

Energy

Introduction

In last week's activity, we calculated the motion of objects by studying the forces on them. Using Newton's Second Law of Motion, we were able to relate the forces on an object to the acceleration it experienced. The presence of constant forces led to constant accelerations, which corresponded to linearly-increasing velocities with respect to time. Comparisons made between the velocities we measured and those calculated from a study of the forces on the objects allowed us to validate Newton's equation F = ma.

In this week's activity, we are going to investigate almost the same experimental setup as last week, but from a different perspective. This time, we will relate the position and velocities of objects to their energies and not to the forces on them. In the absence of external forces to the system, no net work will be done on the objects, which means that the total energy of the system will remain a constant. Using this fact, we will once again find a mathematical relationship between the velocity and time, which will allow for comparisons between theory and experiment.

Two Types of Energy

All forms of energy can be classified into one of two categories: kinetic energy or potential energy. Kinetic energy is the energy that an object has because of its motion relative to its surroundings. It has the ability to do work on other objects by applying a force to those objects in order to change its velocity. For example, a cue ball that is rolling across the table has kinetic energy relative to the other billiard balls on the table that are static. If the cue ball hits another ball, it applies a force to the second ball over a very small distance. This force changes the cue ball's velocity while it transfers energy to the static ball, and thus, it does work.

The other category of energy is potential energy. This is the energy that is stored in a system by virtue of forces between objects that are separated by some distance. If the objects are allowed to move under the influence of the force between them, then work is done as the force displaces the objects from their initial positions, and energy is transferred. A prime example of this is gravitational potential energy. A ball that is dropped from rest at some height above the Earth's surface will begin to accelerate downward as gravity pulls it. As the ball accelerates, gravity is working on it to convert its potential energy into kinetic energy (the ball's height decreases as its velocity increases). If the ball is dropped from a higher altitude, then gravity operates over a greater distance, thereby generating more kinetic energy, i.e. the gravitational potential



Fig 1: Parachutist jumping from an MV-22 Osprey (*DOD*)

energy of an object depends upon the height through which an object is allowed to fall.

Kinetic Energy

From class, we know that the energy of an object is equal to its ability to do work. If an object is moving, i.e. has kinetic energy, this work is done when the object applies a force to its surroundings. By Newton's Laws of Motion, we know that this force will 1) be equal in magnitude to the force on the object and 2) will cause the object to accelerate. Let us consider such an object of mass \mathbf{m} that is being accelerated under the influence of a constant force \mathbf{F} . The work that is being done on the object is given by the formula

$$W = F \Delta x = (ma) \Delta x = m (a \Delta x)$$

Since the force is constant, we know that

$$a\Delta x = \frac{v^2 - v_o^2}{2}$$

Thus, we can re-write the work done on the object as

$$W = \frac{1}{2} m (v^2 - v_0^2) = \frac{1}{2} m v_0^2 - \frac{1}{2} m v_0^2$$

This equation shows that the work done on an object, which is a measure of the kinetic energy of the object, is equivalent to a change in the quantity $\frac{1}{2} \text{ mv}^2$. Thus, we define the kinetic energy of an object as K.E. = $\frac{1}{2} \text{ mv}^2$, and the equation above becomes

$$W = \Delta K.E$$

Gravitational Potential Energy

One of the more important forms of potential energy in our lives is gravitational potential energy. This is the energy that comes about because of gravitational forces between objects, such as yourself and the Earth. Newton's Universal Law of Gravity tells us that the gravitational force between two objects depends upon the inverse of the square of the distance between the objects and is, therefore, not a constant force with displacement. However, if we limit the movements of the objects to small values compared to the total distance between them, then we can consider the force of gravity to be a constant. Objects falling near the surface of the Earth fall into this category. The distance between the center of the Earth and the center of any object near the surface is over 4,000 miles. If we limit the object to fall a distance of less than a mile, then the change in the force of gravity over that distance is less than .05%. Thus, we can take it to be a constant. In this situation, Newton's Universal Law of Gravity reduces to the expression:

F_{gravity} = mass x acceleration due to gravity near Earth = mass x 9.80 m/sec²

Since the force is a constant, the potential energy is merely this force multiplied by the distance through which the object falls. If we allow H to be the height through the object will fall, then the potential energy of the object is:

P.E. = $F_{\text{gravity}} \times H$ = mass x 9.80 m/sec² x H

An example of a device that takes advantage of gravitational potential energy is a hydroelectric dam (Figure 2). This device uses water the potential energy of water "stacked up" behind a dam to move a turbine that is located at some height below the water. The turbine is connected to a generator, which creates electricity as it turns. This form of electricity is very important in today's society, especially in certain regions of the world, such as the U.S. Pacific Northwest (see Additional Reading below). However, it does have a very serious environmental impact, as the



Figure 2: Diagram of a hydroelectric facility that uses gravitational potential energy

placement of the dam converts a river ecosystem to a lake ecosystem. This can have a large effect on the movement of sediments that are necessary for sustained agriculture, as well as aquatic life forms such salmon and trout which need to be able to move freely along the length of the river.

Energy Transfer

In the absence of outside forces, the total amount of potential and kinetic energy in a system will remain the same. This is due to the First Law of Thermodynamics, which states that "Energy can neither be created nor destroyed; it can only be transferred from one form to another." In mathematical terms, this can be written as

$$\Delta E = W + Q$$

where ΔE is the change in the energy of an object, W is the work done on the object, and Q is the heat added to the object. In laymen's terms, this means that the only way to change the energy of an object is to either do work on it or add heat to it (it should be noted that having the object do work on its surroundings, or allowing the object to give off heat, are equivalent to having negative values for W or Q).

Therefore, if there is no work done on the system or any heat added to it, then $\Delta E = 0$. This means that any loss in kinetic energy must create an equal increase in potential energy, and vice versa. For instance, a ball that is initially still at some height above the floor will increase its velocity, and hence its kinetic energy, as it falls to lower heights, which decreases its gravitational potential energy. Given what we have previously discussed, mathematically, this means

$$mgy = \frac{1}{2} mv^2$$

or

$$v = \sqrt{2gy}$$

where y is the amount of distance the ball has dropped and v is the velocity of the ball.

Model

This week's activity will test the First Law of Thermodynamics using a suspended mass that is attached via a thin string to a "frictionless" cart that is free to move along a tilted track (see Figure 3). If the suspended mass, the cart's mass, and the slope of the track are appropriately chosen, the mass will be pulled downward by gravity with a constant net force. This, in turn, will pull the cart up the track with



Fig. 3: Picture of experiment setup

a constant acceleration. In the initial configuration, the suspended mass has potential relative to the floor, while the cart has no potential energy relative to the low point on the track. Since neither is moving, there is no kinetic energy. Thus, the system has a total energy initially given by

$$E_i = m_s g H_s$$

where m_s is the suspended mass and H_s is the height that the mass is initially above the stopping point. When the suspended mass has gotten all the way to the bottom ($H_s = 0$ m), it has no potential energy. However, the cart now has potential energy due to its elevated position relative to the low end of the track. The mass and the cart also have kinetic energy, as both accelerated under the influence of gravity. The final energy is, therefore, given by

$$E_f = m_c g H_c + \frac{1}{2} (m_s + m_c) v_f^2$$

where m_c is the mass of the cart, H_c is the height of the cart above its initial starting height, and v_f is the final velocity of the cart and suspended mass. By setting initial and final energies equal to one another, we can solve for the final velocity as

$$v_f = \sqrt{\frac{2(m_s H_s - m_c H_c)g}{m_s + m_c}}$$

This theoretical velocity can be compared to the measured velocity of the activity below to see how well energy is transferred.

Additional Reading

The links below provide more information about hydroelectric dams and the agencies that oversee them.





Topic: Bureau of Reclamation – Reports and Data **Summary**: Information about the generation and capacity of hydroelectric facilities under the Bureau's control. **Link**: http://www.usbr.gov/power/data/faclcap.html

Activity

Figure 3 shows a potential set up for this activity. Just as with last week's activity, the method of measuring velocities and the type of system used will depend on what is available at your institution. The dynamics cart is initially leveled on the table before blocks are placed under the feet on the end of the track closest to the edge of the table. The pulley needs to be raised such that the string that connects the suspended mass to the cart is level to the track. If the block is sufficiently short in height, then the 50 gram mass holder on the end of the string should still be able to accelerate the cart up the track. With this setup, do the following steps:

- 1. Measure the mass of the cart.
- 2. Connect one end of the string to the cart and the other to the hanging mass (start with 50 grams). Place the cart on the track and run the string over the pulley. Pull the cart as far back along the track so that the mass hanger almost touches the bottom of the pulley. Hold the hanger in place above the ground to prevent the system from moving. Measure the location of the cart on the track (using scale on track if it exists) and the height of the track at the front of the cart.

- Move the cart up the track until the hanging mass is 10-20 cm above the ground. Measure the location of the cart on the track and the height of the track at the front of the cart. Calculate the height through which the hanging mass falss and increased height of the cart from beginning to end.
- 4. Pull the cart back down to its starting (lowest) point.
- 5. Initiate the computer software that will be used to measure the cart's velocity.
- 6. Start recording data and release the cart so it can be pulled by the falling mass hanger.
- 7. Stop the data recording when the cart has passed the high point previously marked Have someone in the group stop the cart before the mass hits the ground.
- 8. Record the velocity of the cart as it passes the highest point on the activity sheet.
- 9. Repeat this process 4 more times and average the measured velocities.
- 10. Repeat the data acquisition procedures above for total masses in 50 gram increments up to 250 grams.
- 11. After this, add the bar weight to the cart. Repeat the above procedure.
- 12. Finish the activity sheet by answering the questions.

ESA21: Environmental Science Activities

Name:

Mass of the Cart = m_c = _____ gm Height through which the hanger falls = H_s = _____ cm Height change of cart = H_c = _____ cm Mass of bar weight = _____ gm

Cart Only

| Hanging | | F | inal Velocit | Avg v _f | Theor. v _f | % Diff. | | |
|---------|-------|-------|--------------|--------------------|-----------------------|---------|--|--|
| Mass | Run 1 | Run 2 | Run 3 | Run 4 | Run 5 | | | |
| 50 gm | | | | | | | | |
| 100 gm | | | | | | | | |
| 150 gm | | | | | | | | |
| 200 gm | | | | | | | | |
| 250 gm | | | | | | | | |

Cart + Bar Weight

| Hanging | | F | inal Velocit | Avg v _f | Theor. v _f | % Diff. | | |
|---------|-------|-------|--------------|--------------------|-----------------------|---------|--|--|
| Mass | Run 1 | Run 2 | Run 3 | Run 4 | Run 5 | | | |
| 50 gm | | | | | | | | |
| 100 gm | | | | | | | | |
| 150 gm | | | | | | | | |
| 200 gm | | | | | | | | |
| 250 gm | | | | | | | | |

1. What are the possible sources of random errors in this experiment? How have you attempted to account for them?

2. What are some possible characteristics of the experimental apparatus that are not accounted for in the model? Could these characteristics account for the differences between measured and theoretical accelerations?