ESA21
Environmental Science Activities for the 21st Century

Ballistic Pendulum

Introduction

The previous two activities in this module have shown us the importance of conservation laws. These laws provide extra “tools” that allow us to analyze certain aspects of physical systems and to be able to predict the motion of objects in the systems without using more complicated analysis. Even in situations wherein we cannot exactly solve the motion, these laws are incredibly useful. For instance, if someone shows us an incredibly complicated device that can seemingly produce electricity with no energy input whatsoever, we know not to invest money in this device, as it must be a sham since it violates the conservation of energy principle.

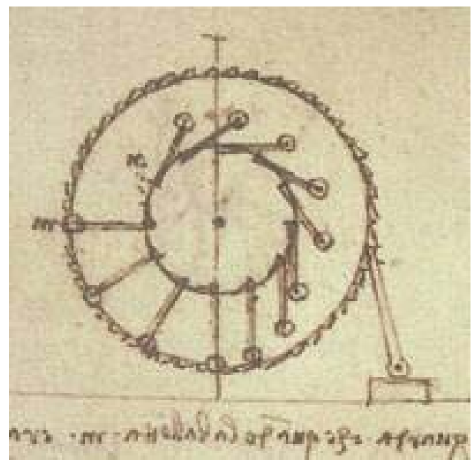

Fig 1: Davinci’s drawing showing the impossibility of perpetual motion

However, both of these conservation laws are theoretical constructs that rarely, if ever, hold 100% true in the real world. The experiments that were run showed proof of this, as the experimental results did not match the theoretical model. Energy is lost as it is transferred from kinetic to potential, and vice-versa. Momentum was not conserved during the collisions, as they turned out to be neither perfectly elastic or perfectly inelastic. It turns out that there is another law at work that limits these other conservation laws: the Second Law of Thermodynamics.

The Second Law of Thermodynamics and Efficiency

The First Law of Thermodynamics tells us that the energy involved in any transfer must be conserved. This would seem to mean that we should never run out of energy and should pay no heed to anybody talking about an energy crisis. The problem is that this is not the only law that governs energy transfers. While the total amount of energy does not change, the second law of thermodynamics puts limits on the amount of usable energy that can be transferred. One of the consequences of this law is that the total amount of usable energy that comes out of any process will be less than the total amount of energy that went into the process. The difference between the total amount of energy input and the usable energy output is expended as waste heat. Take, for example, a ball that is dropped from some height above the ground. As it falls, air acts upon the ball to slow it down. In doing so, some of the initial potential energy of the ball is converted to greater kinetic energy of the molecules of air, which makes them slightly warmer.

This brings us to the issue of efficiency, which is a measure of the amount of usable energy that is generated during any type of transfer. If a transfer is very efficient, then the amount of usable energy that is generated is almost equal to the total amount of energy that went into the transfer. This means that very little waste energy will be produced. An inefficient transfer, conversely, is one in which most of the energy going into the process is converted to waste heat. For example, a fluorescent light bulb converts about 20% of the electrical energy that runs through it into visible light energy. While this may not sound like a very efficient transfer, it is much better than the 5% efficiency of an incandescent light bulb, which most people use.

When discussing the efficiency of a process, we have to make sure and not forget all of the transfers that might need to take place in order to get to the one under investigation. A great example of this occurs when comparing the efficiencies of electric and internal combustion engine powered cars. The efficiency of the electric motor in a car is about 90%, while the efficiency of the internal combustion engine is only about 25%. However, these efficiencies are not the only things that need to be considered when comparing the two devices. How is the electricity that charges the car created? Where does the gasoline come from that powers the internal combustion engine? What types of transmission systems does each car have? There are many steps and energy transfers that take place in getting each type of car to move, and each one of these has its own individual efficiency. For instance, the average coal burning electric plant is only about 30-35% efficient in generating electricity (some newer natural gas plants are closer to 50-60%). This fact greatly reduces the overall efficiency of an electric car. When we consider the total efficiency, from getting the energy from its natural source to the car moving down the highway, we find that the electric car is only about 20% efficient, while the internal combustion engine automobile is about half that at 10%[1].

Ballistic Pendulum

Fig. 2: Ballistic pendulum

In lab this week, we are going to look at a series of energy conversions to see how efficiency works. Figure 2 shows a picture of the ballistic pendulum that we will use in this activity. The device is quite simple to operate. Pushing back the spring-loaded piston on the projectile section stores potential energy that can be used to propel a ball. Once the ball has been propelled out of the launcher by pulling the trigger, it collides inelastically with the pendulum, thus transferring momentum to it. The pendulum then swings upward until all of its kinetic energy is converted to potential energy. The angle measuring system on the side of the device stops it at this height, allowing for measurements of the amount of potential energy stored. This potential energy can then be compared to the initial kinetic energy to see how efficient the energy is converted from one form to another.

To find out what the theoretical efficiency of this process is, let us take the velocity of the ball leaving the gun piston to be v_{b1} the mass of the ball to be m_b, the mass of the pendulum to be m_p, and the height to which the ball-pendulum rises to be h_f. With this, the measured efficiency of the energy transfers will be

$$\text{Efficiency}_{\text{exp}} = \frac{PE_{\text{final}}}{KE_{\text{initial}}} = \frac{2(m_b + m_p)gh_f}{m_b v_{bi}^2} \tag{1}$$

To calculate what the theoretical efficiency is, we need to use the conservation of linear momentum. We know that v_{bi} is related to velocity of the ball and pendulum after the collision (v_p) by

$$v_{bi} = \frac{m_b + m_p}{m_b} \; v_p \tag{2}$$

If the pendulum does not lose energy while it is rising, then the kinetic energy at the bottom should equal the potential energy at the top, giving us the relationship

$$gh_f = \frac{1}{2} v_p^2 \tag{3}$$

If we substitute both Equation 2 and 3 into Equation 1, the theoretical efficiency becomes

$$\text{Efficiency}_{\text{theory}} = \frac{(m_b + m_p)v_p^2}{m_b v_{bi}^2} = \frac{m_b}{m_b + m_p}$$

This equation shows that as the mass of the pendulum goes to 0, then the efficiency of this energy transfer should go to 100%. This makes sense, as a massless pendulum would have no momentum to change during the collision. Furthermore, as the mass of the pendulum becomes large, the efficiency of the energy transfer would tend toward 0. Again, this makes sense, as an infinitely large pendulum would not move after the collision, thus absorbing all of the energy.

Reference

1 Energy: Its Use and the Environment, 2nd Edition by Roger A. Hinrichs, Saunders College Publishing, Orlando, 1996.

Activity

The activity this week relies on a commercially-available piece of equipment: a ballistic pendulum. There are several different manufacturers of these devices (Cenco and Pasco are two of the more popular models). All of them operate in basically the same manner, i.e. a spring loaded gun that shoots a ball into a cup attached to the end of a pendulum.

In order to test our model, we will first need to know the velocity of the ball leaving the end of the plunger (v_{bi}). One way to do this would be to shoot the ball off of the end of a level table and to measure how far the ball travels before it hits the ground. The velocity of the ball v_{bi} is related to the distance D that the ball travels before it hits the ground by the equation

$$v_{bi} = \sqrt{\frac{D^2 g}{2H}} ,$$

where g is the acceleration due to gravity and H is the initial height of the ball above the ground. The velocity can also be measured using photogates in the following manner.

1. Place the ballistic pendulum on the tabletop such that the shot ball will have an unobstructed path to the floor. Remove the pendulum portion of the equipment.
2. Place the photogate near the end of the plunger such that the released ball will sail through the photogate opening, with the middle of the ball passing between the LED light/detector.
3. Place the ball on the plunger and cock the mechanism for firing.
4. Start the software that controls the photogate.
5. Fire the gun, making sure that no one is in the path of the ball and that appropriate measure have been taken to stop the ball after it hits the ground.
6. Record the velocity of the ball
7. Repeat this procedure 4 times and average the velocities. Record these results on the activity sheet.

Once we have the velocity of the ball, we are ready to proceed to the second half of the experiment. Remove the photogate and turn off the software before proceeding.

1. Measure the mass (remove the two brass masses from the bottom of the pendulum before doing this) and the length of the pendulum. Return the pendulum to the device such that the ball will be captured by the ball when it leaves the end of the plunger.
2. Measure the mass of the ball. Load the ball into the launcher.

3. Making sure that the path is clear, fire the ball into the pendulum. Measure the angle to which the pendulum swings. Use this angle to determine the height to which the pendulum and the ball went ($H = L (1 – \cos \theta)$, where L is the length of the pendulum). Record this data on the activity sheet.
4. Repeat this procedure 4 more times.
5. Measure the mass of the two brass masses and record their sum on the activity sheet. Attach them to the bottom of the pendulum.
6. Repeat steps 3-4.
7. Calculate the efficiency of the energy transfer and compare this to the theoretical value.

Name:

Mass of the ball = m_b = ______ gm
Mass of the pendulum = m_p = ______ gm

Velocity of ball

Velocity	Run 1	Run 2	Run 3	Run 4	Run 5	Avg. v_{bi}
v_{bi}						

Angle

Angle	Run 1	Run 2	Run 3	Run 4	Run 5
θ_f (no mass)					
θ_f (added mass)					

Height

Angle	Run 1	Run 2	Run 3	Run 4	Run 5	Avg. h_f
h_f (no mass)						
h_f (added mass)						

No Added Mass   Added Mass

$$\text{Efficiency}_{\text{exp}} = \frac{2(m_b + m_p)gh_f}{m_b v_{bi}^2} =$$ ______________ ______________

$$\text{Efficiency}_{\text{theory}} = \frac{m_b}{m_b + m_p} =$$ ______________ ______________

Percent difference = ______________ ______________

1. How close is the model efficiency to the actual efficiency?

2. What are some possible characteristics of the experimental apparatus that are not accounted for in the model? Could these characteristics account for the differences between measured and theoretical accelerations?