students, showing that more student-to-student responses occurred in the Triangles than in the Painted Cubes episodes.

My analytical scaffolding to individual students reached a total of 65 sentences. The high frequency of analytical scaffolding occurred 40 times in QFM, which is slightly changed from 41 instances in the Painted Cubes episodes. The analytic scaffolding “stimulating students to think deeper about strategies” (TSM) occurred 9 times, an increase from 6 occurrences. While the frequency of “asking or probing question” (QM) was 7, an increase from 1. Similarly OM increased from 14 to 20 times. The increase in teacher questioning both to individual students and to the class highlights the difference in teacher analytical scaffolding between the Triangles and the Painted Cubes tasks.

After their experience during the Painted Cubes, student responses and questions directed to me in the Triangles episodes took a different form (see Table 13). The exchanges reflected the students’ progression toward higher level thinking in mathematics as the number of DMS instances decreased to 25, MVM frequency slightly increased from 21 to 26, and SVM occurrences climbed from 1 to 28 instances. In this episode, students “responded to question/comment from student” (RMS) 3 times, an increase from none in the Painted Cubes episodes. The coding of RMS indicates those students began to respond to other students during our whole-classroom discussion.

Comparing student analytical scaffolding and student math-talk in the Triangles episodes (see Tables 9 and 13), there were 25 instances of DMS compared to the 18 times students explained at level 0. There were 26 and 28 instances of MVM and SVM respectively compared to the 29 times
students responded with brief description (level 1) and 29 times students began to give fuller descriptions of their answer (level 2) in the math-talk coding schemes. The new appearance of 3 instances of RMS is compatible to leading in math-talk at level 1, which was coded 9 times. The comparison on both results based on math-talk and analytical scaffolding coding schemes gave similar results showing that the Triangles episodes reflect the emerging math-talk learning community. In our whole-classroom exchanges students began to respond with brief descriptions of their thinking (MVM), and they explained their method (SVM) when prodded.

Social Scaffolding in the Painted Cubes Task Episodes

Showing that the students and I provided a variety of social scaffolding for each other, Table 12 reveals a total of 54 sentences coded as social scaffolding in the Painted Cubes episodes. The student responses comprised 5 sentences, all with social speech acts directed to me, except one instance that invited the whole class to think outside the box. Other social utterances were recorded: “We can try” (Mae); “Yeaaa” (Troy) on two occasions, “I got it” (Nina).

My social scaffolding to individual students and the whole class reached a total of 49 sentences. Social scaffolding occurred with the frequency of 15 times in “declaration of non-math” (DN). “Open invitation with non-math question” (OMN) occurred 7 times, and “asking non-Math question” (QN) appeared 7 times. The frequency “facilitating discourse” (UPN) equals 3. The high frequency of 17 times in “management-discipline” (MG) reflects that I had to regulate one aspect of the math-talk norms “listen to what others say.”

Though I did have success eliciting student participation, the challenge of changing the classroom environment is real. Below is an example of the negotiation in a whole-classroom discussion when I pursued my students to express their thinking more completely during our discussion of problem number 2 in the Painted Cubes episodes.
Teacher: What pattern did you see? Can someone see the pattern now?

Nina: Two.

Mae: Three.

Teacher: Two, three, two, three. So every one of them has two, three. What is two, three?

Ramos: What is two, three?

Teacher: Let me give you another clue. Okay guys, can we go higher so we can have a better formula? That’s good. two, three is in every expression. What is two times three?

Ramos: Six.

Teacher: Six, so this twelve, the first output has six, alright. How should I say it?

Nina: It’s divisible.

Teacher: Say that again Nina

Nina: Divisible--

Teacher: By?

Nina: Divisible by six.

Teacher: Listen up. Nina said every output is divisible by six, alright. Can we go higher?

Todd: Yes.

Mae: We can try.

Teacher: So, besides six, what do we call the number that goes into?

Ben: Greatest common denominator.

Teacher: What is the greatest common factor between twelve, twenty four, thirty six, forty eight?

Todd: Thirty two.

Todd: Twelve.

Teacher: The greatest?

Todd: It’s twelve! (He knows that his answer is correct).

Teacher: So, all of this, alright, listen up, we just found. What is twelve?

Todd: The greatest common factor in the series of numbers.

Troy: I said that. I said the greatest common factor!

Lines 8 and 12 contain social and analytical scaffolding in the negotiation of how to participate in a whole-classroom discussion to find the pattern in the Painted Cubes task. Although the class had math-talkers who consistently contributed to class discussion, persuading all students to participate proved a difficult task. Line 34 records that Troy voiced his frustration because I missed his contribution. The above findings report that 15% of the spoken sentences (49 coded
sentences out of 335 total exchanges) were teacher social scaffolding addressed to elicit contributions from all students in our math-talk learning practice.

**Social Scaffolding in the Triangles Episodes**

Representing the Triangles episodes, Table 13 shows a total of 55 sentences coded as social scaffolding occurrences. The student responses were comprised 15 sentences directed to me. There were 11 instances of student “declaration of non-math” (DMS-Social). Students gave more responses in social speech to declare: “I am done” (Dora); “I did that” (Novi); “Exactly, I said that already” (Ramos). The above classroom behaviors show the trend from the traditional environment where the students believed that the goal was to find the answers then the job would be done (Silver et al., 1990). Social scaffolding was provided to encourage students to explain their strategy. The negotiation for math-talk norms is shown in these exchanges:

```
35 Teacher: Write down your good strategy. Please write down your strategy. Yes?
36 Dora: I’m done.
37 Teacher: Okay.
38 Ben: I’m done.
39 Teacher: With the strategy and the answer?
40 Ben: I am.
41 Teacher: But what is the strategy, you just wrote the problem.
42 Nina: Oh, what strategy?
43 Teacher: How do we prove these two triangles are congruent?
44 Please explain your strategy.
45 Todd: Why don’t you just add up 6 and 8 and then (inaudible)
46 Teacher: Guys we try to finish this real quick with your input and idea. Alright, Todd said just add up the number, what do you think? Any ideas beside add up?
47 Jed: What do you mean by add up?
```

My social scaffolding to individual students and the whole-class reached a total of 40 sentences. The frequencies of social scaffolding occurred ten times for OMN. The instance of QN occurred five times, while the frequency of UPN equals five. The frequency of my utterance “Please listen” (MG) decreased from 17 to 10 instances, as well as DN from 15 to 10 instances. These lower
frequencies of MG and DN than those in the Painted Cubes episodes may indicate that students began to listen to one another and contribute their mathematical thinking by listening, repeating, completing, or commenting on each others’ responses (Hufferd-Ackles et al., 2004).

Summary and Conclusions of Baseline Data

The analysis of the Painted Cubes and Triangles task episodes reveal several important aspects in building a math-talk learning community. First, the majority of students in this class seldom participated in exchanges even when called upon. During whole-classroom discussions, students depended predominantly on me for mathematical ideas rather than other students. Consequently, students rarely used discourse as a means to build their own understanding, predict outcomes, or question other student’s ideas. The baseline data, therefore, describes an emerging math-talk learning community with very low participatory classroom environment.

Second, the examination of the Painted Cubes and Triangles tasks through field notes and reflective journals, as part of the action research process, helped me to identify some factors that can foster and constrain student opportunities for participation. Explaining, as one of the math-talk components, is a counterpart of questioning. After recognizing my performance gap in the Painted Cubes task, I began to focus and increase my questioning on student thinking that required at least a brief description of their thought process. I paid attention to practice leading in math-talk at level 1 where the teacher helps facilitate students listening to and helping each other.

Third, the small group discussion that occurred in the Triangles task allowed students to focus on the problems and to build strategies of proving congruent triangles. To some degree, students learned to talk to one another during group work, increasing the sharing of a variety of ideas, arguments, questions, and revisions (Hatano & Inagaki, 1991). The success in small group discussion continued to the whole-classroom discussion in which students were able to demonstrate
more listening to and responding to each others’ comments. In addition to the instances of responses with minimal volunteering (level 1), the students provided a fuller description of their thinking (level 2). In contrast, students did not have the opportunity to create productive peer discussion before joining a bigger circle for whole-classroom discussion in the Painted Cubes episodes. As the results reflect, the absence of small group discussion impeded students contributing responses in a whole-classroom discussion (Cobb et al., 1993).

Although students made progress by responding with some volunteered thoughts, their responses tended to be directed primarily at the teacher. Students rarely responded to each other directly during a whole-classroom discussion. I played the role of the sole questioner in our mathematical discourse where only certain math-talkers were willing to answer. The first and the second episodes involved the same *nine* math-talkers out of 28 students, and only added three new math-talkers.

Part 2: Content-Related Comics Strategy

During the last two weeks of the study, I introduced content-related comics activities. Each content-related comic activity was used to introduce a learning task. “A Square by Any Other Name” from the Luann comic (Evans, 2002) was used to introduce the Diagonals and Quadrilateral task. Similarly, “A Stone’s Throw Away” from the Rhymes with Orange comic (Price, 2009) was paired with the Mean Absolute Deviation (MAD) learning task. The math-talk codes for the overall Diagonals and MAD episodes are shown in Tables 15 and 17. In addition to comparing the math-talk between the Diagonals and MAD learning tasks, the analysis also included the comparison between warm-up episodes before and after the introduction of content-related comics strategy. The comparison is described after the analysis of the content-related comic “A
Stone’s Throw Away.” The math-talk codes for each warm-up episode from the Diagonals, MAD, Painted Cubes, and Triangles tasks are shown in Tables 14, 16, 18, and 19 respectively.

**What Math-Talk Learning Community Level is Apparent in the Diagonals Learning Task?**

To describe the math-talk components and levels used in a whole-classroom discussion, every sentence in the Diagonals episodes was coded based on the Coding Scheme of Math-Talk Components and Levels (see Appendix A, Table A1). The math-talk codes for the warm-up of the Diagonals episode are shown in Table 14, next to Table 15 which is the math-talk codes for the overall Diagonals episodes.

**The Content-Related Comic in the Diagonals Episodes**

The comic proposed that “Every square is a rectangle but not every rectangle is a square,” followed by several prompts to open a Q&A session:

a. What do you think about this comic?

a. Can you explain what does it mean that every square is a rectangle but not every rectangle is a square?

b. What is the definition of a square? A rectangle?

c. What other definition includes a square and rectangle?
1. Look at the shape Gabriel made on his geoboard. Explain how you know his shape is a square.

2. Use a geoboard to make a shape that meets each description, if possible. If it is not possible, explain why not.

a. A rectangle that is not a square  
b. A square that is not a rectangle

Figure 13. Content-Related Comic “A Square by Any Other Name”

Note. From Cartoon Corner (p. 50), by G. Evans, 2002, In A. Reeves (Ed.), Reston, VA: NCTM. Copyright 2011 by NCTM Copyright Clearance Center. Posted on Kennesaw State University intranet with permission. (The complete content-related comic “A Square by Any Other Name” can be found in Appendix B.)
Table 14

Coding of Math-Talk During Warm-up Episode in the Diagonals Task

<table>
<thead>
<tr>
<th></th>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Non Math</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AQ</td>
<td>BE</td>
<td>CP</td>
<td>DL</td>
<td>AQ</td>
</tr>
<tr>
<td>Students</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Teacher</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Subtotal</td>
<td>1</td>
<td>S = 0</td>
<td>T = 0</td>
<td></td>
<td>S = 15</td>
</tr>
</tbody>
</table>

Math-talk lines = 43  Non-Math-talk lines = 8  Total = 51

*Note.* AQ = Questioning, BE = Explaining, CP = Proposing, DL = Leading Discourse, S = Students, T= Teacher

Table 15

Coding of Math-Talk During Diagonals Task

<table>
<thead>
<tr>
<th></th>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Non Math</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AQ</td>
<td>BE</td>
<td>CP</td>
<td>DL</td>
<td>AQ</td>
</tr>
<tr>
<td>Students</td>
<td>0</td>
<td>36</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Teacher</td>
<td>13</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>36</td>
</tr>
<tr>
<td>Subtotal</td>
<td>1</td>
<td>S = 36</td>
<td>T = 18</td>
<td></td>
<td>S = 56</td>
</tr>
</tbody>
</table>

Math-talk lines = 238  Non-Math-talk lines = 26  Total = 264

*Note.* AQ = Questioning, BE = Explaining, CP = Proposing, DL = Leading Discourse, S = Students, T = Teacher
As shown in Figure 14, the problems in this content-related activity require students to draw and compare different shapes of polygons. Students’ math-talk included questioning, explaining, proposing mathematical ideas, and leading discourse at level 1. Students also gave math-talk in the form of explaining and leading discourse at level 2. Immediately following the reading of the comic, the students responded with several comments and a question, as recorded in lines 7 and 8 in the excerpt below. Wes and Mae spontaneously commented, and Troy followed by attempting to make sense of the comic (lines 7, 8, 11, and 13). I made a transition in line 16, and we began with the problem in line 18. Although the students did not find the comic funny, they continued with the learning task. The warm-up involved 7 math-talkers—Dan, Jed, Mae, Patel, Ramos, Wes, and Troy—of which, Patel was contributing for the first time.

1 Mae (reading a comic): “Why do you have such a hard time with math? I don’t know, math like bonks around in my head. My teacher said every square is a rectangle but not every rectangle is a square. Man that one is still bonking around.”
2 Teacher: Thank you Mae.
3 Wes: I don’t get it.
4 Mae: Is it a joke?
5 Teacher: Kinda, he’s just confused like some of my students might be. Anyone have questions on the comic?
6 Troy: Yeah, that doesn’t make any sense to me.
7 Teacher: Why doesn’t it make any sense?
8 Troy: Because it’s not funny.
9 Teacher: Because it’s not funny, alright. You have to make a funny comic then, about mathematics. It’s kinda tough. We’re going to use that comic to do the problems. Number 1 explain how you know this shape is a square.
10 How do you know this is a square guys?
11 Jed: It has four sides.
12 Ramos: All angles are 90 degrees.
13 Teacher: I will write here: it has four sides; all angles are 90 degree. How do we know it is ninety degrees?
14 Mae: Because it’s a right angle; all sides are congruent.
15 Teacher: All sides are congruent. But I am not sure we can claim it is 90 degree. How do we know that?
16 Mae: It makes an L.
Teacher: It makes an L. Alright. I want to point out these lines. What do you call this? The one I draw? (I continued to draw the other diagonal)

Troy: Diagonal.

Mae: Right angle.

Teacher: Troy said these are the diagonals, and Mae said they make right angle. Can you say more? What do you see on the diagonals?

Dan: They intersect?

Teacher: They bisect each other and remember when they make right angle? Mae you know this one.

Mae: Diagonals are parallel. No--I forgot.

Troy: Perpendicular!

Teacher: Diagonals of a square are perpendicular. I want you to write all of these. Now we are going to do number two. Explain, is it possible to draw a rectangle that is not a parallelogram?

Wes: No.

Teacher: Alright, so you said it is not possible, why it is not possible?

Wes: Because a rectangle is a parallelogram.

Teacher: Because a rectangle is a parallelogram. You need to explain more.

Wes: It has four sides. You can’t lose a shape that has four sides.

Troy: The sides never meet. They never intersect.

Teacher: Can you say again please. Never intersect?

Troy: Like, they’re not angled, they are like this, not like that (Troy demonstrated his shape using a square).

Teacher: Alright, sides never intersect means it’s parallel. Very good, these sides are parallel.

Patel: and Congruent

Teacher: Right. The opposite sides are congruent and parallel. So we can say a rectangle satisfies the definition of parallelogram. Now, when you have a rectangle, can you tell about its diagonal?

Students: Congruent.

Teacher: Diagonals are congruent.
In this opening discussion, I used CP by building upon Troy and Mae’s explanations (lines 30 and 31), which prompted Dan to join our discussion (line 35). Similarly, I used Wes and Troy’s ideas (lines 50, 52, and 54) to justify the statement “A rectangle is not a parallelogram.” After I clarified Troy’s description, Patel completed my clarification (line 58).

Troy supported and explained Wes’ idea in his own words, thus demonstrating DL in math-talk at level 2, where students clarify other students’ ideas for themselves and others. Both Troy and Wes exhibited confidence about their ideas and shared their own thinking and strategies even if they differed from others’. Evidence of students moving to level 2 was demonstrated by Mae and Troy in lines 38 and 39. Mae was confident that she knew the vocabulary for when two lines form a right angle. She wanted to recall the word from past experience. Even though she forgot the words, Mae consistently engaged in discussion, and took a risk to contribute her answer publicly. Both students proposed ideas that lead the discussion of diagonals in a square. Mae’s incomplete answer prompted Troy to provide the correct vocabulary. The codes in Table 14 indicate that I elicited students’ responses in 6 instances (4 at level 1 and 2 at level 2) by using another student’s idea as shown in lines 32, 36 and 40.

Questioning and Explaining in the Diagonals Episodes

The coding result in the Diagonals episodes indicates that I implemented math-talk components at levels 1 and 2 more than level 0. I asked level 1 questions in 36 instances and level 2 questions in 24 instances, in comparison to 13 instances at level 0 (see Table 15). Students demonstrating BE at level 0 was coded in 36 instances, which appeared to be high, and was related to the Diagonals task including definitions and vocabulary of properties of diagonals attributed to each quadrilateral. Consequently, the initial class discussion involved questions and
responses beginning at level 0 to prepare students for understanding and using the appropriate geometric terms.

As the excerpt from “A Square by Any Other Name” comic indicates, students began to explain their thinking with brief descriptions. Students were able to defend their answer and method as indicated in lines 44, 47, and 50. Although diagonals are relatively new concepts in the freshmen mathematical experience, students responded and continued to participate with levels 1 and 2 explanations at 41% and 9% respectively out of the total amount of student math-talk. Students engaged in questioning me 3 times and another student 1 time (Table 15). Fewer questions than responses addressed during classroom discussion may reflect the fact that students still reserved their traditional view that mathematics activity was about getting the answer, and they rarely questioned the reason behind the answer.

The field notes indicate that students worked successfully in their groups and a representative of each group went to the whiteboard to write the diagonal characteristics of their assigned quadrilateral. Kay was among the presenters. Though ready to share, she said “I cannot explain it,” and Dan came to her aid by reading the written answers. In this mini presentation, I coached my students to ask the audience for questions. Even though their own peers were presenting, the students continued to address me with their questions. To encourage the building of a math-talk community, I directed them to ask questions directly to the presenter.

Proposing and Leading in the Diagonals Episodes

The codes in Table 15 indicate that I elicited students’ responses in 25 instances (level 1) by using another student’s idea as shown in lines 22, 27, and 32. Meanwhile, the students used DL 3 times in the warm-up and by the end of the Diagonals episodes; they contributed DL 7 times at level 1 and 4 times at level 2. The content-related comics may help the students make
connections with previous ideas. For example, Todd referred to the problem presented in the content-related comic activity and argued, “Because on the first one you said, every rectangle is a parallelogram, and we said that is true, okay? And now, you said every parallelogram is a rectangle.” Todd’s math-talk matched with CP at level 2, in which students exhibit confidence about their ideas and share their own thinking and strategies that sometimes guide the direction of the math lesson (Hufferd-Ackles et al., 2004).

I observed that the students were hesitant to share their answers on the board because they were concerned that their work would be wrong. As a result, I had to emphasize frequently that the answer to the problem is not the goal of our learning, and that everyone should share math work to help us better understand the process.

My reflection indicates my reluctance to press students with deeper questions because they wanted to end our whole-classroom discussion. The following excerpt contains an interaction, showing that I should use more opportunities and strategies than what I did to elicit students’ responses. For example in line 67 below, I should have followed up Ramos’ response with a question. However, I was easily distracted with Nina’s question, which needed my attention. The exchanges took place during the closing activity when time was pressing and students were anxious to pack up their materials. We were discussing problem number 8 in the Diagonals task that asks students to compare and contrast the property of diagonals as shown in Figure 14. Prior to this whole-classroom discussion, students determined which quadrilateral(s) could be constructed based on specific information about the diagonals of the quadrilateral(s). For example, given diagonals are perpendicular, construct and name quadrilaterals that satisfy this information.
8. Below are two trapezoids. Draw in the diagonals for these trapezoids.

   a. Diagonals are ______________________________________

   b. What can you summarize about the diagonals of trapezoid A?

   c. What can you summarize about the diagonals of trapezoid B?

\[\text{Figure 14. Problem Number 8 in the Diagonals Learning Task.}\]

Math-talk Learning Community Level 1 in the Diagonals Learning Task

The Diagonals episodes reflect a math-talk learning community at level 1. The Diagonals episodes comprised 264 lines of exchanges, less than the 302 lines or 335 lines of episodes in the baseline data. There were 20 instances of non-math-talk from me, and 6 instances from the students (see Table 15). The fewer math-talk lines in the Diagonals episodes may indicate that the productive peer discussion in small groups allowed more participants to focus on and successfully complete the task with less support from me. In particular, students’ math-talk level 1 in the Triangles episodes (baseline data) represented 42% of the total level 1 math-talk. In the Diagonals episodes, with the content-related comic, students also contributed 42% of their math-talk at level 1 from all math-talk components used in this episode. Meanwhile, teacher non-math-talk is 20 instances compared to 49 and 40 instances in the Painted Cubes and Triangles task respectively. The smaller number of teacher non-math-talk suggests that students contributed math-talk in the Diagonals task with less teacher guidance.
### Table 16

**Coding of Math-Talk During Warm-up Episode in the MAD Math-talk**

<table>
<thead>
<tr>
<th></th>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Non Math</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>AQ</td>
<td>BE</td>
<td>CP</td>
<td>DL</td>
<td></td>
</tr>
<tr>
<td>Students</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
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<td></td>
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<td></td>
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<td>Teacher</td>
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<td>0</td>
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<td>T = 8</td>
<td>S = 6</td>
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<td></td>
<td></td>
<td></td>
<td>T = 12</td>
</tr>
</tbody>
</table>

Math-talk lines = 34  Non-Math-talk lines = 17  Total = 51

*Note.* AQ = Questioning, BE = Explaining, CP = Proposing, DL = Leading Discourse, S = Students, T= Teacher

### Table 17

**Coding of Math-Talk During MAD Task**

<table>
<thead>
<tr>
<th></th>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Non Math</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AQ</td>
<td>BE</td>
<td>CP</td>
<td>DL</td>
<td></td>
</tr>
<tr>
<td>Students</td>
<td>6</td>
<td>18</td>
<td>0</td>
<td>0</td>
<td></td>
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<tr>
<td></td>
<td>8</td>
<td>28</td>
<td>1</td>
<td>8</td>
<td></td>
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<td></td>
<td>4</td>
<td>18</td>
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</tr>
<tr>
<td>Teacher</td>
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<td>4</td>
<td>2</td>
<td>2</td>
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<td></td>
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<td>22</td>
<td>12</td>
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<td>T = 77</td>
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<td>S = 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>T = 0</td>
</tr>
</tbody>
</table>

Math-talk lines = 240  Non-Math-talk lines = 52  Total = 292

*Note.* AQ = Questioning, BE = Explaining, CP = Proposing, DL = Leading Discourse, S = Students, T= Teacher
What Math-talk Learning Community Level is Apparent in the MAD Learning Task?

To describe the math-talk components and levels used in a whole-classroom discussion, every sentence in the Mean Absolute Deviation (MAD) episodes was coded based on the Coding Scheme of Math-Talk Components and Levels (see Appendix A, Table A1). The math-talk codes for warm-up of the MAD episode are shown in Table 16, next to Table 17 which is the math-talk codes for the overall MAD episodes.

Figure 15. Content-Related Comic “A Stone Throw Away”

Note. From “Cartoon Corner” by H. Price, H, 2009, Mathematics Teaching in the Middle School, 15, p. 10. Copyright 2011 by NCTM Copyright Clearance Center. Posted on Kennesaw State University intranet with permission. (The complete content-related comic “A Stone Throw Away” can be found in Appendix B.)
The Content-Related Comic in the MAD Episodes

The comic, as shown in Figure 15, began with the sentence “A stone’s throw away is an idiom” (related to measurement of distance) followed by several prompts to open Q&A session:

1. How far is “a stone’s throw away?”
2. How many ways can we measure a distance?
3. Which way is faster and accurate?

Table 16 shows that students’ math-talk included explaining, proposing mathematical ideas, and leading discourse at level 1. Students also demonstrated questioning, explaining, and proposing mathematical ideas at level 2. The students contributed substantially by sharing their ideas in our whole-classroom discussion, enabling me to focus more on CP using student-initiated ideas in addition to the combination of AQ and BE.

Similar to the comic in the Diagonals episodes, students did not find the comic funny. After Nina read the comic, Ben, Troy, Grant, Mae, Nina, Ramos, and Todd made comments on the comic. Students gave two interpretive remarks and made one inquisitive statement as shown in lines 4, 17, and 24. These behaviors show students on the path to becoming math-talkers as they began to grow in their inquiries and level of participation. The social interactions during the comic activity helped students make connections to the previous activity as indicated by Ben’s query in line 24. He connected the question posted in problem number 1 with the data collection activity (see MAD lesson plan in Appendix B). This behavior resembles the teacher’s inquiry and probing of the reason behind the solution. An excerpt of the discussion surrounding the comic is provided below:

1 Teacher: What is in the cartoon?
2 Nina: Real estate.
3 Teacher: Real estate.
4 Mae: Buying a house.
Teacher: Sorry?
Grant: A broken window with a hole!
Teacher: Buying a house, that’s good, there’s a broken window.
Wes: A stone’s throw.
Teacher: A stone’s throw made the hole.
Nina: And it’s only a stone’s throw away.
Teacher: And it’s only a stone’s throw away from the house.
Alright, which comment do you think is funny coming from this comic?
Ramos: None.
Teacher: None? Well you were laughing before. When it said a stone’s throw away?
Tess: The school’s not far away from there.
Teacher: The school’s not far away from?
Tess: From the house, I mean…
Teacher: Thank you for your comments. So now we…maybe after you read this you understand why we did some experiments yesterday. Alright, let’s read this.
(a paragraph of problem)
Ben: Why exactly did we use meters yesterday, why didn’t we use yards?

*The Social Scaffolding Analysis of the Content-related Comic Activity*

The coded sentences in the Diagonals and MAD episodes suggest that content-related comic activities stimulated classroom interactions in which students more willingly shared their thinking publicly. The above proposal seeks leverage from the analysis of social scaffolding.

Students and I engaged in providing a variety of social scaffolding classified as non-mathematical declaration, questions, invitation, and classroom management. One indicator of the improvement of classroom environment is the decrease in my speech act coded as MG, or literally requesting, “Please listen up,” to focus students’ attention to our discussion at hand. The amount of my utterance associated with MG in both the Diagonals and MAD episodes is lower than the amount recorded during the baseline. The lower number of MG reflects the improvement of classroom social norms where students listened more to each other and felt more comfortable expressing their ideas.
Having described the content-related comic warm-ups, I will now contrast the findings to one without a content-related comic, by revisiting the opening exchanges in the Painted Cubes and Triangles episodes. The math-talk codes for the warm-up in the Painted Cubes and Triangles tasks are shown in Tables 18 and 19. The following exchanges took place during the warm-up activity as the students and I were discussing the formula of the small cubes (edge length one centimeter) used to make the large cube (see Painted Cubes lesson plan in Appendix B).

26 Teacher: Guys, listen up, let’s go back to this. Guys, please. My class, let’s finish this up, right, so we can wrap it up.
27 We already elaborate these two small cubes would become solution for problem a, and we solve that as 2 times 2 times 2, alright? Okay, please guys. Let’s participate. Now, if the input is n, you wrote output n times n times n. Now listen up.
29 Teacher: Therefore, if it’s x, what will be your output?
30 Todd: X
31 Todd: 2y
32 Teacher: If it’s n, n times n times n. If it’s x?

The warm-up in the Painted Cubes episode contained 35 lines of exchanges (Table 18) in which five math-talkers participated as I elicited students’ BE. I pressed the whole class with five requests of “listen up” to facilitate the continuation of our whole-class discussion. The opening discussion of the Painted Cubes episode shows a lack of spontaneity from the participants. Their participation required persuasion, and I gave ample support to encourage students to explain their thinking, which was brief and consisted mostly of one word answers. The students and I engaged in a more traditional teacher-centered class, and although I asked questions to elicit student’s BE, I pre-determined the answer. Thus, I predominantly directed students’ contributions to our whole-classroom discussion and maintained the position as the main source of ideas, which limited volunteered thoughts and new ideas from students.
### Table 18

**Coding of Math-Talk During Warm-up Episode in the Painted Cubes Task**

<table>
<thead>
<tr>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Non Math</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AQ</td>
<td>BE</td>
<td>CP</td>
<td>DL</td>
</tr>
<tr>
<td>Students</td>
<td>1</td>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Teacher</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Subtotal</td>
<td>S = 9 T = 7</td>
<td>S = 6 T = 8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Math-talk lines = 30  Non-Math-talk lines = 5  Total = 35

*Note. AQ = Questioning, BE = Explaining, CP = Proposing, DL = Leading, S = Students, T = Teacher*

### Table 19

**Coding of Math-Talk During Warm-up Episode in the Triangles Task**

<table>
<thead>
<tr>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Non Math</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AQ</td>
<td>BE</td>
<td>CP</td>
<td>DL</td>
</tr>
<tr>
<td>Students</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Teacher</td>
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<td>0</td>
</tr>
<tr>
<td>Subtotal</td>
<td>S = 2 T = 0</td>
<td>S = 4 T = 15</td>
<td>S = 4 T = 11</td>
<td>1</td>
</tr>
</tbody>
</table>

Math-talk lines = 36  Non-Math-talk lines = 3  Total = 39

*Note. AQ = Questioning, BE = Explaining, CP = Proposing, DL = Leading, S = Students, T = Teacher*
During the warm-up episode in the Triangles task, I posed a problem asking how to determine that the set of numbers \{3, 4, 6\} may represent the lengths of the sides of a triangle. The activity would lead students to discuss corresponding parts of congruent triangles which we began from side-side-side congruence theorem.

Teacher: That’s good guys. So you proved that it is possible because of these two numbers. Alright, Dan has another explanation; will you repeat your explanation, please?

Dan: Uh, two shorter sides add up to be more than the longest side.

Tess: That’s exactly what I just put.

Teacher: To be more than the longest side. Guys, that’s it for our warm up. If you’re talking, please stop talking. Determine whether it is possible to draw a triangle with sides 3, 4, 6. Yes, it is possible. Two shorter sides add up to be more than longest side, and we show it 3 plus 4 is 7.

The above excerpt, taken from the total 39 sentences (Table 19), reflected an improvement in student participation as they began to explain their thinking. The number of my “listen up” requests was lower than in the Painted Cubes episodes. However, to draw students’ attention to our whole-classroom discussion still required ample AQ. Table 19 records 7 instances of teacher AQ at level 1 and 6 at level 2, and 8 occurrences of BE at level 1 and 3 at level 2. Students relied heavily on me as the main source of ideas in our whole-classroom discussion.

The Comparison Results between the Warm-Up Episodes

Student response during the comic warm-ups suggest that a content-related comic tightly linked to the task helps the participants see mathematics tasks as more appealing and connected to their interests; thus, they feel inclined to participate spontaneously without worrying about being evaluated for a right or wrong answer. Students began to initiate their ideas voluntarily from the non-mathematical discussion using the comic that may help the whole-classroom
discussion shift smoothly to the mathematical content. Student-initiated ideas stirred more peer interaction compared to when I called upon them with questions in our whole-classroom discussion. This observation reflects the horizontal flow of information proposed by Hitano and Inagaki (1991). As shown in Tables 14 and 16, students responding to other students (DL) occurred during the warm-up episodes where content-related comics were used. In contrast, this kind of interaction did not occur during warm-up activities in the baseline episodes (Tables 18 and 19).

Their interest in expressing their thinking at the beginning of our discussion indicated student-initiated ideas rather than teacher-directed answers. I used student ideas six times (14% out of 43 math-talk components) in the Diagonals warm-up activity (Table 14). My use of CP for levels 1 and 2 yielded 9 instances, as shown in the warm-up for MAD task (Table 16). That is 26% out of 34 math-talk components, whereas CP was coded 7 times at level 2. In contrast, the warm-up episode in the Painted Cubes (Table 18) records 3 instances of teacher CP at level 1 (10% out of 30 math-talk components) and the Triangles (Table 19) records 2 instances of teacher CP at level 2 (6% out of 36 math-talk components).

Students’ non-math-talk contribution positively enhanced social interactions as students responded to other students without teacher probing. Thus, the comic creates social interactions among participants faster and involves more of the math-talkers than other warm-up activities. My reflective journal notes student non-math-talk shifted to say something about the topic at hand instead of declining to join discussion by saying, “I don’t know.”

*Questioning and Explaining in the MAD Episodes*

The codes from the MAD episodes are shown in Table 17. There was a change in the number of responses from level 0 to levels 1 and 2, which can be compared to the results in the
Diagonals episodes. The approach to elicit students’ explaining their thinking by asking more questions (in the MAD episodes, I asked 32 questions compared to 24 in the Diagonals episodes) at level 2 resulted in an increase in students’ explanations at level 2 from 10 to 18 times (see Tables 15 and 17).

The activity begins with a set of data (see Table 20 below). In the warm-up activity, students calculated the five-number summary and drew a box-and-whisker plot which we continued to use for understanding variability.

**Table 20**

<table>
<thead>
<tr>
<th>Throwing distance (meter)</th>
<th>Find Five # Summary Statistics</th>
</tr>
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<tbody>
<tr>
<td>3</td>
<td>Median =</td>
</tr>
<tr>
<td>4</td>
<td>Minimum =</td>
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<tr>
<td>5</td>
<td>Maximum =</td>
</tr>
<tr>
<td>6</td>
<td>Lower Quartile =</td>
</tr>
<tr>
<td>5</td>
<td>Upper Quartile =</td>
</tr>
<tr>
<td>11</td>
<td>Inter Quartile Range =</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Note. Table 20 lists the mean of throwing distance from each group who experimented and collected data to measure how far they could throw a tennis ball. Students recorded the distance to the nearest meter for their team’s 5 throws.

The increase of student responses at level 2 shows evidence that the students and I focused on providing more detailed descriptions and methods to support answers. For example, Nina proposed ideas in lines 9, 11, and 14 during the MAD episodes shown below. The exchanges
took place after Todd drew his box-and-whisker plot representing the median of throwing distance from our class data collection.

1 Teacher: Alright, let’s check our data. What is the median from our data?
2 Nina: 5.
3 Rosa: 5.
4 Teacher: 5? Now lower quartile?
5 Todd: That’s 4.
6 Nina: No!
7 Teacher: Be careful.
8 Nina: No, you have to do 3 and 4, and what’s in the middle is 3.5.
9 Teacher: Very good, we are verifying what is given here.
10 Nina: And the other one upper quartile is 10.
11 Teacher: 10. Why do we have to do adding and dividing, do you know?
12 Nina: Because there’s not a median [inaudible]
13 Teacher: The center has two numbers, so we add them and divide by 2.
14 Todd: We count the median, or the middle in with the rest of them, that’s why there’s 4.
15 Teacher: Alright, now, let’s do the scale. The scale must be correct when you draw it.
16 Todd: It is correct.
17 Teacher: So the upper quartile is on 10, so that’s not right.
18 Todd: No it ain’t.
19 Teacher: Alright, can you revise it a little bit? Make your scale more accurate.

The above excerpt indicates students became more confident in arguing other students’ ideas and defending their own solution. For example, Nina corrected the median in Todd’s box-and-whisker plot and explained how she got a different answer. Todd initially defended the value of his median, but after I showed that the scale units were inaccurate he agreed to revise his drawing to give a more accurate five-number summary.

My reflection on the above episode pointed to my limited probing of students’ responses (lines 1, 2, and 5). I did not know which answer was correct due to lack of preparation. Therefore, I could not be an effective moderator in front of the arguing students. The emphasis
on preparation is paramount, as even with good preparation I did not ask enough follow-up questions. The challenge in constructing meaning through discourse is that students have unpredictable reactions. Many times these prove to be valuable ideas to build upon for further discussion, requiring a swift and correct response from me. In addition to more thorough preparation and practice of classroom instructions, I need to improve my skills in orchestrating a dynamic whole-classroom discussion. My teaching and learning challenged my ability to move to the next level of constructing and negotiating meaning with my students.

In building a math-talk community, I have encouraged students to use the words “revising our answer” when correction is needed. Students have adopted this phrase and sometimes one of them says, “That answer needs to be revised!” instead of “Wrong answer!” The above excerpt contained several level 2 responses where students began to explain steps in their thinking by providing fuller descriptions and to defend their answers and methods (Hufferd-Ackles et al., 2004).

To discuss the variability or changes in data, the box-and-whisker plot from the warm-up activity was used in the closing activity. I asked students the following questions:

1. Tell us how to determine the variability of a set of data?
2. Give examples of variability in our daily life.
3. Explain how knowing the variability of data distribution is useful.

The level 2 explanations were apparent in the MAD episodes as students continued to demonstrate their understanding throughout the MAD task as shown in the excerpt below.

26 Teacher: Thanks Todd. Alright, Ben, what kind of variability do we have here? [I pointed at the box-and-whisker plot]
27 Ben: High.
28 Teacher: High variability?
29 Ben: Actually, it’s low because they’re all close together.
30 Except for 19
Teacher: From this number to 19. Maybe we can consider it again. Is it high or low variability?
Ben: It’s going to be low.
Todd: No, it’s going to be high.
Teacher: Okay, alright, do you agree that this is high variability?
Troy: Yeah.
Teacher: Can you explain why do you agree?
Student: No.
Teacher: Mae, put that away please [she is holding a plastic box]. Can you explain here why we have high variability, or maybe you said low variability?
Mae: That is high variability.
Teacher: Can you explain why?
Mae: I forgot. I did.
Teacher: Guys, can you explain this, why it is high variability here? Rosa, you can say something here. Why high variability?
Tara: When you split it down the middle, the numbers on this side, or whatever’s in the middle, that’s your highest variability.
Teacher: Tara said, when you split it down the middle. Okay, what else did you say Tara?
Tara: I told you.
Teacher: Yeah, split it in the middle--
Tara: Then split that down the middle--and it will give you your highest variability.
Teacher: Okay, which one?
Tara: You’re having 10, so between 9 and 11, is 10 variability.
Teacher: Between 9--Okay, do you mean lower quartile, upper quartile Tara? Alright, is it between lower quartile and upper quartile that we know higher variability?
Tara: Yes
Teacher: Ben, Nina, Dora, Vera, and Tess. Do you agree with High variability?
Rosa: The box is big.
Teacher: What is big?
Rosa: The box.
Teacher: The box is? Alright, Between the lower quartile and the upper quartile, you have quite a big range. This is 3 and a half and 10, what is the difference?
Rosa: Six and a half?
Teacher: Six and a half. Alright, so 6 and a half. That makes a good reason to say this is high variability. Alright, from Tara and Rosa you got a high variability,
To get more students to explain their mathematical thinking on how to determine the type of variability shown in a box-and-whisker plot, I had to probe students with several questions (lines 40, 44, and 46). Tara interjected her ideas loudly to break up the long negotiation in which I kept pressing students to comment on variability. Tara’s explanation was not yet clear as a description of variability, and Rosa, a new math-talker, joined the discussion to clarify Tara’s explanation.

Student responses were longer and more creative as they expressed their thinking in our whole-classroom discussion. Ben gave his response on variability and included 19 as an outlier (line 30). Tara used “split in the middle and the number on the sides” as an expression to find lower and upper quartile (line 49). Rosa used “box” (line 66), which is appropriate for the inter-quartile range given by a box-and-whisker plot graph. The MAD learning task involved interpreting our data, finding the value of the MAD and the five-number summary consisting of minimum, maximum, lower and upper quartile, and median. The task included definitions of MAD, measure of spread, measure of central tendency, and comparison of inter-quartile values across four box-and-whisker plots. These definitions and the presentation of data were new in the freshmen statistics experience. However, students’ responses using explanations at levels 1 and 2 indicate that they had become more comfortable in sharing their thinking than they were during the baseline episodes.

The number of questions that they addressed in two episodes indicate the increase in student participation. For example, the comparison of AQ in Tables 15 and 17 shows that level 0 increased from none to 6 questions, level 1 increased from 3 to 8 questions, and level 2 increased from 1 to 4 questions. Moreover, students began to address questions to other students during whole-classroom discussion, marking the shift from teacher as sole questioner to the students
serving as questioner as well. Students demonstrating their active role became evident in our last closing activity in the MAD task, as indicated in the exchanges below. The topic of discussion involved the measure of variability as shown in Figure 16.

### MEASURE of VARIABILITY

1. The box plots shown represents pulse rates per minute for random samples of 100 people in each of four age groups.

2. Complete the chart below for each group:

<table>
<thead>
<tr>
<th></th>
<th>Newborns</th>
<th>6-yr olds</th>
<th>15-yr olds</th>
<th>35-yr olds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Range</strong></td>
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<td><strong>Q1</strong></td>
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<td><strong>Median</strong></td>
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<tr>
<td><strong>Q3</strong></td>
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<tr>
<td><strong>Interquartile</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Range</strong></td>
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<tr>
<td><strong>Maximum Value</strong></td>
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<tr>
<td><strong>Minimum Value</strong></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

*Figure 16. Measure of Variability in the MAD Learning Task*

*Note. Adopted from the North Metro Mathematics Collaborative (NMMC, 2008), Kennesaw State University*
Teacher: Can we look the data at the table. Can you explain the data? (Novi just wrote her five-number summary for 4 box-and-whisker plots on the whiteboard)

Novi: (Pointing to the row and the column) The 15 years old and 35 years old they have the same inter-quartile. And-

Dan: What is the IQR?

Novi: 25

Teacher: Can you explain how the pulse rates as people get older?

Mae: It gets slower

Ben: The what is slower?

Novi: The pulse rate, the baby up here 160 and the 35 year old is 70.

Teacher: Alright, you heard Novi and Mae. You have to compare two data alright? That’s what box plot is, to compare.

Student: Can you do number 3?

Teacher: Number 3, what is the question? What group age gives a low variability? Alright, how do we decide low variability guys?

Dora: Close together.

Teacher: Close together, can you explain how to decide the data are close together?

Ben: Is it the range?

Dora: Oh inter-quartile.

Teacher: Inter-quartile. Right. Guys, listen up please. So, what group age gives a low variability?

Patel: 15 years old and 35 years old.

Teacher: And you know that they have…

Patel: The IQR is 25

Dan: It is the smallest IQR.

Novi described the inter-quartile range in the problem with a complete strategy using the row and the column of her table (line 4). She justified her explanation of the slower rate data (line 11). As a presenter, Novi demonstrated explaining at level 3. I also noted how Dan, Ben, Dora, Mae, and Patel were supportive during Novi’s presentation. These math-talkers moved to the center of our whole-classroom discussion to support their peers.

Proposing and Leading in MAD Episodes

Students continued to support each other by repeating, completing, and responding to others’ remarks (lines 10, 11, 22, 25, and 27). Students’ contributions in DL were complemented by their 8 and 4 instances of AQ, at levels 1 and 2 respectively, where students became more
engaged by asking questions of other students’ work, compared to 3 and 1 instance in the Diagonals episodes. In the MAD episodes the shift from the teacher to students as the main source of ideas was marked by 8 instances of CP at level 2 (Table 17). Students demonstrating the act of proposing mathematical ideas appeared in lines 30 (Ben) and 56 (Tara), and during warm-up in lines 9, 11, 14 (Nina), and 17 (Todd) in which students’ thinking guided the direction of the math lesson in this episode.

Math-Talk Learning Community Level 2 in the MAD Learning Task

In terms of a math-talk learning community, the MAD episode reflected level 2 math-talk components. Novi went to the whiteboard to complete the five-number summary table from four box-and-whisker plots. Early in the warm-up activity, Novi explained to me each of these values to show her confidence and skills in interpreting box-and-whisker plots. Novi rose to lead the closing discussion that involved Tara, Novi, Rosa, Patel, and Grant who joined the active math-talkers Nina, Todd, Ben, Troy, Dora, Ramos, Tess, Jed, and Wes. As a community the class began to understand the tabulated data, MAD values and five-number summary, through sharing, comparing, and interpreting the results of their statistics with less teacher guidance than occurred in the baseline episodes.

Flow of Information

To describe the flow of vertical and horizontal information in a whole-classroom discussion, every sentence in the Diagonals and MAD episodes was coded to indicate the interactions from teacher to student (vertically), teacher to class (vertically), student to teacher (vertically), or student to student (horizontally) as described in the Coding Scheme of Analytical and Social Scaffolding (see Appendix A, Table A2). Figures 17 and 18 present the flow of
vertical and horizontal information (Hatano & Inagaki, 1991; Nathan & Knuth, 2003) that occurred during the Diagonals and MAD episodes. Figure 17 indicates horizontal flow of information where student-to-student talk occurred across the nodes. Student-to-student talk began when Tess argued Todd’s idea during the warm-up related to the comic “A Square by Any Other Name” as recorded in the exchanges below:

Wes: Every parallelogram is a rectangle—that is false.
Teacher: Explain please, why it is false that every parallelogram is a rectangle
Todd: It can too.
Student: It can’t be a rectangle, or anything with four sides that are parallel.
Troy: Yeah, that’s what I was saying. It can be a square, a trapezoid, a kite
Teacher: But parallelogram must be like this, alright? (pointing to the picture)
Todd: No.
Teacher: This is parallelogram and why it is false that parallelogram is a rectangle?
Tess: Because it’s not a rectangle, it’s a different shape.
Teacher: Alright, so why is this not a rectangle?
Tess: Because it goes like that (she also moves both of her hands).
Troy: What are you talking about?
Tess: It doesn’t have a 90 degree angle.
Teacher: Guys, she said a parallelogram does not have a 90 degree angle.
Todd: That makes no sense.
Teacher: Alright, let’s take a look, why does not?
Todd: Because on the first one you said, every rectangle is a parallelogram, and we said that is true okay? And now, you say every parallelogram is a rectangle.
Teacher: Let’s discuss that.
Todd: Oh, I see.
Teacher: Every parallelogram is a rectangle is false, because it does not have a right angle. So that [parallelogram] does not satisfy the rectangle. But rectangle satisfies the parallelogram.

The frequency of occurrences indicating vertical and horizontal flow of information is presented in Tables 21 and 22. In the Diagonals episodes, there were three groups whose
Dashed black is analytical scaffolding from teacher.

Dashed red is social scaffolding from teacher.

Black is analytical scaffolding from math-talker

Red is social scaffolding from student.

*Figure 17. The Diagonals Episodes Flow of Information*

*Note.* The number in the circle indicates student’s seating chart in the class roster (Table 4).

The line thickness represents relative frequency of the responses.
Dashed black is analytical scaffolding from teacher.

Dashed red is social scaffolding from teacher.

Black is analytical scaffolding from math-talker.

Red is social scaffolding from student.

*Figure 18.* The MAD Episodes Flow of Information

*Note.* The number in the circle indicates student’s seating chart in the class roster (Table 4). The line thickness represents relative frequency of the responses.
Table 21

*Vertical and Horizontal Information in the Diagonals Episodes*

<table>
<thead>
<tr>
<th>Vertical (Teacher-to-Student)</th>
<th>Vertical (Teacher-to-class)</th>
<th>Vertical (Student-to-teacher)</th>
<th>Horizontal (Student-to-student)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>Counts</td>
<td>%</td>
<td>Counts</td>
<td>%</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Analytical</td>
<td>28</td>
<td>72</td>
<td>21</td>
<td>55</td>
</tr>
<tr>
<td>Social</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>30</td>
<td>78</td>
<td>26</td>
<td>70</td>
</tr>
</tbody>
</table>

Table 22

*Vertical and Horizontal Information in the MAD Episodes*

<table>
<thead>
<tr>
<th>Vertical (Teacher-to-Student)</th>
<th>Vertical (Teacher-to-class)</th>
<th>Vertical (Student-to-teacher)</th>
<th>Horizontal (Student-to-student)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>Counts</td>
<td>%</td>
<td>Counts</td>
<td>%</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Analytical</td>
<td>28</td>
<td>83</td>
<td>19</td>
<td>56</td>
</tr>
<tr>
<td>Social</td>
<td>3</td>
<td>10</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>93</td>
<td>29</td>
<td>86</td>
</tr>
</tbody>
</table>
members responded to students outside their groups. Troy (8) responded to Wes (24), and Ben (10) responded to Mae (2). Likewise, Mae responded to Wes, Ben, and Troy as well. The math-talkers in the baseline episodes—Dan (13), Grant (28), and Tess (6)—continued to participate, while new math-talkers Kay (5), who shared her group’s answer on the whiteboard, and Patel (15) joined the class discussion. In addition, Mae (2), Nina (7), and Jed (25) presented their group work on the whiteboard. After the groups presented their answer, we began our whole-classroom discussion in which the members of each group explained and defended their answer. Vertical flow of information includes social scaffolding instances (dashed red line), indicating my invitation for students to join our discussion (see Figure 17).

The coded sentences representing social scaffolding from teacher-to-student were recorded as 2% of 266 total coded sentences (see Table 21). Among those whom I invited, only Kay was willing to contribute her group work while Fort, Ken, and Sue failed to participate.

The flow of information in the MAD episodes is diagrammed in Figure 18. The math-talkers formed three circles of interactions. In these three circles, Mae was the hub of interactions among Ben (10), Dan (13), Dora (1), Grant (28), Nina (7), Novi (27), Todd (4), Troy (8), and Wes (24). Todd served as the center of interactions for Ben, Mae, and Nina. In the MAD episodes, Novi moved to the center of our discussion and became the hub of student-to-student talk between Ben, Dan, Grant, and Mae. Acting as a central figure in each of these circles, Mae, Todd, and Novi stimulated student-to-student talk (horizontal flow of information) that involved students listening, asking questions, and clarifying other’s ideas (see Figure 18).

In the MAD episode, Rosa (18), a new math-talker, clarified Tara’s explanation on high variability (lines 56, 59, 66, and 68). In general, the diagram shows that more students responded to other students’ ideas, stimulating more student-to-student talk, as opposed to when I prodded
them with questions (see Figure 18). Overall, the pattern of the flow of information indicates that the students’ role as the center of our discussion became more apparent. The results support the description of the MAD episodes as a level 2 math-talk learning community.

The emergence of student-to-student talk in our whole-classroom discussion, diagrammed in Figures 17 and 18, gave evidence that student interactions increased over the baseline horizontally during a whole-classroom discussion. In the Triangles episodes, the amount of sentences coded as student-to-student talk is 3 (1%) with 109 (36%) sentences of student-to-teacher talk (see Table 11). In the Diagonals episodes, out of 266 coded sentences, student-to-student talk is recorded as 4% (10 times) of the transcript, compared to student-to-teacher utterances making up 38% (101 times) of the transcript (see Table 21). Student-to-student talk appeared in the MAD episodes (see Table 22). Out of 292 coded sentences, student-to-student talk is 5% (15 times) compared to 30% (86 times) of student-to-teacher talk.

Analytical and Social Scaffolding in the Content-Related Comics Data

To describe the analytical and social scaffolding used in the whole-classroom discussion, every sentence in the Diagonals and MAD episodes was coded based on the Coding Scheme for Analytical and Social Scaffolding (see Appendix A, Table A2). The students and I provided a variety of social and analytical scaffolding for each other in the Diagonals and MAD episodes as shown in Tables 23 and 24 respectively.
Table 23

*Coding of Analytical and Social Scaffolding During Diagonals Task*

<table>
<thead>
<tr>
<th>Scaffolding</th>
<th>Teacher to Student</th>
<th>Teacher to Class</th>
<th>Student to teacher</th>
<th>Student to student</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QM</td>
<td>TM</td>
<td>QFM</td>
<td>TVA</td>
</tr>
<tr>
<td><strong>Analytical</strong></td>
<td>16</td>
<td>1</td>
<td>37</td>
<td>18</td>
</tr>
<tr>
<td><strong>Social</strong></td>
<td>4</td>
<td>2</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>20</td>
<td>3</td>
<td>45</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 24

*Coding of Analytical and Social Scaffolding During MAD Task*

<table>
<thead>
<tr>
<th>Scaffolding</th>
<th>Teacher to Student</th>
<th>Teacher to Class</th>
<th>Student to teacher</th>
<th>Student to student</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QM</td>
<td>TM</td>
<td>QFM</td>
<td>TVA</td>
</tr>
<tr>
<td><strong>Analytical</strong></td>
<td>4</td>
<td>2</td>
<td>36</td>
<td>23</td>
</tr>
<tr>
<td><strong>Social</strong></td>
<td>8</td>
<td>2</td>
<td>18</td>
<td>8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>12</td>
<td>4</td>
<td>54</td>
<td>31</td>
</tr>
</tbody>
</table>

| | Analytical = 83 Social = 10 | Analytical = 56 Social = 30 | Analytical = 86 Social = 12 | Analytical = 15 Social = 0 |
Analytical Scaffolding in the Diagonals Episodes

Table 23 shows that a total of 266 sentences were coded in the Diagonals episodes which comprised analytical and social scaffolding with a ratio of 238 to 28. These coded spoken sentences consist of 111 student responses with 101 and 10 instances directed to me and another student respectively. My analytical scaffolding to individual students reached a total of 72 sentences. The highest analytical scaffolding from teacher-to-student occurred 37 times in asking follow-up question (QFM), which is slightly lower than in the Triangles episodes (40 instances). The vertical flow from teacher-to-class included mathematical explanations to scaffold students’ thinking coded as explaining and eliciting student ideas (TRE), which occurred 24 times and is lower than the 32 times in the Diagonals episodes. An increase is shown in “revoicing” (TVA): 18 times compared with 9 previously.

The coding of analytical scaffolding includes student questions and responses as they engaged in classroom discourse (see Table 23). Students responded to other students (RMS) ten times, an increase from 3 times in the Triangles episodes; thus, there were more math-talkers sharing and supporting each other’s mathematical ideas. Overall, students responded with MVM (41 times) and SVM (10 times), both of which were higher than in the previous task. Students explained their thinking and description of methods (SEM) 9 times, an increase from 1 instance in the Triangles episodes.

Comparing student analytical scaffolding analysis and student math-talk in the Diagonals episodes (see Tables 15 and 23), the appearance of student math-talk at level 2 was compatible with SVM, SEM, and QM. Both coding schemes show students’ tendency to respond with fuller descriptions and to defend their answers. However, the dominant analytical scaffolding, MVM
(41), which is compatible with BE (46) at level 1, indicates that a math-talk learning community at level 1 with some degree of participation is apparent in the Diagonals episodes.

Analytical Scaffolding in the MAD Episodes

With a total of 292 coded sentences, Table 24 represents a variety of analytical (240) and social scaffolding (52) in the MAD episodes. The total responses comprised 86 sentences in student analytical scaffolding directed to me and 15 responses directed to other students. The later instances showed more student-initiated responses occurred as they responded to other students’ answers. In particular, when Novi contributed her five-number summary, several other students asked questions and commented.

My analytical scaffolding to individual students reached a total of 83 sentences. The frequency of analytical scaffolding indicates there were 23 occurrences of TVA compared with 18 previously. A similar increase also happened in the Diagonals episodes (from 9 to 18 times). The use of TVA, or revoicing, in the Diagonals and MAD episodes helped the chain of thinking continue. For instance, problem number 2 in the MAD task presents two sets of data as shown in Figure 19. The students had to calculate the difference of each value and the mean of a distribution step by step for both sets of data and justify the measure of variability from each set of data. Revoicing can be observed in the following excerpt related to problem number 2:

1   Teacher: Based on the investigation of Mary Kate and Ashley…alright, what is the results about student of different age, estimating their teacher’s age. Which group is more consistent?
2  Dora: Mary Kate?
3  Teacher: Why do you say Mary Kate?
4  Dora: Because she had thirty students gave ages twenty-eight, twenty-eight.
5  Teacher: Okay, she has the students that give data twenty eight twice. Do you agree students?
6  Todd and Ramos: Yeah.
7  Teacher: Why do you call that consistent?
8  Dora: Because it’s the most often.
9  Dan: Low variable.
Teacher: Because it’s the most often showed up. Alright, guys, besides the fact that you have repeating data, what else, what is the reason you say consistent?

Tess: Less spread out.

Teacher: Less spread out, they are close together. Now listen up please, can you give another example? Low variability or high variability.

Ben: Temperatures.

Teacher: Temperatures! This week, can you explain is the temperature has low or high variability?

Novi: High

Ben: It’s low. High high high... In the morning is cold, then it gets hot all day.

Revoicing appears in lines 7, 13, and 17, and involved 6 students. I revoiced Dora’s reason for consistent data to ask deeper questions about their understanding of the concept. Revoicing became a useful tool to align student thinking, to focus on the application of measure of spread, and to connect this thinking to a specific example from their daily experience. As a part of analytical scaffolding, revoicing allowed me to encourage more participants to contribute their ideas.

In comparison to the Diagonals episodes, the classroom exchanges in the MAD episodes appeared to have a different composition of student responses and questions directed to me (see Tables 23 and 24). The codes in Table 23 reflect that students progressed toward a higher level of mathematics thinking as the number of DMS instances decreased to 12 and MVM decreased from 41 to 17. RMS was coded 12 times, which slightly increased from 10 in the Diagonals episodes. The horizontal flow of information among the students, including 3 instances of “question to another student” (QSM), did not occur in any other transcript. The appearance of student-to-student talk supports the evidence from math-talk coding categories that students become more inclined to share their responses, ideas, and questions.
One way of comparing variability between groups is to look at the mean absolute deviation (MAD). The mean absolute deviation is the arithmetic average of the absolute values of the difference between each value and the mean of a distribution. The larger the value of the MAD the more spread out the values are from the mean. When comparing variability of data distributions using the MAD, a distribution with a larger MAD has more erratic values, while a distribution with a smaller MAD has more consistent values.

**Figure 19.** Problem Number 2 in the MAD Learning Task

*Note.* Adopted from the North Metro Mathematics Collaborative (NMMC, 2008), Kennesaw State University.
Comparing student analytical scaffolding and student math-talk in the MAD episodes (see Tables 17 and 24), the increase in SVM from 10 to 18 times is similar to the increase in BE at level 2 from 10 in the Diagonals to 18 in the MAD episodes. Both coding schemes support students’ tendency to volunteer thoughts and articulate more information when probed.

The twelve instances of RMS were compatible to leading in math-talk at level 1, which was only coded 8 times. Although students demonstrated the act of explaining or clarifying other students’ ideas in their own words, student explanations needed teacher guidance to obtain the correct explanation. The students used QMS a total of 13 times, marking an increase of student participation in addressing questions. In addition, students exhibited 10 instances of “explaining their thinking and description of methods” (SEM) in the MAD episodes, which is comparable to 8 instances of DL at level 2 under the math-talk categories. Many times these contributions in the MAD episodes guided the direction of the math lesson. Thus, students participated with a higher level of thinking and involved more of their peers than in the Diagonals episodes. Having compared the results in math-talk components and levels, flow of information, and analytical scaffolding coding schemes, the MAD episodes show evidence of an emerging level 2 math-talk community.

*Social Scaffolding in the Diagonals Episodes*

Table 23 shows that a total of 28 sentences were coded as social scaffolding occurrences in the Diagonals episodes, of which student responses comprised 6 sentences, all with directed social speech acts to me. My social scaffolding to individual students and the whole-class reached a total of 21 sentences. In coding social scaffolding, MG occurred 5 times. The frequency of MG occurrences reflects the fact that I had to tell students to “listen to what others say” less than the 17 times in the Painted Cubes activity. Students engaged in providing social
scaffolding classified as both non-mathematical declaration and questions. The excerpt below is an example of how students responded to alternate “I don’t know” responses:

26 Teacher: What do you think about the diagonal of the trapezoid?
27 Jed: They do not bisect each other.
28 Teacher: They do not bisect. So what is the reason?
29 Troy: Whoa, what?
30 Teacher: Please pay attention. Maybe you can explain. They are not bisecting each other, what is the explanation?
31 Troy: That’s not a square.
32 Nina: It’s not in the middle.
33 Teacher: Alright, the intersection is not in the middle and what can you say in complete sentence?
34 Jed: I don’t know
35 Dan: It’s not in the middle. The intersection is not on the midpoint.
36 Teacher: Good. Now Jed, you can tell about the diagonals of this one?
37 Jed: That is a kite. A kite that I made. (The students laughed including Jed). Hey, I participated!

An improved math-talk learning community was evidenced in line 34, which contained social and analytical scaffolding in the negotiation of how to describe the diagonals of a trapezoid, and utilized the vocabulary term midpoint from the Diagonals task. The classroom social norms served as important factors in supporting student participation and the expression of their ideas. Having practiced several episodes with my students, I had learned it is important to make them feel significant to our class discussion. Jed’s participation was especially remarkable since he disliked doing his math work in the past; but he was now a math-talker in our class discussion. His peers cheered when Jed presented a correct answer. To include Jed in our class discussion was positive both socially and analytically. Jed’s expression (line 41) articulated that the students should know the expectation of the classroom social norms we created at the beginning of this research.
Social Scaffolding in the MAD Episodes

A total of 52 sentences were coded as social scaffolding occurrences in the MAD episodes, as shown in Table 24. The student responses were comprised 12 sentences directed to me and 15 instances to other students. Compared to the baseline episodes, students gave more responses in social speech related to the comic. In addition, the social participation in the content-related comic activity allowed students to ask questions and respond publicly with laughter, a rare experience in a mathematics classroom. Students gave non-mathematical responses that related to the topic can be found in the excerpt below:

Grant: How many people live within a stone’s throw of your home?
Teacher: Are there any here that is a stone’s throw away from our classroom?
Ramos: A landmark?
Troy: The football field.
Nina: A landmark must be a famous site, is that what it says? So football field is not. We don’t have a famous site here.
Teacher: Alright, how many people live within a stone’s throw away from your home?
Ben: What if we don’t live anywhere?
Teacher: So you live on big, big property, like a farm?
Ben and Mae: Yeah.
Teacher: Alright, so, then you don’t have. Maybe your barn is a stone’s throw away. The outhouse. Yesterday I asked you what is your estimation of stone’s throw away, and after we do this, is it close enough? Because you said like 7 feet, and you said like a million feet didn’t you?
Dan: Alright, after we make our experiment, is it still true?
Teacher: This is why we collect our own data, do we have an outlier?
Ben: No.
Ramos: 19.
Teacher: 19. Yeah, it the biggest. A lot of different numbers.
Mae: What does convey mean?
Teacher: What does it want to tell you, a stone’s throw away?
Ben: Like how far something is away
Teacher: Away, a distance, very good. Any more… guy’s please. You can participate.

Teacher: Throwing away stones, so just literally, a stone’s throw away. Yeah, what does it meant to you when I say that place is a stone’s throw away?

Student: Near.

Teacher: So a close distance, alright. Okay, good.

The frequency of MG remained low at 8 instances, while DN increased from 2 to 18 instances. The lower frequency in MG than in the baseline, indicates that students began to listen to and understand one another, and contribute their mathematical thinking by clarifying, completing, or commenting upon other student’s responses (Hufferd-Ackles et al., 2004). The increase in DN was related to non-mathematical questions and comments students made during the content-related comic activity. The social scaffolding instances increased as social interactions between the students and I, and among the students themselves, grew in our math-talk learning community.

Reporting even more growth in student participation, Table 24 lists 8 instances of student asked “non-mathematics questions” (QNS). The non-mathematical questions correspond to students’ attempts to have their voices heard in the class discussion. Some of these questions were: “What number are we on?” “Is that one 19?” “That is confusing!” These non-mathematical remarks and questions represented student social participation as a way to interact with their peers so they could succeed in understanding the problem presented on the whiteboard. In this case, student social participation stimulated other students to answer, resulting in the social and mathematical scaffolding influencing each other.

Summary of Content-Related Comic Description Data

The description of the findings across the learning tasks after the application of the combined frameworks is presented in Figure 20.
Figure 20. The Growth of Our Math-Talk Learning Community Across the Learning Tasks

Note. + content-related comic is incorporated; Level 1* contained student-to-student talk.

The description of the Diagonals and MAD episodes help to identify some changes in classroom exchanges related to the introduction of content-related comics activity. The first section, description of data, presents the findings when math-talk coding was applied to the Diagonals and MAD episodes. The results suggest, through the growth of student math-talk, that students seemed more confident participating and supporting other students’ ideas, as evidenced in students’ explaining with fuller description of their thinking and contributing to our discussion by completing other students ideas. In the second section, these findings were supported by the flow of information diagrams that charted the interactions designated as teacher-to-student, teacher-to-class, student-to-teacher, and student-to-student (see Coding Scheme of Analytical and Social Scaffolding in Appendix A).

The coding by the analytical and social scaffolding frameworks is presented in the second section, which focuses on the detailed description of analytical scaffolding diagrammed in Figures 17 and 18. The social scaffolding recorded in both Diagonals and MAD episodes exhibited an increase in student non-mathematical spoken sentences related to the building of our classroom social norms.
Part 3: Data Analysis

The Data Description section shows how the math-talk and analytical and social scaffolding coding schemes were applied to characterize the growth of a math-talk learning community within the nine-week study. The purpose of this section is to further analyze the pattern of math-talk in the baseline and content-related comics episodes.

In performing qualitative data analysis, I imported data into an Excel 2007 spreadsheet. The main spreadsheet consisted of student names and each sentence spoken (or a group of sentences related to the same thought) with its coding categories (see Table 6 in chapter 3). Sentences relating to the same idea were usually mine because I repeated myself or re-phrased my words to clarify a question or persuade a response from the students. Similarly, students used several sentences to explain their solutions, all of which generally expressed the same thought or methods. To analyze data, Excel’s filter feature was used to create groups of selected data, called nodes. For example, by selecting students’ questioning (S-AQ) at levels 0 through 3 via mouse clicks, Excel’s filter allows me to see only the data under these categories. Then, I used each node filtered from the main spreadsheet to seek the common and conflicting themes that emerged from the extracted data (Creswell, 2003, 2007).

The analysis is organized into two parts. Part one includes analysis of the pattern of math-talk in the baseline episodes: the Painted Cubes and Triangles episodes. Part two includes analysis of the pattern of discourse in the Diagonals and MAD episodes. In describing the emerging math-talk pattern, I compare the Triangles episodes (baseline data) to the Diagonals, and the Diagonals to the MAD episodes.
Figure 21. Bar Graph of Teacher Math-Talk
Figure 22. Bar Graph of Student Math-Talk
Overview of the Patterns of Math-Talk

Math-talk at levels 0 and 1 predominated in the Painted Cubes and Triangles episodes respectively. After the introduction of the content-related comics, math-talk at level 1 (with active participation) and level 2 were apparent in the Diagonals and MAD episodes. Figures 21 and 22 illustrate the outcome of AQ, BE, CP, and DL labeled on the horizontal axis at level 0 through 3 for teacher and students. Each code is paired with a number indicating the level of math-talk. For example, AQ-0 means questioning at level zero. The percentages in Figures 21 and 22 are calculated by dividing the total number of codes for each AQ, BE, CP, and DL for teacher and students. For example, in the Painted Cubes (blue bar), a total of teacher AQ codes at level 1 is divided by the denominator 128, which represents the total amount of teacher math-talk. In Figures 21 and 22, the blue, red, green, and purple bars represent the Painted Cubes, Triangles, Diagonals, and MAD tasks respectively.

Figure 22 shows how student math-talk changed across the four tasks. Students’ AQ-2, BE-1, and CP-2 increased across the tasks. Math-talk at BE-3 occurred in the MAD task only; at the same time, teacher’s BE-1 and BE-2 decreased as shown in Figure 21. In the next section, I will explain how teacher and student discourse changed throughout the tasks. To support the explanation of this progression from the baseline to the end of the study, Figures 21 and 22 are broken up into the individual graphs of Figures 24 through 37 representing teacher and student math-talk for each task. The percentages in Figures 24 through 37 are calculated in the same way as in Figures 21 and 22.
Figure 23. Bar Graph of Math-Talk in the Painted Cubes
Figure 24. Teacher Math-Talk Level 1 in the Painted Cubes Episodes.

Figure 25. Student Math-Talk Level 1 in the Painted Cubes Episodes
Figure 26. Teacher Math-Talk Level 1 in the Triangles

Figure 27. Student Math-Talk Level 1 in the Triangles

Figure 28. Teacher Math-Talk Level 2 in the Triangles

Figure 29. Student Math-Talk Level 2 in the Triangles
Pattern of Math-Talk in the Baseline Episodes

Figure 23 shows raw numbers of math-talk and non-math-talk codes in the Painted Cubes episodes. Figures 24 and 25 show percentages of codes in AQ, BE, CP, and DL at level 1 for teacher and students respectively. Math-talk components at level 2 did not appear in the Painted Cubes episodes. The qualitative analysis across the episodes focuses on levels 1 and 2 since the baseline data indicated a math-talk learning community at level 1 was achieved after the Triangle episodes and level 3 math-talk was never achieved in the baseline.

The Painted Cubes episodes showed the vast majority of student responses were centered at level 0 (Figures 23 through 25). Recognizing my weakness, I stepped up the level of AQ, BE, and CP on my part to elicit more students explaining their thinking in the Triangles episodes, so to push our math-talk to a higher level. My reflection on the Triangles episodes identified a need to carefully listen to student responses and invite their ideas as a way to improve my CP at level 1. Student contributed CP at levels 1 and 2 was lower than AQ, BC, and DL, showing the students’ tendency to not volunteer or propose mathematical ideas as a new strategy or method in our whole-classroom discussion. The low amount of teacher DL utilized while encouraging students to clarify and ask questions about other student’s work indicates a teacher-directed instruction in the Triangles episodes. Consequently, the classroom interactions did not explore student ideas since I was the main source of ideas.

The analysis of the baseline episodes showed an increase in students’ mathematical thinking from mostly centered at levels 0 in the Painted Cubes to levels 1 and 2 in the Triangles episodes. Students were not inclined to express their ideas publicly in the Triangles task, unless persuaded. Figures 26 through 29 show that teacher’s AQ and BE are consistently high as well as students’ BE
at levels 1 and 2. This pattern of teacher math-talk suggests that pressing questions combined with explanations were necessary to encourage students to explain their thinking.

In the Triangles episodes, the data indicate students needed support in complying with the new expectations of our math-talk norms: to figure out an approach to a problem, discuss, argue, and justify their ideas. For example, in addition to asking questions, I explained the problem further to encourage students to explain their thinking. Teacher support was evident by the appearance of a high frequency of teacher AQ (19% and 21%) and BE (21% and 13%) at levels 1 and 2 in the Triangles episodes (Figures 26 and 28). Meanwhile, teacher’s CP at level 2, which indicates the teacher’s attempt to build on student ideas by repeating and extending their contributions to a discussion and involve other participants, did not stimulate an increase in students’ CP and DL (Figures 28 and 29). Student contributions of CP and DL at level 1 were similar to those in the Painted Cubes. Although students engaged more in proposing ideas (CP) and repeated what other students say (DL) at level 2, these contributions did not stimulate student-to-student talk in the Triangles task. The analysis of horizontal and vertical flow of information charted in Figures 11 and 12 in the Data Description section show student-to-student interactions did not change significantly from the Painted Cubes, because only a few horizontal interactions occurred among the math-talkers in the whole-classroom discussion.

**Pattern of Math-Talk in the Diagonals Episodes**

The Diagonals and MAD episodes were presented with a content-related comic activity. Data obtained from the Triangles episodes is hereafter called the baseline data as the results represent the growth accomplished by the class prior to the content-related comic strategy. In the Data Description section, Table 14 lists that students contributed math-talk in 111 spoken-sentences out of the 264 used in the Diagonals episodes. In comparison, the Triangles episodes included 112
Figure 30. Teacher Math-Talk Level 1 in the Diagonals

Figure 31. Student Math-Talk Level 1 in the Diagonals

Figure 32. Teacher Math-Talk Level 2 in the Diagonals

Figure 33. Student Math-Talk Level 2 in the Diagonals
instances of student math-talk out of 302 spoken-words. At the same time, my total math-talk decreased to 127 compared with the 135 occurrences in the Triangles episodes. The codes, followed by the explanation below, seem to indicate that students performed better in the Diagonals episodes in response to a similar amount of teacher math-talk.

One issue in stimulating classroom discourse is finding the best approach to elicit student responses. The baseline data reveal that students responded to my AQ, BE, and CP with BE at levels 1 and 2. However, this approach did not stimulate student-to-student talk. Therefore, in addition to probing deeply into student thinking and supporting explanations from students, I have to focus on generating students’ CP and DL, encouraging students to come forward with their ideas and taking on the role of questioner.

Figures 30 through 33 show the percentages of teacher and student math-talk at levels 1 and 2 in which a variety of math-talk components were observed. In Figures 30 and 32, a pattern of math-talk emerges, showing that I used a lower amount of BE than the amount of teacher BE in the Triangles episodes to probe student thinking and elicit their strategies at levels 1 and 2. The decrease of teacher’s BE at levels 1 and 2 indicates that students began to initiate their thinking voluntarily in our whole-classroom discussion. I used BE 12% in the Diagonals compared to 21% in the Triangles to elicit students’ BE, resulting in students’ BE 41% at level 1 (Figure 31), the highest number of student responses in all episodes. The decrease in teacher BE to 3% at level 2 in the Diagonals episodes from 13% at level 2 in the Triangles (Figures 28 and 32) means that students received less probing even on challenging questions (teacher AQ at level 2 is 19%). The tendency to receive less probing suggests the beginning of students initiating their ideas voluntarily, providing fuller information and defending their answers and methods.
The comparison between level 2 math-talk in the Triangles and Diagonals episodes (Figures 29 and 33) shows a decrease from 32% in the Triangles episodes to 19% in the Diagonals episodes. This decrease of students’ math-talk components in quantity should be considered with the growth of students’ initiatives to contribute and to give their own ideas publicly and exhibit more participation in CP (4%) at level 2 than CP (1%) at level 2 in the Triangles episodes (Figures 29 and 33). Four percent result in students’ CP at level 2 is the important measure considering the challenge to create a discourse learning environment in my Math I Support in which students did not propose their ideas voluntarily. The small increase in students’ CP at level 2 in our whole-classroom discussion initiated further conversations that encouraged other students to talk as indicated by DL 6% and 4% at levels 1 and 2 (Figures 31 and 33). Thus, students contributing mathematical ideas enhanced classroom interaction more effectively than teacher eliciting student participation. The transcript and the videotape recording documented that students responded (with math-talk or non-math-talk) quicker to their peers than to my probing questions considering I usually rephrased my questions and added few seconds of wait time.

The fact that students’ CP and DL occurred with fewer BE and zero DL from me at both levels 1 and 2 may be linked to the rise of student-initiated ideas during the warm-up with a content-related comic activity. The warm-up with content-related comic activity “A Square by Any Other Name” showed that I used student ideas 6 times, an increase from 3 and 2 times in the Painted Cubes and Triangles’ warm-ups respectively. Students also proposed ideas 2 times in the Diagonals’ warm-up. Although I did not employ DL, students began to demonstrate DL to clarify and question other students’ ideas in the warm-up. These actions, CP and DL, did not appear in the baseline warm-ups. The content-related comic activity
could become a *window* that attracts more students to express their thinking spontaneously and continue to contribute to our whole-classroom discussion.

The increase of students’ initiatives and willingness to express their ideas under less teacher probing was apparent by the result of the Diagonals episodes charted as flow of information in Figure 17 (Data Description section). In the whole-classroom discussion, students began to respond to each other. The increase in student responses directed to other students created a horizontal flow of information showing that students began to share their thoughts and clarify other students’ ideas. Consistent with the horizontal flow of information, the theme of student analytical scaffolding shifted to more frequently using MVM (minimal volunteering thought) and SEM (description of methods). In addition, coding revealed ten instances of student RMS, verifying that more student-to-student talks took place in the Diagonals than in the Triangles episodes. Clarifying and extending student’s ideas through TVA (revoicing) doubled in the Diagonals episodes compared to the Triangles episodes. The copious amount of revoicing indicated that student-initiated ideas increasingly became a part of our whole-classroom with math-talk components at levels 1 and 2, though I still maintained the role of questioner to guide student discussion. In exercising revoicing as a tool to involve more math-talkers, I increasingly built students’ ideas as the core of our mathematical discussion.

The codes in Figures 30 and 32 show the absence of teacher DL in the Diagonals task; however, this absence of teacher’s DL did not impede students in leading discourse, which appeared 6% and 4% at levels 1 and 2 respectively. The content analysis indicates that my discourse pattern using the combination of math-talk AQ, BE, and CP persists throughout the diagonals episodes.
Figure 34. Teacher Math-Talk Level 1 in the MAD Episode

Figure 35. Student Math-Talk Level 1 in the MAD Episode

Figure 36. Teacher Math-Talk Level 2 in the MAD Episode

Figure 37. Student Math-Talk Level 2 in the MAD Episode
Pattern of Math-Talk in the MAD Episodes

The teacher math-talk pattern in the MAD episodes indicates AQ and CP were used to persuade students to contribute their ideas. Figures 34 to 37 present the percentages of teacher and student math-talk at levels 1 and 2 in the MAD episodes. Teacher math-talk appears to have the lowest BE at levels 1 and 2 compared to other episodes. In the MAD episodes, I used BE 1% and 5% of 139 codes at level 1 and 2 respectively. This phenomenon exhibits that most of our whole-classroom discussions were built upon student ideas. Students came forward with their ideas beginning in the warm-up activity in which they proposed mathematical ideas 1 time at level 1 and 2 times at level 2. Students led discourse by clarifying other students’ ideas 2 times at level 1. Meanwhile, I used CP 2 and 7 times at levels 1 and 2 respectively. The comic enhanced classroom interaction as in the Diagonals episodes. Students willingly expressed their ideas spontaneously through the content-related comic activity.

Comparing the MAD and Diagonals episodes (Figures 31 and 35), I noted the decrease of students’ BE at level 1, 27% from 41% at level 1, balanced with the increase in BE at level 2, 18% from 9%. The first and only appearance of math-talk at level 3 occurred in the MAD episode (2%). The growth toward level 2 (Figure 37) is supported with the increase in the other student math-talk components from those observed in the Diagonals episodes (Figure 33). Students demonstrated both AQ (4%) and CP (8%) at level 2, supporting the evidence of students’ role shifting to the center of our whole-classroom discussion (Figure 37). The MAD episodes also provided the highest level of participation. A student, Novi, gave level 3 responses when she described a complete strategy, defended, and justified her answer with little prompting from the teacher. In addition, the highest frequency of occurrences in all teacher math-talk is CP
at level 1 (16%) and at level 2 (27%), which means the flow of information also came from the students. Thus, I was no longer the main source of ideas.

The result from the analytical scaffolding theme is consistent with the above analysis. The analysis in student analytical scaffolding revealed the growth in students’ SVM and SEM. Students increasingly showed their active role as a source of ideas that directed the discussion by demonstrating SII (interjecting idea) 3 times, and QSM (question of peer’s work) 3 times. Inviting more students to clarify and extend another student’s ideas through revoicing continued to flourish in the MAD episodes, which appeared in 23 instances compared to 18 previously recorded in the Diagonals episodes.

In addition, these findings support my approach of using the combination of AQ, BE, and CP as a discourse pattern to facilitate students sharing more ideas in our whole-classroom discussion. The proportion of this combination suggests lower teacher BE (1% and 5%) than AQ (13% and 23%), followed by the highest number of CP (16% and 27%) for levels 1 and 2. The change in teacher DL from 9% to 1% at levels 1 and 2 respectively may relate to the change in students’ DL decreasing from 8% to 1% at levels 1 and 2.

My reflection revealed that throughout the episodes I seldom used DL (encouraging students to clarify and ask questions about other student’s work) to complement AQ, BE, and CP. Given the length of time of this study, I had limited chances to evaluate the effectiveness of utilizing AQ, BE, CP, and DL in different percentages. In particular with regard to DL, I felt the tension between creating a discourse-rich learning environment and my students’ resistance to try multiple strategies and methods in our whole-classroom discussion. Practicing DL at levels 1 and 2 can be the key to step up to level 3 in our math-talk learning community. The challenge of moving to a math-talk learning level 3 requires preparing my students to be confident math-
talkers. Overall, the teacher should practice using AQ, BE, CP, and DL to facilitate the range of student responses in AQ, BE, CP, and DL at levels 0, 1, 2, and 3 at the appropriate time.

The Climate of Student Social Participation

From the perspective of social scaffolding, the climate of student social participation improved over the study. Data indicate teacher-to-class spoken sentences coded as MG, to manage classroom discussion, decreased from 17 out of 335 sentences (the Painted Cubes) to 10 out of 302 sentences (the Triangles). The videotape recording also documented that the noisy classroom gradually became quiet when Dan, Tess, and Todd explained their thinking in the Triangles episode. In the next 7 minutes, there were 8 math-talkers involved in making the plan to prove order matters in congruent triangles.

The absence of noise during the warm-up activity indicates that the other students were listening. The listening and focusing on the mathematical plan were supported by student questions about the proposed plan and what unit of measurement to use. The quotes below reflect students paying attention to the proposed plan:

1 Mae : Are we doing 5 and 7? [5 and 7 are the length of a proposed triangle] I thought we did that yesterday.
2 Liz : Do we do it in centimeters?

Liz was a non-math-talker in whole-classroom discussion. Liz and her group regularly engaged in getting the assignment done—the consistent goal of their mathematical activity. The non-mathematical question from Liz represents her minimal participation in our discussion. In the Triangles task, students finished the triangle constructions and were able to compare their triangles to their peers. Students were required to respond with explanations involving geometry vocabulary and problem solving skills. Table 13 tallies 15 times in which the students used the social scaffolding to respond and ask questions indicating they interacted more than in the
Painted Cubes episodes (4 times). Based on the above analysis, active participation among students improved as the students listened to and cooperated with one another and the teacher in a whole-classroom discussion.

Student interaction increased during content-related comics activities as students gave non-math-talk spontaneously. It appears that participation in non-math-talk became a stimulus for math-talk by enabling students to feel more comfortable interjecting with comment or answer whether right or wrong. Student non-math-talk in the Diagonals and MAD episodes tended to appear first and most of the time during the warm-ups which included a comic. The analysis of non-math-talk or social scaffolding indicates that students gave topic-related explanations, questions, ideas, and arguments during the content-related comic activity. For example, Nina explained and argued during the warm-up activity in the MAD episode:

<table>
<thead>
<tr>
<th>4</th>
<th>Teacher: Are there any here that is a stone’s throw away from our classroom?</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Ramos: A landmark?</td>
</tr>
<tr>
<td>6</td>
<td>Troy: The football field.</td>
</tr>
<tr>
<td>7</td>
<td>Nina: A landmark must be a famous site, is that what it says?</td>
</tr>
<tr>
<td>8</td>
<td>So football field is not. We don’t have a famous site here.</td>
</tr>
</tbody>
</table>

The above non-math-talk has the potential to become a mathematical discussion compared to another response in the baseline:

<table>
<thead>
<tr>
<th>11</th>
<th>Teacher: Alright, can someone tell me why we did this kind of plan.</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>Jed: Cause I want a grade.</td>
</tr>
</tbody>
</table>

Jed gave a non topic-related answer testing my authority and our new standards to learn mathematics. Thus, the content-related comic can be a tool to create productive social participation. In this study, social participation is defined as students contributing non-math-talk related to the topic without being evaluated for a right or wrong answer. The initial analysis in
the Diagonals and MAD episodes from the Data Description indicate student participation in non-math-talk stimulated math-talk, in other word the social and mathematical scaffolding influenced each other.

    My commitment to build a math-talk community began from an unfamiliar experience in which I used math-talk at levels 0 and 1 in the Painted Cubes episodes. Our classroom discourse grew from the traditional IRE pattern, or level 0, to a situation in which a student explained and defended her solutions at level 3. Novi rose to lead the closing activity and her peers supported her and participated in completing the task as a math-talk learning community. At the end of this study, my students showed that they were implementing discourse standards where I was a co-learner.

    Summary of Data Analysis

    The findings from data analysis include a pattern of discourse to elicit students’ responses consisting of AQ, BE, and CP. The classroom community showed growth of student-initiated ideas with less teacher-directed math-talk or analytical scaffolding. This finding is supported by the decrease of teacher BE in the Diagonals and MAD episodes and students taking the role of a questioner and source of ideas as they began to imitate and model the teacher in asking probing questions. The growth in quality was evident as students also contributed AQ, BE, and CP at level 2 in the MAD episodes (see Figure 37). One of math-talkers demonstrated BE at level 3 and stimulated more AQ and BE during a whole-classroom discussion in the MAD episode.

    The social and analytical scaffolding analysis supported the finding that there are more student-to-student interactions in the Diagonals and MAD episodes. Students contributed to our whole-classroom discussion by listening to, responding to, and supporting their peers, which began from the warm-up activity where they expressed their non-math-talk spontaneously.
Students contributing non-math-talk related to the topic being discussed, engendered social participation that stimulated other students to talk and join our mathematical discussion.

To summarize the data analysis of baseline and content-related comic episodes I present: (a) Teacher and student math-talk at level 1, which occurred in the Triangles, Diagonals, and MAD episodes in Figures 38 and 39 respectively, and (b) Teacher and student math-talk at level 2, which occurred in the Triangles, Diagonals, and MAD episodes in Figures 40 and 41 respectively.
Figure 38. Teacher Math-Talk Level 1

Figure 39. Student Math-Talk Level 1

Figure 40. Teacher Math-Talk Level 2

Figure 41. Student Math-Talk Level 2
CHAPTER V
DISCUSSION

Summary of Study

This study was designed and conducted to explore the research question: To what extent does teaching with content-related comics support student participation in mathematical discourse? Theory and research around building a math-talk learning community were applied to my third period Math I Support class. Student engagement in mathematical discourse was examined without comics and with comics through action research, which included participant observational methods during a nine-week instructional period. To analyze the effectiveness of content-related comics in eliciting student participation, the combined theoretical frameworks consisting of a Hufferd-Ackles’ et al. (2004) math-talk learning community and Nathan and Knuth’s (2003) social and analytical scaffolding were employed. The results suggested that the content-related comics helped students become more comfortable and independent in expressing their thinking during class discussion.

Summary of Findings

Three major findings include, first, the comic activity embedded in the mathematics problem appeared to help students engage in the opening discussion through spontaneous comments, questions, and interpretations. Engagement with the content related comics seemed to provide an avenue to participate socially. As suggested by Silver and Smith (1996):

Even teachers who want their students to understand that mathematical ideas are the topics most valued in discussions in their classroom may decide it is prudent to move toward that goal one step at a time. If one sees the development of classroom discourse communities as a journey, then it seems reasonable to begin in a safe, possibly non-
mathematical space, in which students may initially be more comfortable, and then move gradually to settings in which the mathematical ideas are salient in the discussion. (p.24) The employment of content-related comics aligns with the goal of orchestrating mathematical discourse, which is to create an atmosphere in which students can feel safe asking questions and expressing their thinking while contributing to the learning community (Hufferd-Ackles et al., 2004; Silver & Smith, 1996). The absence of safe space in the baseline tasks might have created a tension between the students and myself, resulting in my math-talk dominating our whole-classroom discussions as described through the course of the baseline period.

Second, the findings suggest that facilitating discourse that enabled students to propose ideas spontaneously could increase student-to-student talk. As the analysis indicated, students’ initiative to propose ideas that guided the direction of the math lesson (level 2) stimulated more students to become math-talkers and created student-to-student talk in our discussion. Student-initiated ideas, which began during the warm-up and continued throughout the episodes, are the phenomena that distinguished the Diagonals and MAD episodes from the baseline data. Why students participated more often through peer interaction compared to when I prompted them with questions is explained by Hatano and Inagaki (1991):

In contrast, in horizontal interaction, members’ motivation to disclose their ideas tends to be natural and strong, because no authoritative right answers are expected to come immediately. Therefore, the members often express fearlessly a variety of ideas, which are likely to be examined, sorted out, and elaborated in interaction. (p. 333)

Finally, the findings indicate a complementary relationship between student social participation and mathematical discourse. The participation flourished beginning with non-math-talk that enticed more students to become listeners, math-talkers, presenters, and thinkers. Figure
42 illustrates the complementary relationship in which the smaller gear represents social participation that is easier to rotate with only a small amount of input of students’ non-math-talk. The movement of the smaller gear provides a rotational speed for the bigger gear; it is a stimulus for math-talk by enabling students to feel more comfortable with interjecting comments or answer whether right or wrong. The relationship resembles social participation and math-talk influencing each other.

![Math-Talk Learning Community](image)

Figure 42. Student Social Participation and Math-Talk Influencing Each Other

In this chapter, the organization of discussion comprises (a) implications focused on mathematics educational researchers and teachers, (b) personal reflection, and (c) limitations on the entire study that opens opportunities for future research studies.

Implications

Mathematical Education Research

Two implications for mathematics educational researchers include, first, this study used the combined theoretical frameworks from Hufferd-Ackles et al. (2004) and Nathan & Knuth (2003) as an analytical tool to examine the implementation of classroom discourse. Lloyd et al.
(2005) and Steinbring et al., (1998) indicate the need to develop more analytic tools that are specifically geared toward mathematics classrooms than those currently available to investigate the nature and role of discourse in the learning of mathematics. Aligned with the above view, Hufferd-Ackles et al (2004) propose that a framework can guide teachers’ work to implement discourse practice and to facilitate mathematics researchers understanding of the process in the real classroom.

The combined theoretical frameworks became an analytical tool to perform a multilevel analysis and described different facets of classroom discourse where each is complementary to the other. The multilevel analysis constituted detailed descriptions of the interactions between the students and myself, and among the students themselves, enabling me to identify the level of math-talk learning community for each learning task. The result of a multilevel analysis performed by the combined theoretical frameworks on each learning task is shown in Figure 20, which presents the growth of our math-talk learning community across the learning tasks as described in chapter 4, part 2. In addition, the social scaffolding analysis provided the description of classroom social norms that evolved while the participants were building a math-talk learning community.
Second, the math-talk analysis was performed by coding every sentence (or a group of sentences related to the same ideas), rather than the entire episode, to determine the overall level of the math-talk learning community; hence providing a more detailed description about each math-talk component and level than the later approach as shown by the authors (Hufferd-Ackles et al., 2004) in their article. The use of the math talk framework at the sentence level is compatible with the use of a second framework of analytical and social scaffolding (Nathan & Knuth, 2003), which treats every sentence (or group of sentences related to the same ideas) as one unit of analysis. Therefore, comparing analysis by math-talk and analytical scaffolding at the sentence level provided more information about students’ contributions in our whole-classroom discussion as opposed to comparing the result to the math-talk analysis at the episode level and analytical scaffolding at the sentence level.

Mathematics Teachers

Two implications for classroom teachers include, first, this study provides a concrete example of the advantages of systematically investigating one’s own classroom practice. As described by Herbel-Eisenmann and Cirillo (2009) and proponents of reflective-oriented teaching and learning, teachers need to identify performance gaps between a teacher’s belief and intention and the way she or he actually teaches in the classroom (Giovanelli, 2003; Hopkins, 2002; Hubbard & Power, 1999; Johnson, 2002). Through this action research, I was drawn to systematically examine my own teaching practice. The math-talk analysis of the Painted Cubes learning task indicated the mismatch between my standards-based teaching intention and the actual practice. Considering this initial finding is an important “performance gap” (Herbel-Eisenmann & Cirillo, 2009, p. 20), I made changes in my discourse behaviors by pressing more questions focused on student thinking (level 1) and multiple strategies from different students
(level 2) that resulted in students’ explaining moving to levels 1 and 2 as well. Furthermore, my experience in the Painted Cubes learning task provided an example of reflective action in which a teacher found a performance gap between her beliefs and the actual behaviors in the classroom through one unit lesson.

Lastly, the content-related comic is an ordinary object and easy to implement as a teaching tool to stimulate discourse. Based on the social scaffolding analysis, student interaction increased during content-related comics activity as students gave non-math-talk related to the topic spontaneously, which in turn invited more students to respond as opposed to when I prodded them with questioning, explaining, and proposing ideas. The comic selected for this study had two specific features. First, the content-related comic consisted of a topic that might help the participants see mathematics tasks as more appealing and connected to their interests; thus they would feel more inclined to participate. For example, the students spontaneously commented on the comics “A Square by Any Other Name” and “A Stone’s Throw Away,” indicating that they are “not funny.” Nevertheless, both of the comics related to student and school issues that students could easily say something about. Hence, they continued to contribute their thinking. This phenomenon relates to Clark’s (1998) proposal that students more likely attempt the problem when they are exposed to familiar material than when they do regular Math problems. Proponents of research-based humor in instructional material (Garner, 2006; Ziv, 1988) advised that in all content-related comics activities, the topic of discourse presented was tightly linked to the mathematics skills being taught in class.

Second, the content-related comic was followed by several open-ended questions to open a Q&A session and to facilitate social interaction. The questions capitalize on the idea presented
by the comic to prompt and focus students’ thinking on the topic being discussed. For example in the comic “A Stone’s Throw Away,” students responded to the following questions:

1. How far is “a stone’s throw away?”

2. How many ways can we measure a distance?

The Q&A session became a spring-board to elicit student participation. The emerging pattern of participation indicate that from student social participation in non-mathematical discussion related to the content-related comic activity, the whole-classroom discussion shifted smoothly to the mathematical content of the learning task.

Personal Reflections on Research Methodology

Several research studies have indicated that facilitating students’ explaining of their thinking requires patience and creativity on the part of the teacher (Cobb et al., 1993; Hufferd-Ackles et al., 2004; Silver & Smith, 1996). Silver and Smith (1996) indicate that “teachers have found that a critical aspect of building classroom learning communities in which students are willing to engage in investigation and discourse is the creation of an atmosphere of trust and mutual respect” (p. 22).

In embracing the role of teacher-researcher, I had to be open minded in recognizing my weaknesses in order to improve classroom discourse. The key words “patience” and “creativity” describe the challenges to persuade students to talk in our whole-classroom discussion. One of the challenges occurred when the initial implementation revealed a gap between my new expectation and the students’ old habits. I experienced a stumbling block similar to the warning that to implement discourse-oriented teaching can “involve pushing against a strong tide and almost always create turbulence” (Pimm, 2009, p. 137). The students were the collaborators that I had to work with. Their collaboration was a critical factor in the process of understanding the
task, engaging in small-group discussion, and progressing toward the solution. To understand what kind of collaboration a teacher needs from her students, I first needed to close the gap between my zone and their zone.

When I listened to the early videotaped recording, I found myself regretting the lack of follow-up questions I asked, my failure to connect ideas to deeper mathematical ideas, and not calling other students to participate. Then, I decided I wanted to do better in listening to what my students said because this was our discussion, and every response needed my attention. The beginning of our math-talk learning community actually took place when I began to listen to each of my students’ responses. Jed, who was never interested in his mathematics work before, refreshed our classroom interactions. To value Jed’s and other student’s contributions required more than simply expecting them to say something. I needed to be on their level of thinking and to consider the potential of their responses as mathematical ideas, which required scaffolding in the learner’s zone proximal development (Erickson, 1996; Hufferd-Ackles et al., 2004; Palinscar et al., 1993). The students and I discussed a problem until we all agreed on the solution and I was no longer standing in the front of the class. We were in the same zone to share our thinking, to practice questioning, explaining, proposing mathematical ideas, and revising each other ideas. The students were always around me, and I frequently stooped my head to listen to a student’s soft answer. Even other students wanted to know what was said. I found myself vulnerable amid the conflicting answers, pressing questions, and feeling unsure where the discussion was heading, as all of this was a new experience. In short, I became their co-learner.

As the data pointed to my discourse practices, I realized that although I wrote the lesson plan and worked out the problems, I must revisit and prepare the problems thoroughly and frequently as if I were presenting a case in court. I had to master the lesson in order to orchestrate
A classroom discussion in which students’ ideas could be guided effectively. The data also made known that students played important roles in the way ideas were exchanged in our whole-classroom discussion. Hence, students’ ideas and strategies sometimes guide the direction of the math lesson (Hufferd-Ackles et al., 2004). They were math-talkers who began at level 0 and moved as a community to level 2. I valued them as pioneers in building our math-talk learning community. At the end of the study, Novi performed at level 3 in presenting the closing problems. She led the class in closing the MAD learning task, giving me the message that the class was implementing the discourse standards.

Limitations and Future Research

The current study has limitations that were unavoidable but may open possibilities for future research. First, due to the small sample size, this study may not be generalized to all secondary mathematics students. The time allocated to complete this research was limited; therefore, the progression and flow of our classroom discourse did not yield a math-talk learning community level 3. In spite of these limitations, this research provided insight into the use of comics as a teaching tool from real classroom practice. Teaching with content-related comics offers a way to establish discourse community. The results provide the preliminary seeds for future investigations into the usefulness of comics in mathematical teaching and learning. Considering the goal of making mathematics more accessible to young students, especially in secondary levels, this study can be extended to investigate the impact of content-related comic activity in a long-term interaction and larger population.

Second, another concern of this research is the validity of the findings. The participants were students in my Mathematics I Support class; thus, their responses tended to yield to the
teacher’s expectations. For example, students might have felt they needed to participate during our whole-classroom discussion because I told them the goal of my study.

Third, the learning tasks selected for this study derived from the Georgia Performance Standards (GPS, 2008) curriculum that aligned with the NCTM standards (2000). The learning tasks were designed to engage students in thinking and reasoning about important mathematical ideas which would be sufficient to build a math-talk learning community as opposed to incorporating the content-related comic activities prior to the learning tasks. Although one can argue that the comic did not have any relation to the increase in student math-talk by reasoning that the students became more comfortable talking over time, this action research documented that a few instances of students’ proposing at level 2 during the warm-up appeared to open the path of student-to-student talk in the entire episodes. Future research using a control group can investigate the possible effects of the nature of the tasks on math-talk, and whether it could have improved without the comics.

Finally, this study also provided data about classroom interaction around mathematics that could be examined to explore the influence of discourse on student learning. This study could be extended to focus on the learning outcomes as a result of the improved discourse. Using the data collected in this study, researchers may identify student learning by classifying what makes something true or reasonable in mathematics (NCTM, 1991). A method to analyze the data can use sociomathematical norms proposed by Cobb and Yackel (1996). “Examples of sociomathematical norms include what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation” (Cobb & Yackel, 1996, p. 213).
The teaching standards document (NCTM, 1991) asserts that students need guidance and encouragement in order to participate in the discourse of a collaborative community. This research study that investigated the content-related comic activity is an example of new strategies needed to promote communication. The content-related comic activity, which was tightly linked to the task, helped students begin to participate and enabled me to facilitate a more student-centered discussion than our past experience. In this study, the teacher and students used discourse that enabled individual students to listen to each other, to explain their mathematical solutions, and to argue other students’ mathematical ideas, or simply put, to become math-talkers. These findings lead to the inquiry: if discourse is a process to learn mathematics, what is the impact of discourse in students’ understanding of mathematics? Future study to investigate how the discourse practices enhance students’ learning is needed.
REFERENCES


Appendix A: Framework and Coding Schemes
Table A1

*Coding Scheme of Math-talk Components and Levels*

<table>
<thead>
<tr>
<th>A. Questioning</th>
<th>B. Explaining mathematical thinking</th>
<th>C. Proposing mathematical ideas</th>
<th>D. Leading discourse for learning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Level 3</strong></td>
<td><strong>Teacher</strong>: AQ-3</td>
<td><strong>Teacher</strong>: CP-3</td>
<td><strong>Teacher</strong>: DL-3</td>
</tr>
<tr>
<td></td>
<td><strong>Student</strong>: SAQ-3</td>
<td><strong>Student</strong>: SCP-3</td>
<td><strong>Student</strong>: SDL-3</td>
</tr>
</tbody>
</table>

**A. Questioning**

*Teacher expects students to ask one another questions about their work. The teacher’s questions still may guide the discourse.*

*Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions and listen to responses. Many questions are "Why?" questions that require justification from the person answering. Students repeat their own or other’s questions until satisfied with answers.*

**Teacher follows along closely to student descriptions of their thinking, encouraging students to make their explanations more complete; may ask probing questions to make explanations more complete. Teacher stimulates students to think more deeply about strategies.*

*Students describe more complete strategies; they defend and justify their answers with little prompting from the teacher. Students realize that they will be asked questions from other students when they finish, so they are motivated and careful to be thorough. Other students support with active listening.*

**Teacher allows for interruptions from students during her explanations; she lets students explain and "own" new strategies. (Teacher is still engaged and deciding what is important to continue exploring.) Teacher uses student ideas and methods as the basis for lessons or mini extensions.*

*Students interject their ideas as the teacher or other students are teaching, confident that their ideas are valued. Students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons.*

**The teacher expects students to be responsible for co-evaluation of everyone’s work and thinking. She supports students as they help one another sort out misconceptions. She helps and/or follows up when needed.*

*Students listen to understand, then initiate clarifying other students’ work and ideas for themselves and for others during whole-class discussions as well as in small group and pair work. Students assist each other in understanding and correcting errors.*
<table>
<thead>
<tr>
<th>A. Questioning</th>
<th>B. Explaining mathematical thinking</th>
<th>C. Proposing mathematical ideas</th>
<th>D. Leading discourse for learning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teacher continues to ask probing questions and also asks more open questions. She also facilitates student-to-student talk, e.g., by asking students to be prepared to ask questions about other students’ work. (Q-2)</strong>&lt;br&gt;Students ask questions of one another's work on the board, often at the prompting of the teacher. Students listen to one another so they do not repeat questions. (SQ-2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Teacher probes more deeply to learn about student thinking and supports detailed descriptions from students. Teacher open to and elicits multiple strategies.</strong>&lt;br&gt;Students usually give information as it is probed by the teacher with some volunteering of thoughts. They begin to stake a position and articulate more information in response to probes. They explain steps in their thinking by providing fuller descriptions and begin to defend their answers and methods. Other students listen supportively</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Teacher follows up on explanations and builds on them by asking students to compare and contrast them. Teacher is comfortable using student errors as opportunities for learning.</strong>&lt;br&gt;Students exhibit confidence about their ideas and share their own thinking and strategies even if they are different from others. Student ideas sometimes guide the direction of the math lesson.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Teacher encourages student responsibility for understanding the mathematical ideas of others. Teacher asks other students questions about student work and whether they agree or disagree and why.</strong>&lt;br&gt;Students begin to listen to understand one another. When the teacher requests, they explain other students' ideas in their own words. Helping involves clarifying other students' ideas for themselves and others. Students imitate and model teacher's probing in pair work and in whole-class discussions.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Level 2**

<table>
<thead>
<tr>
<th>Coding</th>
<th>Coding</th>
<th>Coding</th>
<th>Coding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher: AQ-2</td>
<td>Teacher: BE-2</td>
<td>Teacher: CP-2</td>
<td>Teacher: DL-2</td>
</tr>
<tr>
<td>Student: SAQ-2</td>
<td>Student: SBE-2</td>
<td>Student: SCP-2</td>
<td>Student: SDL-2</td>
</tr>
<tr>
<td>A. Questioning</td>
<td>B. Explaining mathematical thinking</td>
<td>C. Proposing mathematical ideas</td>
<td>D. Leading discourse for learning</td>
</tr>
<tr>
<td>----------------</td>
<td>-------------------------------------</td>
<td>--------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>Teacher questions begin to focus on student thinking and focus less on answers. Teacher begins to ask follow-up questions about student methods and answers. Teacher is still the only questioner. As a student answer a question, other students listen passively or wait for their turn.</td>
<td>Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in explanations herself. Students give information about their math thinking usually as it is probed by the teacher (minimal volunteering of thoughts). They provide brief descriptions of their thinking.</td>
<td>Teacher is still the main source of ideas, though she elicits some student ideas. Teacher does some probing to access student ideas. Some student ideas are raised in discussions, but are not explored.</td>
<td>Teacher begins to set up structures to facilitate student listening to and helping other students. The teacher alone gives feedback. Students become more engaged by repeating what other students say or by helping another student at the teacher's request. This helping mostly involves students showing How they solve a problem</td>
</tr>
<tr>
<td>Level 1</td>
<td>Level 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher: AQ-1</td>
<td>Teacher: BE-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student: SAQ-1</td>
<td>Student: SBE-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher: CP-1</td>
<td>Student: SCP-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher: DL-1</td>
<td>Student: SDL-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher is the only questioner. Short frequent questions function to keep students listening and paying attention to the teacher. Students give short answers and respond to the teacher only. No student-to-student math talk.</td>
<td>No or minimal teacher elicitation of student thinking, strategies, or explanations; teacher expects answer-focused responses. Teacher may tell answers. No student thinking or strategy-focused explanation of work. Only answers are given.</td>
<td>Teacher is physically at the board, usually chalk in hand, telling and showing students how to do math. Students respond to math presented by the teacher. They do not offer their own math ideas.</td>
<td>Teacher repeats student responses (originally directed to her) for the class. Teacher responds to students' answers by verifying the correct answer or showing the correct method. Students are passive listeners; they attempt to imitate the teacher and do not take responsibility for the learning of their peers or themselves.</td>
</tr>
<tr>
<td>Teacher: AQ-0</td>
<td>Teacher: BE-0</td>
<td>Teacher: CP-0</td>
<td>Teacher: DL-0</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Student: SAQ-0</td>
<td>Student: SBE-0</td>
<td>Student: SCP-0</td>
<td>Student: SDL-0</td>
</tr>
</tbody>
</table>

**Note.** Coding Scheme of math-talk components and levels is adapted from the math-talk theoretical framework formulated by Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81–116. This study used the terms “proposing mathematical ideas” and “leading discourse for learning” that are originated from the terms “source of mathematical idea” and “responsibility for learning” respectively in the math-talk framework by Hufferd-Ackles et al. (2004) to express the same actions.
Table A2

*Coding Scheme of Analytical and Social Scaffolding*

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Scaffolding</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS</td>
<td>Teacher to student (Vertical flow of information)</td>
<td>A</td>
</tr>
<tr>
<td>QM</td>
<td>Ask question (Probing)—Math</td>
<td>A</td>
</tr>
<tr>
<td>TM</td>
<td>Tell and show students how to do math</td>
<td>A</td>
</tr>
<tr>
<td>QFM</td>
<td>Ask follow-up questions</td>
<td>A</td>
</tr>
<tr>
<td>TVA</td>
<td>Revoicing</td>
<td>A</td>
</tr>
<tr>
<td>TSM</td>
<td>Stimulate student to think more deeply about strategies</td>
<td>A</td>
</tr>
<tr>
<td>TIM</td>
<td>Use student ideas and methods as the basis for lesson</td>
<td>A</td>
</tr>
<tr>
<td>QN</td>
<td>Ask question—non math</td>
<td>S</td>
</tr>
<tr>
<td>MG</td>
<td>Management—discipline, admin, homework</td>
<td>S</td>
</tr>
<tr>
<td>UPN</td>
<td>Set up structures to facilitate discourse</td>
<td>S</td>
</tr>
<tr>
<td>TC</td>
<td>Teacher to class (Vertical flow of information)</td>
<td>A</td>
</tr>
<tr>
<td>OM</td>
<td>Open invitation—Math question, challenge, yes-no</td>
<td>A</td>
</tr>
<tr>
<td>TRE</td>
<td>Explaining and eliciting student idea</td>
<td>A</td>
</tr>
<tr>
<td>DM</td>
<td>Declaration of math principle, fact, rule</td>
<td>A</td>
</tr>
<tr>
<td>OMN</td>
<td>Open invitation—non Math question, challenge, yes-no</td>
<td>S</td>
</tr>
<tr>
<td>DN</td>
<td>Declaration of non-math principle, fact, rule</td>
<td>S</td>
</tr>
<tr>
<td>Code</td>
<td>Description</td>
<td>Scaffolding</td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td><strong>ST</strong></td>
<td>Student to teacher (Vertical flow of information)</td>
<td></td>
</tr>
<tr>
<td>DMS</td>
<td>Declaration of math principle, fact, rule</td>
<td>A</td>
</tr>
<tr>
<td>RYN</td>
<td>Respond with Yes or No</td>
<td>A</td>
</tr>
<tr>
<td>MVM</td>
<td>Respond with minimal volunteering thoughts</td>
<td>A</td>
</tr>
<tr>
<td>SVM</td>
<td>Respond with some volunteering thoughts</td>
<td>A</td>
</tr>
<tr>
<td>SEM</td>
<td>Explaining their thinking and description of methods</td>
<td>A</td>
</tr>
<tr>
<td>SII</td>
<td>Interjecting idea</td>
<td>A</td>
</tr>
<tr>
<td>QMS</td>
<td>Ask question—Math</td>
<td>A</td>
</tr>
<tr>
<td><strong>SS</strong></td>
<td>Student to student/class (Horizontal flow of information)</td>
<td></td>
</tr>
<tr>
<td>QNS</td>
<td>Ask question—non Math (break time, homework)</td>
<td>S</td>
</tr>
<tr>
<td><strong>Note.</strong></td>
<td>A = Analytic scaffolding; S = Social scaffolding</td>
<td></td>
</tr>
</tbody>
</table>

Coding Scheme of Analytical and Social Scaffolding is adopted from Nathan and Knuth’s (2003) A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction, 21*(2), 175-207.
Appendix B: Lesson Plan and Content-Related Comics Prompts
Baseline data collection:

1. Painted Cubes Learning Task
2. Triangle Learning Task

Content-related comics data collection

1. Diagonals and Quadrilaterals Learning Task
2. Mean Absolute Deviation (MAD) Learning Task

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**Daily Lesson Plan**

**Subject:** Painted Cubes Learning Tasks

**Section:** 1.7


**Key Vocabulary:** function notation, vertical shift, horizontal shift, y-intercept, family of functions, reflection, vertical stretch.

**Opening:** With multiple sizes of cubes, discuss observations made by students. Ask leading questions (complete task # 1)
**Special needs:** Students who have difficulty in math will be assisted by a poster of a large 3-D cube to visualize the number of small cubes that are needed to make a larger cube. With a partner and teacher assistance, the students will understand the method to count the painted surface (face) of different sizes of painted cubes made of smaller cubes. During the group work, the teacher will monitor and assist the students when they need to move to the next level by building up their mathematical ideas.

**Work Time (Strategies):** Painted Cubes Task problems #2, 3, 6, 7

**Closing:** In groups, students will present their findings to the class.

**Homework:** (suggested): p.41 # 23 – 25 and/or p.43 25 – 27 and/or # 43, 44, or 45 (teacher preference)

**Assessment:** Students and teacher will engage in questioning, explaining, proposing mathematical ideas, and leading discourse for learning. Student work will be collected to see if they are correct and to provide feedback (commentary and grade).

---

**Painted Cubes**

The Vee Company, which produces the Zingo game, is working on a new product: a puzzle invented by one of its employees. The employee, Martin, made a large cube from 1,000 smaller cubes. Each cube had an edge length of one centimeter, and used temporary adhesive to hold the small cubes together. He painted the faces of the large cube, but when the paint had dried, he separated the large cube into the original 1000 centimeter cubes. The object of his puzzle is to put the cube back together so that no unpainted faces show.
The manager responsible for developing Martin’s puzzle into a Vee Company product thought that a 1000 cube puzzle might have too many pieces and decided that he should investigate the puzzle starting with smaller versions.

1. The cube at the right is made of smaller unpainted cubes, each having an edge length of 1 centimeter. All the faces of the large cube are painted yellow.
   a. How many small cubes were used to make the large cube?
   b. If you could take the large cube apart into the original centimeter cubes, how many cubes would be painted on
      i. three faces?      ii. two faces?
      iii. one face?      iv. no faces?

2. Consider large cubes with edge lengths of 3, 4, 5, 6, and 7 centimeters by building and/or sketching models, and answer the same questions that you answered for the large cube of edge length 2. Organize all your answers in a table as shown below.

<table>
<thead>
<tr>
<th>Edge length of large cube</th>
<th>Number of centimeter cubes</th>
<th>Number of small centimeter cubes painted on</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3 faces</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>125</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>216</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>343</td>
<td>8</td>
</tr>
</tbody>
</table>
3. Study the pattern in the table when the edge length of the large cube is used as input and the output is the number of centimeter cubes used in constructing the large cube.

a. Denote this functional relationship \( N \) so that \( x \) represents the edge length of the large cube and \( N(x) \) represents the number of centimeter cubes used to make the large cube. Write an equation expressing \( N(x) \) in terms of \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = x^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.5</td>
<td>-91.125</td>
</tr>
<tr>
<td>-4</td>
<td>-64</td>
</tr>
<tr>
<td>3.25</td>
<td>-34.328</td>
</tr>
<tr>
<td>-3</td>
<td>-27</td>
</tr>
<tr>
<td>-2</td>
<td>-8</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>-0.5</td>
<td>-0.125</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.125</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>3.25</td>
<td>34.328</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
</tr>
<tr>
<td>4.5</td>
<td>91.125</td>
</tr>
<tr>
<td>6</td>
<td>216</td>
</tr>
</tbody>
</table>

b. If we want to consider this relationship in general, not limiting ourselves to Martin’s puzzle, what should we use for the domain of the function \( N \)?

Comments:
This learning task opens with a description of a puzzle consisting of a large cube made from 1000 centimeter cubes, but, since we are not limiting ourselves to Martin’s puzzle, there is no upper limit to the edge length.

Solution:
If the cube is to be made using more than one smaller cube, the smallest edge is 2. So, the domain is \( \{2, 3, 4, 5, \ldots \} \), the set of positive integers greater than or equal to 2.

c. Let \( f \) be the function such that the formula for \( f(x) \) is the same as the formula for \( N(x) \) but the domain is all real numbers. Make a table for values of \( f \). Include some \( x \)-values that are in the domain of the function \( N \) and some that are not in the domain of \( N \), especially some negative numbers and fractions.

d. Sketch the graphs of \( f \) and \( N \) on the same axes for \(-10 \leq x \leq 10\). How does the graph of \( f \) help you understand the graph of \( N \)?

Comment: Students would benefit from using technology to graph the functions \( N \) and \( f \) and experiment with using a larger domain window, such as \(-500 \leq x \leq 500\), to see that
the graph of $f$ reveals the shape of the graph of $N$ without having to go to a larger scale.

In the graph at the right below, $N$ is graphed with blue points. The symbols overlap and cover the graph of $f$ for $x$ greater than or equal to 2.

**Solution:**

The graphs of $f$ and $N$ are shown below. The graph of $N$ consists of exactly those points on the graph of $f$ which have integer values of 2 or greater for their $x$-coordinates. The graph of $f$ helps in seeing the pattern to the points on the graph of $N$; that they lie on a curve with a particular shape.
Daily Lesson Plan

Subject: Congruent Triangles

Week 2

Day: 3 - 4


Standard(s): MM1G3 c. Students will understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS).

Key Vocabulary: SSS, SAS, ASA, AAS, Corresponding parts of triangles, corresponding sides, corresponding angles, and included sides and included angles.

Opening: Have students complete the launch for the Pennant Triangles investigation.

Congruence activity: How to determine two triangles are congruent?

Special needs: Students who have difficulty in math can use hands-on triangles to help them plan out their story problem, or they can also draw out the problem instead of writing it out. Students will be paired with a partner and assisted to begin the construction of triangles.

Work Time (Strategies): Group work. Students will complete the Triangle.


Closing: Have students report to class the different methods they found to prove two triangles congruent. Present summary of table of congruent triangles. Make a conjecture: once it is known that two triangles are congruent, what can be said about the parts of the triangle?
**Homework:** Finish summary of Triangle Congruence activity.

**Assessment:** Presentation by groups.

My assessment will be both informal and formal. The informal assessment will be done through my asking and answering questions of and from the students as they work to prove congruent triangles. Formal assessment will come as I collect the problems to see if they are correct and to provide feedback given as commentary and grade.

---

**Daily Lesson Plan**

**Subject:** Congruent Triangles  
**Standard(s):** MM1G3 c. Students will understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).  
**Key Vocabulary:** SSS, SAS, ASA, AAS, HL, corresponding parts of triangles, corresponding sides, corresponding angles, and included sides and included angles.  
**Opening:** Review all methods to prove triangles congruent.

How to use the converse of Pythagorean theorem to determine a triangle is a right triangle.

**Special needs:** Students who have difficulty in math can use hands-on triangles to help them identify the sides $a$, $b$, and $c$. Students will be paired with a partner and assisted to use Pythagorean theorem.

**Work Time (Strategies):** Group work. Given two sides of a right triangle, how do you determine that two right triangles are congruent?
Students will complete the learning task of problem # 12 to 16.

**Closing:** Students will present conjecture to prove two right triangles are congruent and problem # 16.

**Homework:** Review notes on congruence triangles

**Assessment:** Presentation and participation. Student work will be collected to check their understanding.

---

**Notes on Triangles Learning Task**

**This task provides a guided discovery for the following:**

- **SSS, SAS, ASA, AAS, and HL Congruence Postulates and Theorems can be used to prove triangles are congruent.**
- **SSA and AAA are not valid methods to prove triangles are congruent.**
- **Corresponding parts of congruent triangles are congruent.**
- **Congruent triangles can be used to solve problems involving indirect measurement.**

**Supplies Needed:**

There are many ways students can approach this task and the supplies needed depend upon the method you choose for your students.

- **Hands-on manipulatives like spaghetti noodles, straws, pipe cleaners, d-stix, etc. can used to represent the lengths of the sides. Protractors will be needed to create the indicated angles between the sides and clay or play dough can be used to hold the sides together.**
- **Students can use compasses, straightedges and protractors to construct the triangles.**
- **Geometer’s Sketchpad, or similar software, is a good tool to use in these investigations.**
Triangles Learning Task

The students at River Park High School decided to make large pennants for all 8 high schools in their district. The picture above shows typical team pennants. The River Park High students wanted their pennants to be shaped differently than the typical isosceles triangle used for pennants and decided each pennant should be a scalene triangle. They plan to hang the final products in the gym as a welcome to all the schools who visit River Park High.

Jamie wanted to know how they could make sure that all of the pennants are congruent to each other. The students wondered if they would have to measure all six parts of every triangle to determine if they were congruent. They decided there had to be a shortcut for determining triangle congruence, but they did not know the minimum requirements needed. They decided to find the minimum requirements needed before they started making the pennants.

Comment:

As students construct triangles to meet the given restrictions, it may become necessary for them to use a protractor and/or ruler to determine the measures of angles and/or sides not given to convince themselves the triangles are not congruent.

As soon as students begin discussing their triangles, encourage them to use correct geometric terms. They will need to use words like corresponding sides, corresponding angles, included sides and included angles. This common vocabulary will make sharing information about the triangles much easier.
1. Every triangle has ____ parts, _____ sides and _____ angles.

2. First they picked out 3 sides and each person constructed a triangle using these three sides.
   Construct a triangle with sides of 3 inches, 4 inches, and 6 inches. Compare your triangle to other students’ triangles. Are any of the triangles congruent? Are three sides enough to guarantee congruent triangles? Explain.

   \[ \text{Sample Answers:} \]

   All the triangles are congruent to each other. Even though some triangles might be rotated in a different direction or reflected, they are all congruent.

   Three sides are enough to guarantee the triangles are congruent.

   Nothing about the triangle can be changed when all the sides have to be a specific length.

3. Next the class decided to use only 2 sides and one angle. They chose sides of 5 inches and 7 inches with an angle of 38°. Using these measures, construct a triangle and compare it to other students’ triangles. Are any of the triangles congruent?

   \[ \text{Comments:} \]

   Note that the students are not told where to put the given angle. There are three possibilities. They can place the angle between the given side lengths, opposite the 5 inch side, or opposite the 7 inch side. Groups may approach this problem differently. Some groups may only choose to investigate the case where the angle is between the two given sides. If so, that is fine as the next problem will push them to look at other possibilities. However, some groups may want to immediately investigate all three options.
**Solutions:**

The solutions below include all three cases.

**Case 1: Angle between the two given sides**

If you put the angle between the two given sides all the triangles will be congruent. Even though some triangles might be rotated in a different direction or reflected, they are all congruent.

**Case 2: Angle opposite the 5 in. side**

As is the example above, this does not always create two congruent triangles.

This can also be illustrated in the following manner. Draw the 7 inch side and the 38° angle.

Since the given angle is at O, the 5 inch side must use N as one of its endpoints. There are an infinite number of segments having a length of 5 inches with N as one endpoint. These can all be represented by constructing a circle using N as the center with a radius of 5 inches. We are interested in where the side of unknown length intersects the circle. Those points of intersection meet all three conditions: the angle is 38°, the side adjacent to the angle is 7 inches and the opposite side is 5 inches. In this case there are two different triangles which satisfy those conditions.
Case 3: Angle opposite the 7 in. side

Comments:

This is the triangle everyone will get when the angle is opposite a side of 7 inches.

It is important to note that this case will give you congruent triangles.

Ask students to notice the similarities between this case and case 2. In both cases the angle is not included between the two given sides. So, the same pieces of information are given: side of 5 inches, side of 7 inches and an angle of $38^\circ$. But since we did not always get congruent triangles in case 2, and those triangles are not congruent to the triangles here, this information does not guarantee congruency.

This may confuse some students but it is critical for them to gain an understanding of this concept. If the constraint sometimes creates congruent triangles and sometimes does not, we cannot use it to prove two triangles are congruent.

Ask students: What is the key difference between case 2 and case 3? (e.g. The angle is opposite the larger given side in case 3.) Why do we only get one triangle now? (See drawing below.) If students have time, they might want to investigate the third case a little more to see if anything special is happening there.
The sketch below, using a circle with a radius of 7 inches shows why only one triangle can be created when the 7 inch side is opposite the given angle.

4. Joel and Cory ended up with different triangles. Joel argued that Cory put her angle in the wrong place. Joel constructed his triangle with the angle between the two sides. Cory constructed her sides first then constructed her angle at the end of the 7 in. side not touching the 5 in. side. Everybody quickly agreed that these two triangles were different. They all tried Cory’s method, what happened? Which method, Joel’s or Cory’s, will always produce the same triangle?

Comments:

If a group investigated all three cases for #3, this will look very familiar.

Sample Answers:

Joel’s method will always work. Cory’s method sometime works and sometimes doesn’t. (For further explanation, see solutions for Case 2 and Case 3 in #3).

5. Now the class decided to try only 1 side and two angles. They chose a side of 7 in. and angles of 35° and 57°. Construct and compare triangles. What generalization can be made?
Comments

This problem is similar to #3. Note that students are not told where to put the given side. They have three options. They can place the side between the given angles, opposite the 35° angle, or opposite the 57° angle. Groups may approach this problem differently. Some groups may only choose to investigate the case where the side is between the two given angles. If so, that is fine as the next problem will push them to look at other possibilities. However, some groups may want to immediately investigate all three options.

Solutions:

The solutions below include all three cases.

Case 1: Side between the two given angles

If you put the side between the two given angles, all the triangles will be congruent. Even though some triangles might be rotated in a different direction or reflected, they are all congruent.

Case 2: 7 in side opposite the 35° angle

The triangle constructed here is not congruent to the triangle constructed in case 1. However, all the triangles constructed in this manner are congruent to each other. (AAS)
Case 3: 7 in side opposite the $57^\circ$ angle

The triangle constructed here is not congruent to the triangle constructed in case 1. However, all the triangles constructed in this manner are congruent to each other. (AAS)

Comments:

Even though the triangles in cases 2 and 3 may look very similar they cannot be congruent to each other. The missing angle is $88^\circ$. That means the 7 inch side, in case 2, has to be the smallest side because it is opposite the smallest angle. In case 3 there is a side that is smaller than the 7 inch side. Therefore, the three sides cannot be congruent to each other.

6. Jim noticed that Sasha drew her conclusion given two angles and the included side. He wondered if the results would be the same if you were given any two angles and one side. What do you think?

See the 3 cases discussed in question #5. It is important for students to understand that AAS does work but the order in which the angles and side are listed is very important.

Solutions:

This would be a good time to discuss the connection between AAS and ASA. For example, in case 2, it can be proven that the third angle of the triangle is $88^\circ$ (using the Triangle Sum Theorem).

So all triangles constructed with angles of $35^\circ$ and $57^\circ$ will also have a third angle of $88^\circ$. This
means the side of 7 inches will always be included between the 35° angle and the 88° angle which leads us to ASA found in case 1.

7. The last situation the class decided to try was to use three angles. They chose angles of 20°, 40°, and 120°. How do you think that worked out? Construct a triangle using these three angles and compare with others. Can they prove two triangles are congruent using the three corresponding angles? Explain why or why not.

**Solution:**

Three angles are not enough information to prove congruence because the side lengths can vary even while the angle measures stay the same. In the picture below the angles are the same in both triangles but the side lengths are not and they are definitely not congruent.

8. Summarize the results using the chart below. Discuss what is meant by the common abbreviations and how they would help to remember the triangle situations you have just explored.

**Comments:**

The significance of the abbreviations has been written with more formality than the students may initially write on their own. This would be a good time to explore and help students formalize the triangle congruence postulates and theorems. Up to this point, the students have dealt with specific cases only. They need to expand this to deal with general cases.
### Solutions:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Explanation Abbreviation</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSS</td>
<td>3 sides lengths</td>
<td>If all sides of one triangle are congruent to all sides of another triangle, the triangles will be congruent.</td>
</tr>
<tr>
<td>SAS</td>
<td>2 sides and the included angle (the angle created by the two sides)</td>
<td>If two sides and the included angle in one triangle are congruent to the corresponding sides and included angle in another triangle, the triangles will be congruent.</td>
</tr>
<tr>
<td>SSA</td>
<td>2 sides and the non-included angle whose vertex lies on the side listed second</td>
<td>This does not guarantee that the triangles are congruent to each other.</td>
</tr>
<tr>
<td>ASA</td>
<td>2 angles and the included side (the side between the two angles)</td>
<td>If two angles and the included side in one triangle are congruent to the corresponding angles and included side in another triangle, the triangles will be congruent.</td>
</tr>
<tr>
<td>AAS</td>
<td>2 angles and the non-included side that is part of the second angle</td>
<td>If two angles and the non-included side in one triangle are congruent to the corresponding angles and non-included side in another triangle, the triangles will be ≠</td>
</tr>
<tr>
<td>AAA</td>
<td>three congruent angles</td>
<td>Creates similar triangles, but not necessarily congruent triangles.</td>
</tr>
</tbody>
</table>

9. SSS, SAS, and ASA are generally accepted as postulates. Look up the definition of a postulate. Discuss the need for postulates in this case. Why can’t we just use SSS as a single postulate? Could we use less than the three given or could we choose a different set as postulates?
10. Prove that AAS is always true. If we can prove a statement is always true, what do you we call it?

Comments:

The next few problems offer good opportunities to emphasize the importance of proofs.

Emphasize the importance of a logical argument and the justification of every statement in a proof. These problems give students a chance to see how the methods discovered above can be applied to determine ways to prove two right triangles are congruent. The solutions below are not formally written proofs; they are logical through processes students might employ to prove the methods are valid.

11. The methods listed in the table, which can be used for proving two triangles congruent, require three parts of one triangle to be congruent to three corresponding parts of another triangle. Nakita thought she could summarize the results but she wanted to try one more experiment. She wondered if the methods might be a bit shorter for right triangles since it always has one angle of 90°. She said: “I remember the Pythagorean Theorem for finding the length of a side of a right triangle. Could this help? My father is a carpenter and he always tells me that he can determine if a corner is square if it makes a 3 – 4 – 5 triangle.” Nakita chose to create a triangle with a hypotenuse of 6 inches and a leg of 4 inches. Does her conjecture work? Why or why not?

12. What if Nakita had chosen 6 inches and 4 inches to be the length of the legs? Does her conjecture work? Why or why not?
13. What are the minimum parts needed to justify that the two right triangles are congruent?

Using the list that you already made, consider whether these could be shortened if you knew one angle was a right angle. Create a list of ways to prove congruence for right triangles only.

Comments:
The students should develop the following theorems here. Make sure they investigate cases that are not specifically mentioned above. Also, make sure they connect their theorems to the congruence postulates and theorems they have already discovered.

14. Once it is known that two triangles are congruent, what can be said about the parts of the triangles? Write a statement relating the parts of congruent triangles.

Comments:
Students need to develop a clear understanding of CPCTC (corresponding parts of congruent triangles are congruent) here. They will use this in the problems that follow. Students need to use correct mathematical terms as they discuss “corresponding” parts of the triangles.

Congruent triangles can be used to solve problems encountered in everyday life. The next two situations are examples of these types of problems.

15. In order to construct a new bridge, to replace the current bridge, an engineer needed to determine the distance across a river, without swimming to the other side. The engineer noticed a tree on the other side of the river and suddenly had an idea. She drew a quick sketch and was able to
use this to determine the distance. Her sketch is to the right. How was she able to use this to determine the length of the new bridge? You do not have to find the distance; just explain what she had to do to find the distance.

16. A landscape architect needed to determine the distance across a pond. Why can’t he measure this directly? He drew the following sketch as an indirect method of measuring the distance. He stretched a string from point J to point N and found the midpoint of this string, point L. He then stretched a string from M to K making sure it had the same center. He found the length of MN was 43 feet and the length of segment LK is 19 feet. Find the distance across the pond. Justify your answer.
Daily Lesson Plan

**Subject:** Properties of Parallelograms  
**Week:** 3 - 4

**Section:** Content-related comics Luann  
**Day:** 7-8


**Standard(s):** MA1G3. d. Students will understand, use, and prove properties of and relationships among special quadrilaterals; parallelogram, rectangle, rhombus, square, trapezoid, and kite.

**Key Vocabulary:** quadrilateral, parallelogram, congruent, diagonal, bisect, consecutive angles, opposite angles, theorem, trapezoid, perpendicular bisector, parallel lines, perpendicular lines.

**Opening:** Using a fun quiz from the internet, review definitions of quadrilateral, rectangle, square, parallelogram, and rectangle. Use the content-related comic Luann for checking understanding on vocabulary and the relationship between members of quadrilateral family.

**Special needs:** Students who have difficulty in math can use hands-on quadrilaterals to help them identify the vocabulary, sides, and diagonals. Students will be paired with a partner and assisted to compare members of quadrilateral family.

**Work Time (Strategies):** Students will draw squares, rectangles, parallelograms, rectangles, trapezoids, and triangles, and explain a statement such as why a rectangle is not a parallelogram (for example). The students will draw the two diagonals in their parallelograms. Students should measure the diagonals and their segments and make conjectures concerning the diagonals.

**Closing:** Students will present their conjectures about the diagonals and formally write them as theorems.

**Homework:** Definition of parallelogram and properties of parallelograms.
Assessment: Students and teacher will engage in questioning, explaining, proposing mathematical ideas, and leading discourse for learning. Student work will be collected to check for their understanding and to provide feedback as commentary and grade.

Fun Quiz


Instruction: Answer the question and define each geometric figure (in your own words).

1. What do you say when you see an empty parrot cage?
   
   **Polygon. A geometric figure formed by connecting three or more line segments end to end to form a closed shape.**

2. What do you call a crushed angle?
   
   **A rectangle. It is a 4-sided polygon where all interior angles are 90°.**

3. What did the Italian say when the witch doctor removed the curse?
   
   **Hexagon. It is a six-sided polygon.**

4. What do you call an angle which is adorable?
   
   **Acute angle. That is an angle less than 90 degrees.**

5. What do you call more than one L?
   
   **Parallel lines. Parallel lines are two lines that do not intersect.**
   
   *(Lines are parallel if they lie in the same plane, and are the same distance apart over their entire length.)*

5. What do you call people who are in favor of tractors?
   
   **Protractors. A protractor is a tool used to measure angles.**
1. Look at the shape Gabriel made on his geoboard. Explain how you know his shape is a square.

2. Use a geoboard to make a shape that meets each description, if possible. If it is not possible, explain why not.

   a. A rectangle that is not a square
   b. A square that is not a rectangle
   c. A rectangle that is not a parallelogram
   d. A parallelogram that is not a rectangle
   e. A quadrilateral that is not a trapezoid
   f. An isosceles triangle that is not equilateral

Subject: Diagonals and Quadrilateral Learning Tasks  
Week 4

Section: 5.9  
Day: 9-10


Standard(s): MA1G3. d. Students will understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.

Secondary focus: Students will articulate the above understanding to peers and teacher.

Key Vocabulary: quadrilateral, parallelogram, congruent, diagonal, bisect, consecutive angles, opposite angles, theorem, trapezoid, perpendicular bisector, parallel lines, perpendicular lines.

Opening: Review vocabulary and construction of a perpendicular bisector in quadrilaterals, congruent segments, diagonals, vertices.

Work Time (Strategies): Given a drawing of diagonals, students will draw a quadrilateral and assign an appropriate name to each member of quadrilateral family. Students will make conjectures concerning the diagonals as property of members of quadrilateral family.

Special needs: Students who have difficulty in math can use hands-on quadrilaterals to help them identify the vocabulary, sides, and diagonals. Students will be paired with a partner and assisted to construct each quadrilateral.

Closing: Problem #14, students will present their conjecture and the justification of the properties of each members of quadrilateral family.

Homework: Property of quadrilaterals.
**Assessment:** Students and teacher will engage in questioning, explaining, proposing mathematical ideas, and leading discourse for learning. Student work will be collected to check for their understanding and to give grade with teacher commentary.

**Task: Constructing Diagonals—Quadrilaterals**

This task provides a guided discovery and investigation of the properties of quadrilaterals.

Students will determine which quadrilateral(s) can be constructed based on specific information about the diagonals of the quadrilateral(s).

**Supplies Needed:**

There are many ways students can approach this task and the supplies needed depend upon the method you choose for your students.

- Hands-on manipulatives like spaghetti noodles, straws, pipe cleaners, d-stix, etc. can be used to represent the lengths of the sides. Protractors will be needed to create the indicated angles between the sides and clay or play dough can be used to hold the sides together.

- Students can use compasses, straightedges and protractors to construct the triangles.

- Geometer’s Sketchpad, or similar software, is a good tool to use in these investigations.

**Comments:**

Sample proofs are given for each problem. The samples provided are not the only correct way these proofs can be written. Students should realize that proofs can be logically organized with differing orders of steps. They should also be given the opportunity to decide which type of proof they prefer writing.
1. Below are two segments (problem # 2) that are perpendicular bisectors.

   a. Perpendicular means ________________________________

   b. Bisector means ________________________________

2. Connect the four end points to form a quadrilateral.

   a. Below What names can be used to describe the quadrilaterals formed using these constraints?

3. Below are two segments that are perpendicular bisectors where both segments are congruent.

   a. Congruent means ________________________________

4. Connect the four end points to form a quadrilateral.

   a. What names can be used to describe the quadrilaterals formed using these constraints?

5. Below are two segments that bisect each other but are not perpendicular.

   Connect the four end points to form a quadrilateral.
a. What names can be used to describe the quadrilaterals formed using these constraints?

6. What if the two segments in #5 above are congruent in length... Then, what type of quadrilateral is formed?

   a. What names can be used to describe the quadrilaterals formed using these constraints?

7. Below are two segments that are perpendicular. One is bisected at its midpoint, but the other is not bisected. Connect the four end points to form a quadrilateral.

   a. Midpoint means ________________________________

   b. What names can be used to describe the quadrilaterals formed using these constraints?
8. Below are two trapezoids. Draw in the diagonals for these trapezoids.

a. Diagonals are ________________________________________

b. What can you summarize about the diagonals of trapezoid A?

c. What can you summarize about the diagonals of trapezoid B?

![Trapezoid A](image1.png) ![Trapezoid B](image2.png)

9. Complete the chart below by identifying the quadrilateral(s) for which the given conditions are true.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Quadrilateral(s)</th>
<th>Explain your reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagonals are perpendicular.</td>
<td>Rhombus</td>
<td>This is always true of a rhombus, square and kite.</td>
</tr>
<tr>
<td></td>
<td>Square</td>
<td>But there are some parallelograms and rectangles that do not have perpendicular diagonals.</td>
</tr>
<tr>
<td></td>
<td>Kite</td>
<td></td>
</tr>
<tr>
<td>Diagonals are perpendicular and only one diagonal is bisected.</td>
<td>Kite</td>
<td>All parallelograms have diagonals that bisect each other so this can’t be true for any of them. The diagonals are not bisected in an isosceles trapezoid so this would have to be a kite.</td>
</tr>
<tr>
<td>Diagonals are congruent and intersect but are not perpendicular.</td>
<td>Rectangle, Isosceles, Trapezoid</td>
<td>This is true for a rectangle but not a square. The square’s diagonals are perpendicular. The isosceles trapezoid is special and this is true of that type of trapezoid.</td>
</tr>
<tr>
<td>Diagonals are perpendicular and bisect each other.</td>
<td>Rhombus</td>
<td>This is true of rhombuses and is also true of squares. But unless we know the diagonals are congruent we do not know if it is a square.</td>
</tr>
<tr>
<td>Diagonals are congruent and bisect each other.</td>
<td>Rectangle</td>
<td>This is always true of a rectangle. But unless we know the diagonals are perpendicular we don’t know if it is a square.</td>
</tr>
<tr>
<td>Diagonals are congruent, perpendicular and bisect each other.</td>
<td>Square</td>
<td>Only a square has all three of these properties. The rectangle’s diagonals are not always perpendicular and the diagonals of rhombuses are not always congruent.</td>
</tr>
</tbody>
</table>
10. As you add more conditions to describe the diagonals, how does it change the types of quadrilaterals possible?

11. Name each of the figures below using as many names as possible (fill in the chart on the next page)

12. State as many properties as you can about each figure of the figures below (fill in the chart on the next page).

Solution:

Answers may vary but they should include the properties listed below. Students may discover more properties than listed here. As long as they can prove it to always be true they should list as many properties as possible.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Names</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td><em>Parallelogram</em></td>
<td><em>Opposite sides are congruent.</em></td>
</tr>
<tr>
<td></td>
<td><em>Rectangle</em></td>
<td><em>Diagonals are congruent and bisect each other.</em></td>
</tr>
<tr>
<td>B</td>
<td><em>Kite</em></td>
<td><em>Diagonals are perpendicular.</em></td>
</tr>
<tr>
<td></td>
<td></td>
<td><em>Angles between non-congruent sides are congruent to each other.</em></td>
</tr>
<tr>
<td>C</td>
<td><em>Parallelogram, square rectangle, rhombus</em></td>
<td><em>Diagonal between congruent angles is bisected by the other diagonal.</em></td>
</tr>
</tbody>
</table>
13. Identify the properties that are **always true** for the given quadrilateral by placing an X in the appropriate box.

14.

<table>
<thead>
<tr>
<th>Property</th>
<th>Parallelogram</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Square</th>
<th>Isosceles</th>
<th>Kite</th>
<th>Trapezoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opposite sides are parallel</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Only one pair of opposite sides is parallel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Opposite sides are congruent</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Opposite angles are congruent</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diagonals are congruent</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diagonals bisect vertex angles</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All $\angle$s are right $\angle$s</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>All sides are congruent</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two pairs of consecutive sides are congruent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

15. Using the properties in the tables above, list the **minimum** conditions necessary to prove that a quadrilateral is:

   a. a parallelogram

   b. a rectangle
c. a rhombus  
d. a square  
e. a kite  
f. an isosceles trapezoid  

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**Daily Lesson Plan**

**Subject:** Analyze Surveys and Samples/  
Calculate summary statistics (mean, median, quartiles, and interquartile range)

**Section:** Content-Related Comics Rhyme with Orange  
Day: 11 – 12


**Standard(s):** Ma1D2. Compare summary statistics (mean, median, quartiles, and interquartile range) from one sample data distribution to another sample data distribution in describing center and variability of the data distributions.

Secondary Focus: Represent the summary statistics in box-and-whisker plot.

**Key Vocabulary:** Mean, Median, Mode, Measure of central tendency, Measure of Dispersion (spread), Range, Quartiles, and Interquartile range, Deviation of the mean, Variability, Mean Absolute Deviation.

**Opening:** Use content related comics to open discussion about the expression “A stone throw away” Why do we need data? Student will collect their own data by measuring the distance of his or her throwing a tennis ball (until the ball hit the ground the first time). Students will compile data and find the five-number statistics summary and construct a box-and-whisker plot.
Special needs: Students who have difficulty in math will be assisted with a picture of a box-and-whisker plot to help them identify and understand the five-number summary. Students will be paired with a partner to find the five-number summary.

Work Time (Strategies): In groups of 3 or 4, students will compile data and find the 5 number statistics summary and construct a box-and-whisker plot.

Closing: Have students/pairs present a whisker-and-box plot.

I will add for our discussion what happens with a box-and-whisker plot if all students are girls? All students are boys?

On the basis of your class data, consider the meaning of “a stone’s throw away.

Why do we need to collect data?

Homework: Finish the learning tasks to find the methods to measure the thickness of paper. Be prepared to compare one method to another. Students need to prepare a question and commentary about other’s method.

Assessment: Students and teacher will engage in questioning, explaining, proposing mathematical ideas, and leading discourse for learning. Student work will be collected to check for their understanding and to give a grade with teacher commentary.
A Stone’s Throw Away

1. How far is “a stone’s throw away”? 
2. How many ways can we measure a distance? 
3. Which way is faster and accurate?

Learning Tasks

"A stone's throw away" is an idiom, a common expression that conveys an idea or image whose meaning is not a literal translation. Many idioms involve math concepts or relationships. What might such expressions mean if they were "taken at face value" (another idiom)?

1. Just how far is "a stone's throw away"? Conduct the following experiment and approximate the distance.

   a. Go to a location designated by your teacher. Work with one or two partners. Take turns standing at a marked home point and throw a tennis ball as far as you can. Your partners need to note where your ball hits the ground. Mark distances in yards or meters. They
should record the distance to the nearest yard or meter for each team member's 3 to 5 throws. Find the mean of your distances, rounding to the nearest whole yard or meter.

Back in the classroom, collect the mean throwing distances of all the students in your class. Determine (1) the median throwing distance for your class and (2) the range of the throwing distances. Display the class data in a box-and-whisker plot or with other types of graphs, as directed by your teacher.

c. On the basis of your class data, consider the meaning of "a stone's throw away":

- Are any houses within a stone's throw of your school? How many?
- Identify several objects or landmarks that are within a stone's throw of your classroom.
- How many people live within a stone's throw of your home?

d. What message is "a stone's throw away" meant to convey?

"When the deer jumped in front of the car, I came within 'a hair's breadth' of hitting it."

i. What message is "a hair's breadth" meant to convey?

ii. How can you measure something as small as the breadth (width or thickness) of a single strand of hair? How about the thickness of one sheet of paper? Design a hands-on method to determine a reasonable approximation for the thickness of one sheet of paper. Complete your experiment, and report to the class on what you found for the thickness of the paper. Explain how you arrived at your conclusion.

4. Complete the following task: Mean Absolute Deviation
Daily Lesson Plan

Subject: Analyze Surveys and Samples to determine MAD 

Compare summary statistics (mean, median, quartiles, and interquartile range)

Section: Mean Absolute Deviation

http://math.kennesaw.edu/.../Comparing%20Data%20Distributions.doc

Standard(s): MA1D4. Students will explore variability of data by determining the mean absolute deviation (MAD) or the average of the absolute values of the deviation.

Secondary Focus: MA1D2. Compare summary statistics (mean, median, quartiles, and interquartile range) from one sample data distribution to another sample data distribution in describing center and variability of the data distributions.

Key Vocabulary: Mean, Median, Mode, Measure of central tendency, Measure of Dispersion (spread), Range, Quartiles, and Interquartile range, Deviation of the mean, Variability, Mean Absolute Deviation

Opening: Students will explain vocabulary on Measure of Central Tendency and Variability.

Work Time (Strategies): In group of 3 or 4, students will compute the difference between each value and the mean of a distribution.” Students need to understand the “difference between each value and the mean of a distribution” represents the variability that will be averaged to find the MAD. Part A. Task #1-4 to understand and calculate MAD requires patience, students need to be familiar with the new and long vocabulary for the MAD concept (remember this task is challenging even for adults). Part B. Task #1 – 3 Measure of Variability is related to student’s
interests. Students will complete the chart on Measure of Variability and record the five-number summary from the box-and-whisker plots.

**Closing:** Have students/pairs present Measure of Variability by comparing the 4 whisker-and-box plots.

When do we need to use five-number summary and when do we need to use Measure of Central Tendency?

**Homework:** Design a survey to propose the need to have a skateboard facility in our campus, or another activity that you want to have in this school. Please identify your sample (all students, teachers and faculty, only freshmen, or other groups).

**Assessment:** Student project on survey. Students and teacher will engage in questioning, explaining, proposing mathematical ideas, and leading discourse for learning. Student work will be collected to check for understanding and to give a grade with teacher commentary.

---

**Mean Absolute Deviation (MAD)**

Central tendency:

Variability:

Comparing measures of central tendency between groups can sometimes lead one to believe that groups are similar. Comparing variability or spread of data between groups can often yield differences that at first were not apparent. It is important to look at the variability of data distributions when comparing differences and similarities between and among groups.

**Average Deviation from the Mean**

1. Determine the difference between each piece of data and the mean of its distribution. What is the average deviation from the mean for each group? Explain your results. Will these results always occur? Why or why not?
2. 

<table>
<thead>
<tr>
<th>Ashley(K teacher)</th>
<th>$x - \bar{x}$</th>
<th>Mary Kate (9th Grade) Teacher</th>
<th>$x - \bar{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
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**Notes.** One way of comparing variability between groups is to look at the mean absolute deviation (MAD). The mean absolute deviation is the arithmetic average of the absolute values of the difference between each value and the mean of a distribution. The larger the value of the MAD the more spread out the values are from the mean. When comparing variability of data distributions using the MAD, a distribution with a larger MAD has more erratic values, while a distribution with a smaller MAD has more consistent values.

3. Explain why taking the absolute value of the differences changes the average variability of the groups?

4. Determine the Mean Absolute Deviation (MAD) for each data set.

| Ashley(K teacher) | $x - \bar{x}$ | $|x - \bar{x}|$ | Mary Kate (9th Grade Teacher) | $x - \bar{x}$ | $|x - \bar{x}|$ |
|-------------------|--------------|----------------|-------------------------------|--------------|----------------|
| 32                |              |                |                               | 26           |
| 28                |              |                |                               | 36           |
| 12                |              |                |                               | 31           |
| 70                |              |                |                               | 28           |
| 31                |              |                |                               | 28           |
| 41                |              |                |                               | 28           |
| 13                |              |                |                               | 34           |
| 28                |              |                |                               | 33           |
| 37                |              |                |                               | 30           |
| 10                |              |                |                               | 26           |
| 30                |              |                |                               |              |
| 28                |              |                |                               |              |
| Sum               |              |                |                               | Sum          |
| Mean              |              |                |                               | Mean         |
5. Based on this Investigation write a summary and statistical assessment for Ashley and Mary Kate’s article that summarizes the results about students of different ages estimating their teacher’s age. Which group is more consistent in guessing their teacher’s age? Include statistics that verify/confirm your conclusions.

**Conclusions:**

Statistics that compare measures of central tendency between different sets of data include ________, ________, and ________.

Statistics that compare measures of variability or dispersion between different sets of data include ____________, ______________, and ____________.

The mean absolute deviation is the mean of ________________________________

**MEASURE of VARIABILITY**

1. The box plots shown represents pulse rates per minute for random samples of 100 people in each of four age groups.

2. Pulse Rates per Minute
Complete the chart below for each group:

<table>
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<tr>
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<th>Newborns</th>
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<th>15-yr olds</th>
<th>35-yr olds</th>
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<tr>
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<td>Q₃</td>
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<tr>
<td>Interquartile Range</td>
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<tr>
<td>Maximum Value</td>
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<tr>
<td>Minimum Value</td>
<td></td>
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</tr>
</tbody>
</table>

3. Compare the results and summarize your findings. What do these plots indicate about pulse rates as people get older?

4. If two data sets have the same interquartile range do they also have the same median? Give a specific example of values for two data sets that illustrate your conclusion.

5. When can the median of a data set be a better measure of central tendency than the mean? Give a specific example.
Appendix C: Internal Review Board Approval
Date Submitted: October 27, 2010

Name: Jeni M. Halimun

Address: 

City, State, Zip 

Email: teacher@school.edu 

Phone #'s Work: Cell# 

Employer or Company Name: River Park High School 

Name of College or University for which the project is to be conducted, if applicable: Kennesaw State University 

Name of major or advising professor: 

Description of research project: The purpose of this study is to explore the effectiveness of content-related comics in eliciting students participation in mathematical discourse. 

Upon the approval of Kennesaw State University Institute of Review Board, the instructional strategy will employ a content-related comics activity prior to the learning task. The components of mathematical discourse will be observed and analyzed using the combined theoretical frameworks of math-talk community (Hufferd-Ackles, Fuson, & Sherin, 2004) and analytical and social scaffolding (Nathan & Knuth, 2003). 

Beginning and Ending Dates for Research Project: 1-3-2011 to 4-30-2011 

How will parental permission be obtained, if required? I will request signed permission forms from parents (attach a copy).
Describe any data requests that will be made of school(s) or school system:
Student attendance record.

Will subjects be paid?  No. Students’ participation in this study is voluntary.

Names of schools to be involved: River Park High School

Number of students to be involved and grade levels: 30 students of Math I Support class. All students are in ninth grade.

Estimated time required of students: Thirty minutes of each observation.

Number of teachers/administrators/support staff to be involved and grade levels: One Math I teacher to observe the activity.

Estimated time required of staff: Thirty minutes of each observation.

Description of questionnaires, surveys, and materials to be used in the project:

Resources: Classroom Discourse The Language of Teaching and Learning (Cazden, 2001), Cartoon Corner Humor-Based Mathematics Activities (Reeves, 2007), I Think Therefore I Laugh (Paulos, 2000), Describing levels and components of a math-talk learning community (Hufferd-Ackles, Fuson, and Sherin, 2004).
Read and sign below:

I understand that information related to employees and students in River County School System is confidential and that River County School System employees and students shall not be identifiable in any research reports.

______________________________  ______________________
Signature                              Date

Please submit to your principal for review and then forwarded to the superintendent for approval.

______________________________  ______________________
Principal Signature                Date

______________________________  ______________________
Superintendent Signature            Date
Appendix D: Parental Consent Forms
January 3, 2011

Dear Parents,

This year, I especially want to create a more supportive learning environment where students feel welcomed and enjoy their learning. To serve this purpose, I will be looking closely at strategies using humorous examples to increase student attention and engagement in learning mathematical concepts. Humor can serve as a bridge between teachers and students by sharing common understanding and psychological bonding. It is my hope that teaching and learning with humor will stimulate student thinking processes and improve classroom discourse which are related to gaining student motivation and engagement. Occasionally, I will be video-taping and audio-taping the classroom interactions. The purpose of these recordings is to allow me to examine the students’ comments more closely within the context in order to catch things I might otherwise miss.

I would appreciate your permission to include your child's mathematics engagement survey, commentary, interview responses, and test scores in the data analysis of this research. I will use fictitious names in my writing to protect your child's privacy. Your child’s participation and the data will become a valuable contribution to the improvement of the mathematics instructional strategy. If you have any questions, please e-mail me at teacher@school.edu Please sign below granting permission to use your child’s survey and test scores collected for purposes of research.

Thank you very much for your kind consideration.

Sincerely,

Jeni Halimun

____________________________       ______________________________  ______________
Student’s Name                                     Parent or Guardian                                 Date

Cc: Dr. Principal
Mrs. Math Department
Appendix E: Copyright, Fair Use, and Research
The guidelines to use copyrighted materials in educational research is posed under the section on "fair use" of the Omnibus Copyright Revision of 1976 (Public Law 94-553) that regulates the use of all copyrighted materials in the United States; use of comic strips falls under this law.

Section 107 (p.44), Limitations on exclusive rights: Fair use.

Notwithstanding the provisions of section 106, the fair use of a copyrighted work, including such use by reproduction in copies or phone records or by any other means specified by that section, for purposes such as criticism, comment, news reporting, teaching (including multiple copies for classroom use), scholarship, or research, is not an infringement of copyright. In determining whether the use made of a work in any particular case is a fair use the factors to be considered shall include-

1. the purpose and character of the use, including whether such use is of a commercial nature or is for nonprofit educational purposes;

2. the nature of the copyrighted work;

3. the amount and substantiality of the portion used in relation to the copyrighted work as a whole; and

4. the effect of the use upon the potential market for or value of the copyrighted work.

The terms and conditions to use comic strips fall within the Fair Use guidelines and this study meets all four stipulations. First, the purpose of this study is for educational use and no one will profit financially. Second, the nature of the material is appropriate for classroom activity with no intention of marketing the comics. Third, one comic strip from any one artist cannot be considered a substantial portion of a copyrighted work as a whole, including journals and books. Fourth, using these comic strips in this study has little or no effect on neither the value nor the potential market of these comic strips. However, to avoid any possibility of potential liability for the University or myself, permission was obtained to use copyrighted materials in this manuscript from the Copyright Clearance Center, Incorporation at www.copyright.com. The written permission is included in this Appendix E.
Step 3: Order Confirmation

Thank you for your order! A confirmation for your order will be sent to your account email address. If you have questions about your order, you can call us at 978-646-2600, M-F between 8:00 AM and 6:00 PM (Eastern), or write to us at info@copyright.com.

Confirmation Number: 10413398
Order Date: 06/14/2011

If you pay by credit card, your order will be finalized and your card will be charged within 24 hours. If you pay by invoice, you can change or cancel your order until the invoice is generated.

Payment Information
Jeni Hallman
jenihallman77@gmail.com
+1 (770)6777726
Payment Method: CC ending in 1013

Order Details

Course: DIGITAL COMMONS @ KENNESAW STATE UNIVERSITY

University/Institution: KENNESAW STATE UNIVERSITY
Instructor: Jeni Hallman
Start of term: 08/17/2011
Course number:
Number of students: 30

Cartoon corner: humor-based mathematics activities: a collection adapted from 'Cartoon Corner' in mathematics teaching in the middle school

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Permission Status: ✔ Granted
Permission type: Use in electronic course materials
Type of use: Post on an academic institution internet
Per Page Fee: $ 0.15
Article/Chapter: A Square by Any Other Name
Date of issue: 2017
Page range(s): 50
Total number of pages: 1
Number of students: 30
($0.27 per student)

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Confirmation Number: 10413411
Order Date: 08/14/2011

Payment Information
Jeni Halimun
jenihalimun77@gmail.com
+1 (770)6097726
Payment Method: CC ending in 1013

Order Details

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MATHMATICS TEACHING IN THE MIDDLE SCHOOL

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| MATHEMATICS                                      |          |
| Author/Editor: NATIONAL COUNCIL OF TEACHERS OF  |          |
| MATHEMATICS                                      |          |
| Volume: 1                                      |          |
| Edition: 1                                      |          |
| Your line item reference: Rhymes-CartoonCorner |          |

Rightsholder terms apply (see terms and conditions)

| Permission Status: | Granted |
| Permission type:   | Use in electronic course materials |
| Type of use:       | Post on an academic institution intranet |
| Per Page Fee:      | $0.15 |
| Article/Chapter:   | Cartoon Corner |
| Date of issue:     | August 2009 |
| Page range(s):     | 10-11 |
| Total number of pages: | 2 |
| Number of students: | 30 |

$ 12.50
($0.42 per student)

Total order items: 1

Order Total: $ 12.50